

1 **Multiple slip effects on dissipative and chemical reactive MHD flow over a**
2 **permeable stretching sheet in the presence of heat source**

3 Bharat Keshari Swain

4 Agarpara College, Agarpara, Bhadrak, Odisha, 756115, India

5 bharatkeshari1@gmail.com

6

7 **Abstract:** The present investigation construes the unsteady MHD flow over a permeable
8 stretching sheet embedded in a porous medium. The effects of viscous and Darcy dissipation,
9 heat source and chemical reaction have been delineated with the existence of multiple slips.
10 Similarity variables are employed for remodeling the governing equations as a system of first
11 order nonlinear ordinary differential equations. The reduced system is dealt with 4th order
12 Runge-Kutta method along with shooting technique. The present results have been verified with
13 earlier works and found a good agreement. The important findings reported herein are: porosity
14 acts as aiding force for the fluid velocity, more dissipative heat leads to higher velocity and
15 temperature, chemical reaction parameter adversely affects the concentration. This analysis is
16 relevant to various industrial processes at boundaries of pipes, walls, and/or curved surfaces.

17 **Keywords:** Viscous dissipation, Darcy dissipation, Heat source, Chemical reaction, multiple
18 slips.

19 **Nomenclature:**

20 a Stretching rate

D_T Thermal diffusivity

21	B_0	Magnetic field of constant strength	E_c	Eckert number
22	C	Concentration of the fluid	f_w	Suction/injection
23	C_f	Local skin friction coefficient	k^*	Mean absorption coefficient
24	C_{fr}	Reduced skin friction	K_p	Porosity parameter
25	C_p	Specific heat	M	Magnetic field parameter
26	C_o	Reference concentration	Nur	Reduced Nusselt Number
27	C_w	Stretching sheet concentration	P_r	Prandtl number
28	C_∞	Ambient concentration	Q	Heat source parameter
29	D_d	Darcy dissipation parameter	R_c	Chemical reaction parameter
30	D_M	Molecular diffusivity	Re_x	Local Reynolds number
31	S_c	Schmidt number	ϕ	Nondimensional concentration
32	S_f	Velocity slip	η	Similarity variable
33	Sh_r	Reduced Sherwood number	ν	Kinematic viscosity
34	S_r	Soret number	α	Thermal diffusivity
35	S_θ	Thermal slip	β_c	Concentration expansion coefficient
36	S_ϕ	Solutal slip	β_T	Thermal expansion coefficient
37	T	Temperature	τ	Stress tensor

38	T_w	Stretching sheet temperature	δ	Unsteady parameter
39	T_0	Reference temperature	λ	Constant
40	T_∞	Ambient temperature	ρ	Density
41	u	x component of velocity	λ_1	Thermal buoyancy parameter
42	v	y component of velocity	λ_2	Solutal buoyancy parameter
43	x,y	Coordinates	σ^*	Stefan-Boltzmann constant

44 **Greek Symbol:**

45 θ Non-dimensional temperature

46 **1. Introduction:**

47 Magnetohydrodynamics (MHD) may be described as the microscopic interaction of
48 electrically conducting fluid and magnetic field. Its application ranges from industries to space
49 science as well as magnetic-therapy. In recent years, MHD flow with the slip condition is of
50 engineering interest. If the viscous force between liquid molecules at the interface is stronger
51 than the force between the molecules of the liquid and molecules of the solid, then the molecules
52 can slide on the surface. The resemblance of the mean free path of particles with the flow field
53 characteristic gives rise to the breakdown of Navier-Stokes equations because the hypothesis of
54 continuum media fails. If the Knudsen number (Kn) remains in $1 < Kn < 10$, the higher order
55 continuum equation like Burnett equation is appropriate. No-slip condition can't be used (Hak,
56 1999) in the range of $0.1 > Kn > 0.001$. For $Kn < 0.001$, the no-slip condition is authentic.

57 Particularly, in case of emulsions, suspensions etc., no-slip condition is insufficient and slip
58 could occur on the boundary.

59 The fluid flow nature under slip condition deviates from the standard flow. The slip flows
60 under various configurations were inspected by several researchers. Raza, Rohni, Omar and
61 Awais (2016) illuminated the performance of slip in a nanofluid flow during rotating channel.
62 Turkyilmazoglu (2013) did the mass transfer analysis in a slip flow of fluid. Akbar and Khan
63 (2014) have studied the impact of thermal as well as velocity slips on biviscosity fluid flow.
64 Hayat, Awais and Hendi (2012) and Reddy (2016) elucidated the impact of slip condition on
65 rotating flow between two porous walls and on MHD peristaltic flow respectively. Mabood &
66 Shateyi (2019) studied radiative MHD unsteady flow with multiple slips. Swain, Biswal and
67 Dash (2021) have investigated the second order slip effect and heat source on dissipative MHD
68 flow of blood through a permeable capillary in stretching motion. They found that skin friction
69 coefficient at the capillary wall decreases with higher order slip.

70 Application of external magnetic field in various MHD flow problems is considerably
71 vital. When this flow is considered through a porous medium, then study becomes more alluring.
72 These forms of engineering issues are more admissible in energy extractions, oil exploration and
73 also the physical phenomenon management within the field of aeromechanics. So several
74 investigations are explored by the celebrated researchers. Sekhar, Reddy, Raju, Ibrahim and
75 Makinde (2018) illustrated the multiple slips impacts on MHD flow through porous medium.
76 Makinde, Khan, Ahmad and Khan (2018) studied unsteady hydromagnetic radiating fluid flow
77 past a slippery stretching sheet embedded in a porous medium. Makinde, Khan and Khan (2016)
78 analyzed the MHD nanofluid flow over a convectively heated permeable vertical plate embedded
79 in a porous medium. Mutuku-Njane and Makinde (2013) illuminated the ramification of MHD

80 flow of a nanofluid over a convectively heated vertical porous plate through porous medium.
81 Swain and Senapati (2015) inspected the effect of mass transfer on free convective flow set in a
82 porous medium in the presence of magnetic field. Uddin (2015) has also made his analysis
83 considering magnetic field and a porous medium.

84 The internal energy in the fluid element is modified by the flow of heat and also
85 the performance of work. The work done consists of the reversible half and dissipated work. we
86 tend to solely take into account the contribution from the mechanically dissipated energy per unit
87 volume (SI unit W/m³), effort aside the contribution of the entropy flux and also the production
88 of entropy stems from the finite temperature distinction in thermal conductivity (Baehr and
89 Stephan (2006)). Anjali and Ganga (2010) investigated the outcome of dissipation for a nonlinear
90 flow with prescribed heat flux. Ferdows, Chapal and Afify (2014) expounded the nanofluid flow
91 considering permeable stretching sheet and clearly discussed the result of viscous dissipation.
92 Venkateswarlu, Satya Narayana and Tarakaramu (2018) investigated the viscous dissipation
93 response on MHD flow over a moving surface with constant heat supply. Hunegnaw and Kishan
94 (2017) elucidated on the repercussion of viscous dissipation in an unsteady MHD flow over
95 stretching sheet. Biswal, Swain, Das and Dash (2022) studied the effect of viscous dissipation in
96 their study of MHD stagnation point flow towards an inclined stretching sheet embedded in a
97 porous medium. Swain, Parida, Kar and Senapati (2020) investigated viscous dissipation and
98 joule heating effect on MHD flow and heat transfer past a stretching sheet embedded in a porous
99 medium. Parida, Swain and Senapati (2021) analyzed viscous dissipative MHD nanofluid flow
100 over a stretching sheet embedded in a porous medium.

101 Chemical reaction is additionally of extended importance in fluid flow at the side of the
102 combined impact of heat and mass transfer. These kinds of studies have large utilizations

103 in several industrial processes and conjointly in physiological flows. Reaction is called to
104 be homogeneous once it happens uniformly through a given part. Otherwise reaction is termed as
105 heterogeneous. Further, chemical reaction is of first order if rate of reaction is directly
106 proportional to the concentration. Possible applications may be found in processes like drying,
107 injury of crops because of state change, water surface evaporation etc. Several researchers have
108 done their research analysis regarding this subject. Chamber and Young (1958) delineated the
109 consequences of homogeneous first order chemical reactions within the neighborhood of a flat
110 plate. Muthucumaraswamy and Janakirama (2008) and Swain, Senapati and Dash (2014) have
111 studied the impact of chemical change in MHD physical boundary layer flow past a vertical
112 plate. Ahmed (2014) and Ahmed and Kalita (2014) resolved numerically the MHD
113 flow issues taking care of chemical reaction. Swain, Senapati and Dash (2017) have conjointly
114 given their interest on resolution the matter of chemical reaction effect on free convective Flow.
115 Swain (2021) has also shown his interest on solving the problem of second order chemical
116 reaction effect on MHD convective flow. He explained the consequences of the second order
117 chemical reaction after solving the pertinent equations by finite difference method.

118 The above studies and applications show the importance of dissipative and chemical
119 reactive MHD flow in the presence of multiple slips. Fluid flow through porous medium and
120 Darcy dissipation, which stems from the resistance offered by viscosity of the fluid and porous
121 matrix of the saturated porous medium are analyzed here. This type of flow is very applicable in
122 irrigation and drainage problems, extraction of particulates from soil, cosmetic and
123 petrochemicals products etc. Further, heat source due to temperature difference which provides
124 thermal power to heat transfer process may be of possible occurrence. The above aspects are not

125 considered by Mabood and Shateyi (2019) and Mabood and Das (2016). This research gap
126 motivates the present study.

127 The novelties of the present study are:

- 128 (i) to study the effects of viscous and Darcy dissipation on MHD flow through
129 porous medium.
- 130 (ii) to analyze the influence of heat source and chemical reaction.
- 131 (iii) to analyze the flow with multiple slip conditions.
- 132 (iv) to discuss the works of Mabood & Shateyi (2019), Mabood & Das (2016) ,
133 Chamkha, Aly and Mansour (2010) and Ali (1994) as particular cases.

134 2. Mathematical Formulation:

135 Present analysis deals with a permeable surface along which the x -axis is taken. The y -
136 axis is drawn normal to x -axis. The fluid flows over a permeable stretching sheet embedded in a
137 porous medium. The sheet is stretched with velocity $U_w(x,t) = \frac{ax}{1-\lambda t}$ in the direction of x -axis.

138 Magnetic field $B(t) = B_0(1-\lambda t)^{-1/2}$ is applied in transverse direction and B_0 is constant initial
139 magnetic field strength. Figure.1 depicts the present flow system. The induced magnetic field is
140 negligible. T_∞ and C_∞ are taken as the ambient temperature and mass concentration respectively.
141 Since the wall is permeable, so multiple slips are analyzed.

142 The following points are assumed in the present analysis.

- 143 (i) The two dimensional viscous flow is unsteady, incompressible, free convective and
144 electrically conducting.
- 145 (ii) The stretching permeable sheet is subjected to transverse magnetic field and
146 embedded in porous medium.
- 147 (iii) There exist velocity slip, thermal slip as well as solutal slip.
- 148 (iv) The induced magnetic field is neglected.
- 149 (v) It is also assumed that no polarization voltage exists.

150 (vi) The diffusive species is chemically reactive with variable concentration at the surface.

151 The governing equations are conferred as (cf. Mabood & Shateyi (2019), Misra & Sinha (2013))

$$152 \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad , \quad (1)$$

$$153 \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B^2(t)}{\rho} + \frac{\nu}{K_p^*(t)} \right) u + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \quad , \quad (2)$$

$$154 \quad \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(1 + \frac{16T_\infty^3 \sigma^*}{3k^*k} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\mu}{\rho C_p K_p^*(t)} u^2 + \frac{Q^*}{\rho C_p} (T - T_\infty), \quad (3)$$

$$155 \quad \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_M \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} - R_c^*(C - C_\infty) \quad . \quad (4)$$

156 The boundary conditions are given by

$$157 \quad \begin{cases} y = 0 : u = U_w(x, t) + U_{slip}, v = V_w, \\ T_w(x, t) + T_{slip}, C_w(x, t) + C_{slip} \\ y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \end{cases} \quad . \quad (5)$$

158 Sheet temperature $T_w(x, t)$ and concentration $C_w(x, t)$ at the surface are taken as

$$159 \quad T_w(x, t) = T_\infty + T_0 \left(\frac{ax}{2\nu} \right) (1 - \lambda t)^{-2}, C_w(x, t) = C_\infty + C_0 \left(\frac{ax}{2\nu} \right) (1 - \lambda t)^{-2} \quad (6)$$

160 where T_0 , the reference temperature and C_0 , the reference concentration and $0 \leq T_0 \leq T_w$,

161 $0 \leq C_0 \leq C_w$. The above expressions are valid for $(1 - \lambda t) > 0$.

162 In equations (2) and (3), $K_p^*(t) = k_1(1 - \lambda t)$ is the time dependent permeability parameter.

163 In (5), V_w represents injection/suction velocity of fluid at the surface and is given by

$$164 \quad V_w = -\sqrt{\frac{av}{1 - \lambda t}} f(0).$$

165 The stream function ψ is expressed as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ that satisfy equation (1). The non-

166 dimensional functions and variable f, θ, ϕ and η are taken as follows:

$$\eta = \sqrt{\frac{a}{\nu(1-\lambda t)}}y, \psi = \sqrt{\frac{a\nu}{(1-\lambda t)}}xf(\eta), T = T_\infty + T_0\left(\frac{ax(1-\lambda t)^{-2}}{\nu}\right)\theta(\eta),$$

167 . (7)

$$C = C_\infty + C_0\left(\frac{ax(1-\lambda t)^{-2}}{\nu}\right)\phi(\eta)$$

168 Now putting (7) in equations (2)-(4), following equations are obtained

$$169 \quad f''' + ff'' - f'^2 - \delta\left(\frac{\eta}{2}f'' + f'\right) - \left(M + \frac{1}{K_p}\right)f' + \lambda_1\theta + \lambda_2\phi = 0 \quad , \quad (8)$$

$$170 \quad \frac{1}{P_r}(1+R)\theta'' + f\theta' - f'\theta - \delta\left(\frac{\eta}{2}\theta' + 2\theta\right) + E_c f''^2 + D_d f'^2 + Q\theta = 0 \quad , \quad (9)$$

$$171 \quad \phi'' + S_c(f\phi' - f'\phi) - S_c\delta\left(\frac{\eta}{2}\phi' + 2\phi\right) + S_c S_r \theta'' - S_c R_c \phi = 0 \quad . \quad (10)$$

172 The reduced boundary conditions are

$$173 \quad f(0) = f_w, f'(0) = 1 + S_f f''(0), \theta(0) = 1 + S_\theta \theta'(0), \phi(0) = 1 + S_\phi \phi'(0), \\ f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0. \quad (11)$$

174 The non-dimensional parameters used in (8)-(11) are given by

$$175 \quad \delta = \frac{\lambda}{a}, \lambda_1 = \frac{g\beta_T T_0}{a\nu}, \lambda_2 = \frac{g\beta_C C_0}{a\nu}, P_r = \frac{\nu}{\alpha}, R = \frac{16T_\infty^3 \sigma^*}{3k^* k}, M = \frac{\sigma B_0^2}{\rho a}, K_p = \frac{ak_1}{\nu}, S_c = \frac{\nu}{D_M} \\ S_r = \frac{D_T T_0}{\nu C_0}, f_w = -V_w \sqrt{\frac{1-\lambda t}{\nu a}}, D_d = \frac{E_c}{K_p}, E_c = \frac{U_w^2}{C_p (T_w - T_\infty)}, Q = \frac{Q^* (1-\lambda t)}{\rho C_p a}, R_c = \frac{R_c^* (1-\lambda t)}{a}$$

$$S_f = A_1 \sqrt{\frac{a}{\nu(1-\lambda t)}}, S_\theta = A_2 \sqrt{\frac{a}{\nu(1-\lambda t)}}, S_\phi = A_3 \sqrt{\frac{a}{\nu(1-\lambda t)}}.$$

A_1, A_2, A_3 are velocity slip factor, thermal slip factor and solutal slip factor respectively.

$A_1 = A_2 = A_3 = 0$, gives no slip conditions.

Here, $f_w = 0$ represents the impermeable surface, $f_w > 0$ produce suction and $f_w < 0$ means injection.

The physical quantities local skin friction coefficient C_f , local Nusselt number Nu and local Sherwood number Sh are defined as

$$C_f = \frac{\mu}{\rho U_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (12)$$

$$Nu = -\frac{x}{k(T_w - T_\infty)} \left[k \left(\frac{\partial T}{\partial y} \right)_{y=0} - \frac{4\sigma^*}{3k^*} \left(\frac{\partial T^4}{\partial y} \right)_{y=0} \right], \quad (13)$$

$$Sh = -\frac{x}{(C_w - C_\infty)} \left(\frac{\partial C}{\partial y} \right)_{y=0}. \quad (14)$$

Putting (7) into equations (12)-(14), the non-dimensional forms obtained are:

$$Cfr = \sqrt{\text{Re}_x} C_f = f''(0), Nur = \frac{Nu}{\sqrt{\text{Re}_x}} = -(1+R)\theta'(0), Shr = \frac{Sh}{\sqrt{\text{Re}_x}} = -\phi'(0), \quad (15)$$

3. Method of Solution:

The coupled nonlinear equations (8) - (10) under the boundary conditions (11) have been worked out using the fourth order Runge-Kutta method with shooting technique. The equations (8)-(10)

191 have been converted to a set of first order differential equation substituting

192 $f = y_1, f' = y_2, f'' = y_3, \theta = y_4, \theta' = y_5, \phi = y_6, \phi' = y_7.$ (16)

193 The reduced equations are

194 $y_3' = -y_1 y_3 + y_2^2 + \delta \left(y_2 + \frac{1}{2} \eta y_3 \right) + \left(M + \frac{1}{K_p} \right) y_2 - \lambda_1 y_4 - \lambda_2 y_6,$ (17)

195 $y_5' = \left(\frac{\text{Pr}}{1+R} \right) \left(-y_1 y_5 + y_4 y_2 + \delta \left(2y_4 + \frac{1}{2} \eta y_5 \right) - E_c y_3^2 - D_d y_2^2 - Q y_4 \right),$ (18)

196 $y_7' = -S_c (y_1 y_7 - y_2 y_6) + S_c \delta \left(2y_6 + \frac{\eta}{2} y_7 \right) - S_c S_r y_5' + S_c R_c y_6$ (19)

197 Boundary conditions are written as

198 $y_1 = f_w, y_2 = 1 + S_f y_3, y_3 = ?, y_4 = 1 + S_\theta y_5, y_5 = ?, y_6 = 1 + S_\phi y_7, y_7 = ?$ at $\eta = 0,$ (20)

199 $y_2 \rightarrow 0, y_4 \rightarrow 0, y_6 \rightarrow 0$ as $\eta \rightarrow \infty.$ (21)

200 For integration, the values of y_3, y_5 and y_7 at $\eta = 0$ are guessed and the step by step integration

201 is accomplished with step length 0.01 adopting shooting technique with MATLAB code having

202 error bound 10^{-3} .

203 **4. Validation:** To validate the present results, the previous studies are taken into account in

204 Tables.1, 2 and 3. Table.1 and Table.2 give the values of skinfriction of previous studies

205 (Chamkha, Aly & Mansour (2010), Mabood and Shateyi (2019), Mabood and Das (2016)) and

206 present results for unsteady parameter and magnetic field parameter respectively. Similarly,

207 some values of Nusselt numer of the present study for Prandtl number are compared to that of

208 the previous studies (Mabood and Shateyi (2019), Ali (1994)) in Table.3. It is found that the
209 present results are in good agreement with the earlier studies, which shows the accuracy of the
210 present study.

211 **5. Results and Discussion:** Numerical solutions to an unsteady MHD flow over permeable
212 stretching sheet with multiple slip effects embedded in a porous medium are obtained. The
213 effects of viscous and Darcy dissipation, heat source and chemical reaction are of main concern.
214 The values of pertinent parameters are considered following Mabood and Shateyi (2019) and
215 Mabood and Das (2016). The repercussion of physical parameters are elucidated through graphs
216 and tables assuming the parameters as

$$217 \quad P_r = 1, S_c = 10, \lambda_1 = \lambda_2 = 0.2, M = 1, S_\theta = S_\phi = S_r = R = 0.5, \delta = 0.2, f_w = 1, S_f = 1, E_c = 0.2, \\ Q = 0.5, K_p = 1, R_c = 0.5$$

218 unless or otherwise stated.

219 Figure.2 illustrates the effect of heat source parameter Q on velocity. It is obtained that the
220 increasing values of Q enhance the velocity. Again it displays the effect of no-slip and first order
221 velocity slip on velocity distribution. It is observed that the decrease in velocity commensurates
222 with the order of slip. This can be attributed to low momentum transport/diffusion within the
223 boundary layer.

224 In Fig.3, porosity parameter K_p acts as aiding force for the fluid velocity i.e. the increasing
225 values of K_p enhance the velocity. This is because with a rise in permeability of the medium, the
226 regime becomes more porous. As a consequence, the Darcian body force decreases the
227 magnitude (as it is increasing proportional to the permeability). The Darcian resistance acts to
228 decelerate the fluid particles in continuous. This resistance diminishes as permeability of the
229 medium increases. So progressively less drag is experienced by the flow and flow retardation is

230 there by decreased. Hence, the velocity of the fluid increases as the permeability parameter
231 increases. On careful observation, again it is found that when K_p is increased from 0.1 to 1, rate
232 of change in velocity is higher and after that rate of increment gets slowdown in the range of $K_p=$
233 1 to10. Again, it is found that velocity slip leads a lower velocity.

234 Figure.4 elucidates the impacts of Eckert number (E_c) on velocity distribution respectively. The
235 parameter is directly proportional to the velocity. As Eckert number comes from kinetic energy
236 of flow and heat enthalpy difference, so improve in Eckert number enhances kinetic energy.
237 Again it is known that temperature is considered as average kinetic energy. Greater viscous
238 dissipative heat rises the temperature. Thus we can say that temperature of the fluid rises. From
239 the Figure.6, it is analyzed that fluid temperature rises as E_c is increased. Due to higher
240 temperature, resistive power to the fluid flow reduces and fluid moves with higher velocity (in
241 fig. 6) for higher values of E_c .

242 In Figure.5, Temperature profiles are drawn for various values of porosity parameter K_p . Increase
243 in both the parameters helps to fall in temperature significantly with and without thermal slip. In
244 case of no thermal slip ($S_\theta =0$), temperature is comparatively more.

245 As heat supply is increased, temperature rises across the flow field Figure.7. Consequently, it
246 yields a thicker thermal boundary layer. It is pertinent to notice that Q measures internal thermal
247 power per unit volume of the fluid. An increase in heat source means an increase in thermal
248 power. It also gives rise to lower temperature for first order thermal slip as compared to no-slip
249 condition. Moreover, with increase in temperature the fluid gets thin and becomes less viscous.
250 Therefore the fluid flows with higher velocity.

251 Figure.8 delineate the impact Chemical reaction parameter (R_c) on concentration profile
252 respectively. It is observed that larger values of Chemical reaction parameter lead to lower

253 concentration. This is due to the fact that destructive chemical reduces the solutal boundary layer
254 thickness and increases the mass transfer. In physical point of view chemical reaction for
255 destructive case is very large. Because of this fact molecular motion is quite higher which
256 enhances the transport phenomenon, thus suppressing the concentration field in the fluid flow.
257 This figure also confirms that decrease in concentration commensurate with the order of solutal
258 slip.

259 Table-4 represents the effects of λ_1 , λ_2 , K_p , E_c , Q , R_c and S_c on skin friction, Nusselt number
260 and Sherwood number. The other parameters are taken as

261 $P_r = 1, S_f = 0.2, \delta = M = 0.5, R = S_\theta = S_\phi = S_r = 0.5, f_w = 0.2$. It is observed that increasing
262 values of λ_1 , λ_2 , and K_p simultaneously increase skin friction coefficient and improve the rate of
263 heat and mass transfer. This results show a good agreement with the results of Mabood and
264 Shateyi(2019). Further, it is noticed that higher values of E_c and Q help to increase the skin
265 friction but reverse trend is shown in case of R_c and S_c .

266 Now higher values of E_c are obtained small temperature difference and so rate of heat transfer
267 decreases. Similarly more the heat source parameter, Q less the rate of heat transfer. Likewise,
268 increase in R_c and S_c leads to the heat transfer rate. Finally rate of mass transfer is analyzed. It is
269 noted that all the parameters are directly proportional to rate of mass transfer.

270

271 **6. Conclusion:** The present study reveals the chemical reactive and dissipative MHD flow over a
272 permeable stretching sheet embedded in a porous medium in the presence of heat source. The
273 governing equations are solved using similarity transformation and Runge-Kutta method of

274 fourth order along with shooting technique. The results are analyzed through graphs and tables.

275 Some of the conclusions are drawn from the results as follows:

- 276 • Higher values of heat source parameter enhance the velocity.
- 277 • Porosity parameter performs as an aiding force for the velocity of fluid.
- 278 • Fluid moves with higher velocity for higher values of viscous dissipation parameter.
- 279 • Higher values of chemical reaction parameter reduce the velocity of the fluid flow.
- 280 • Increasing values of dissipative heat represented by Ec , because of viscous dissipation
- 281 gives rise to significant increase in temperature.
- 282 • Increase in porosity helps to fall in temperature significantly with and without thermal
- 283 slip.
- 284 • Larger values of Chemical reaction parameter lead to lower concentration. Higher values
- 285 of E_c and Q lead to increase the skin friction but reverse trend is shown in case of R_c and
- 286 S_c .

287 The limitation of the present study are:

288 (i) the joulean dissipation, which is generated because of outwardly magnetic

289 field interacting with conducting liquid, has been neglected .

290 (ii) induced magnetic field and hall current are neglected which should be studied in case

291 of strong applied magnetic field strength.

292 The above limitations may be studied and considered as future directions for research.

293 Effect of second slip conditions may also be taken care of and flow may be considered

294 over different type of surfaces like wavy surface.

295

296 **References:**

- 297 Ahmed, S. (2014)., Numerical analysis for magneto hydrodynamic chemically reacting and
298 radiating fluid past a non-isothermal impulsively started vertical surface adjacent to a porous
299 regime. *Ain Shams Engineering Journal*, 5, 923-933.
- 300 Ahmed, S. & Kalita, K. (2014). Unsteady MHD chemically reacting fluid through a porous
301 medium bounded by a non-isothermal impulsively-started vertical plate: A numerical technique.
302 *Journal of Naval Architecture and Marine Engineering*, 11, 39-54.
- 303 Akbar, N.S. & Khan, Z.H. (2014). Heat transfer analysis of the peristaltic instinct of biviscosity
304 fluid with the impact of thermal and velocity slips. *International Communications in Heat and*
305 *Mass Transfer*, 58, 193- 199.
- 306 Ali, M. E. (1994). Heat transfer characteristics of a continuous stretching surface. *Warme-und*
307 *Stoffubertragung*, 29, 227-234.
- 308 Anjali Devi, S .P. & Ganga, B. (2010). Dissipation Effects on MHD Nonlinear Flow and Heat
309 Transfer Past a Porous Surface with Prescribed Heat flux. *Journal of Applied Fluid Mechanics*,
310 3, 1-6.
- 311 Baehr, H. D. & Stephan, K. (2006). *Heat and Mass transfer*, 2nd edition, Springer, pp-280.
- 312 Biswal, M.M., Swain, B.K., Das, M. & Dash, G.C. (2022), Heat and Mass transfer in MHD
313 stagnation point flow towards an inclined stretching sheet embedded in a porous medium. *Heat*
314 *Transfer*, 51, 4837-4857.
- 315 Chamber, P. L. & Young, J. D. (1958). The effects of homogeneous 1st order chemical reactions
316 in the neighbourhood of a flat plate for destructive and generative reactions. *Physics of Fluids*, 1,
317 48-54.

318 Chamkha, A. J. , Aly, A. M. & Mansour, M. A. (2010). Similarity solution for unsteady heat and
319 mass transfer from a stretching surface embedded in a porous medium with suction/injection and
320 chemical reaction effects. *Chemical Engineering Communications*, 197, 846-858.

321 Ferdows, M. , Chapal, S.M. & Afify, A.A. (2014). Boundary layer flow and heat transfer of a
322 nanofluid over a permeable unsteady stretching sheet with viscous dissipation. *Journal of*
323 *Engineering Thermophysics*, 23, 216-228.

324 Gad-el-Hak (1999). *The fluid mechanics of micro devices - the freeman scholar lecture*, ASME.
325 *Journal of Fluids Engineering*, 121, 5-33.

326 Hayat, T., Awais, M. & Hendi, A.A. (2012). Three-dimensional rotating flow between two
327 porous wall with slip and heat transfer. *International Communications in Heat and Mass*
328 *Transfer*, 39, 551-555.

329 Hunegnaw, D. & Kishan, N. (2017). Unsteady MHD heat and mass transfer flow over stretching
330 sheet in porous medium with variable properties considering viscous dissipation and chemical
331 reaction. *American Chemical Science Journal*, 4, 901-917.

332 Mabood, F. & Das, K. (2016). Melting heat transfer on hydromagnetic flow of a nanofluid over
333 a stretching sheet with radiation and second-order slip. *The European Physical Journal Plus*, vol.
334 131, 1-12.

335 Mabood, M. & Shateyi, S. (2019). Multiple slip effects on MHD unsteady flow heat and mass
336 transfer impinging on permeable stretching sheet with radiation. *Modelling and Simulation in*
337 *Engineering*, 2019, 1-11.

338 Makinde, O. D., Khan, W.A. & Khan, Z.H. (2016). Analysis of MHD nanofluid flow over a
339 convectively heated permeable vertical plate embedded in a porous medium. *Journal of*
340 *Nanofluids* 5, 574-580.

341 Makinde, O. D., Khan, Z.H., Ahmad, R. & Khan, W. A. (2018). Numerical study of unsteady
342 hydromagnetic radiating fluid flow past a slippery stretching sheet embedded in a porous
343 medium. *Physics of Fluids*, 30, 083601-083607.

344 Misra, J.C. & Sinha, A. (2013). Effect of thermal radiation on MHD flow of blood and heat
345 transfer in a permeable capillary in stretching motion. *Heat Mass Transfer*, 49, 617-628.

346 Muthucumaraswamy, R. & Janakiramana, B. (2008). Mass transfer effect on isothermal vertical
347 oscillating plate in presence of chemical reaction. *International Journal of Applied Mathematics*
348 *and Mechanics*, 4, 66-74.

349 Mutuku-Njane, W. N. & Makinde, O. D. (2013). Combined effect of buoyancy force and Navier
350 Slip on MHD flow of a nanofluid over a convectively heated vertical porous plate. *The Scientific*
351 *World Journal*, 2013, 1-8.

352 Parida, B.C., Swain, B.K., Senapati, N. (2021). Mass transfer effect on viscous dissipative MHD
353 flow of nanofluid over a stretching sheet embedded in a porous medium. *Journal of Naval*
354 *Architecture and Marine Engineering*, 18, 73-82.

355 Raza, J., Rohni, A.M., Omar, Z. & Awais, M. (2016). Heat and mass transfer analysis of MHD
356 nanofluid in a rotating channel with slip effects. *Journal of Molecular Liquids*, 219, 703-708.

357 Reddy, M.G. (2016). Heat and mass transfer on Magnetohydrodynamic peristaltic flow in a
358 porous medium with partial slip. *Alexandria Engineering Journal*, 55, 1225-1234.

359 Sekhar, K.R., Reddy, G.V., Raju, C.S.K., Ibrahim, S.M. & Makinde, O.D. (2018). Multiple slip
360 effects on magnetohydrodynamic boundary layer flow over a stretching sheet embedded in a
361 porous medium with radiation and joule heating. *Special Topics & Reviews in Porous media: An*
362 *International journal*, 9, 117-132.

363 Swain, B. K. (2021). Effect of second order chemical reaction on MHD free convective radiating
364 flow over an impulsively started vertical plate. *Journal of Nonlinear Modelling and Analysis*, 3,
365 167-178.

366 Swain, B.K., Biswal, M.M. & Dash, G.C. (2021). Effect of the second-order slip and heat source
367 on dissipative MHD flow of blood through a permeable capillary in stretching motion.
368 *International Journal of Ambient Energy*, DOI: 10.1080/01430750.2021.1979649.

369 Swain, B.K., Parida, B.C., Kar, S. & Senapati, N.(2020). Viscous dissipation and joule heating
370 effect on MHD flow and heat transfer past a stretching sheet embedded in a porous medium.
371 *Heliyon*, 6, 1-8. <https://doi.org/10.1016/j.heliyon.2020.e05338>

372 Swain, B. K. & Senapati, N. (2015). The effect of mass transfer on MHD free convective
373 radiating flow over an impulsively started vertical plate embedded in a porous medium. *Journal*
374 *of Applied Analysis and Computation*, 5, 18-27.

375 Swain, B. K., Senapati, N. & Dash, M. (2014). The effect of chemical reaction and thermal
376 radiation on the hydromagnetic free convective rotating flow past an accelerated vertical plate in
377 the presence variable heat and mass diffusion. *Der Chemica Sinica*, 5, 56-66.

378 Swain, B.K., Senapati, N. & Dash, M. (2017). Chemical Reaction Effect on MHD Convective
379 Flow with Heat and Mass Transfer Past a Semi-Infinite Vertical Porous Plate. *Journal of*
380 *Advanced Mathematics and Applications*, 6, 1-8.

381 Turkyilmazoglu, M. (2013). Heat and mass transfer of MHD second order slip flow. Computers
382 and Fluids, 71, 426-434.

383 Uddin, M. S. (2015).Viscous and Joules Dissipation on MHD Flow past a Stretching Porous
384 Surface Embedded in a Porous Medium. Journal of Applied Mathematics and Physics, 3, 1710-
385 1725.

386 Venkateswarlu, B., Satya Narayana, P.V., & Tarakaramu, N. (2018). Melting and Viscous
387 dissipation effects on MHD flow over a moving surface with constant heat source. Transactions
388 of A.Razmadze Mathematical Institute, 172, 618-630.

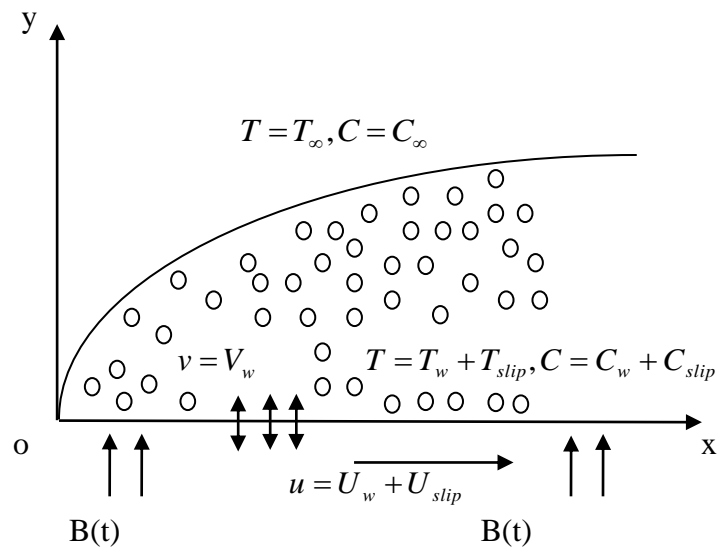


Figure.1. Physical problem

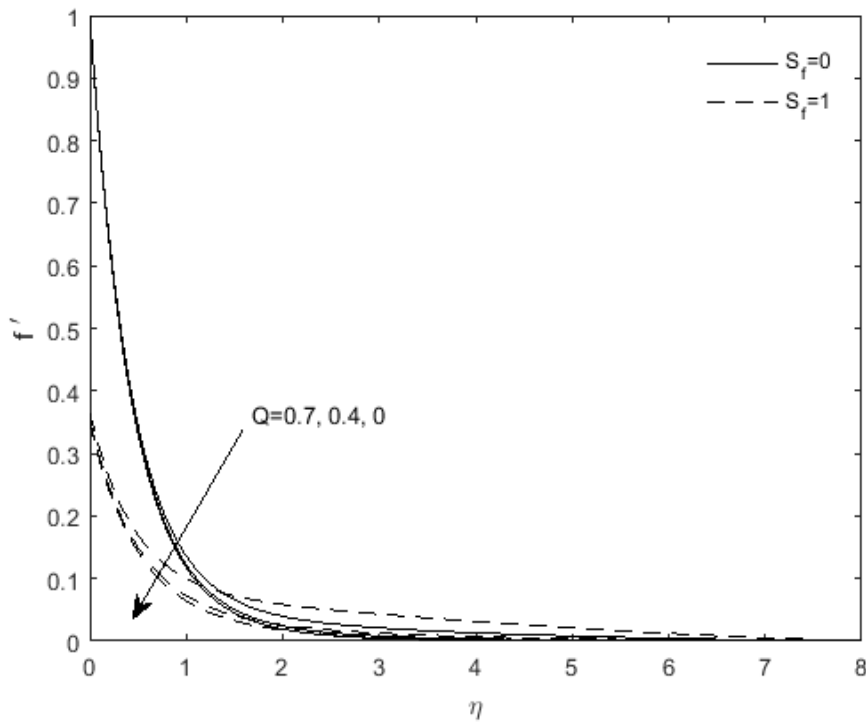


Fig.2. Velocity profile for Q

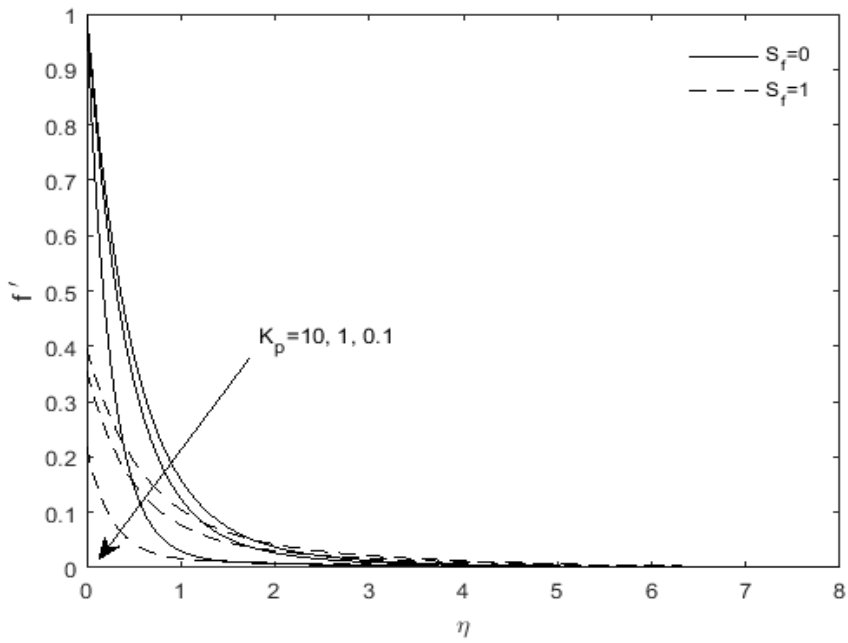


Fig.3. Velocity profile for K_p

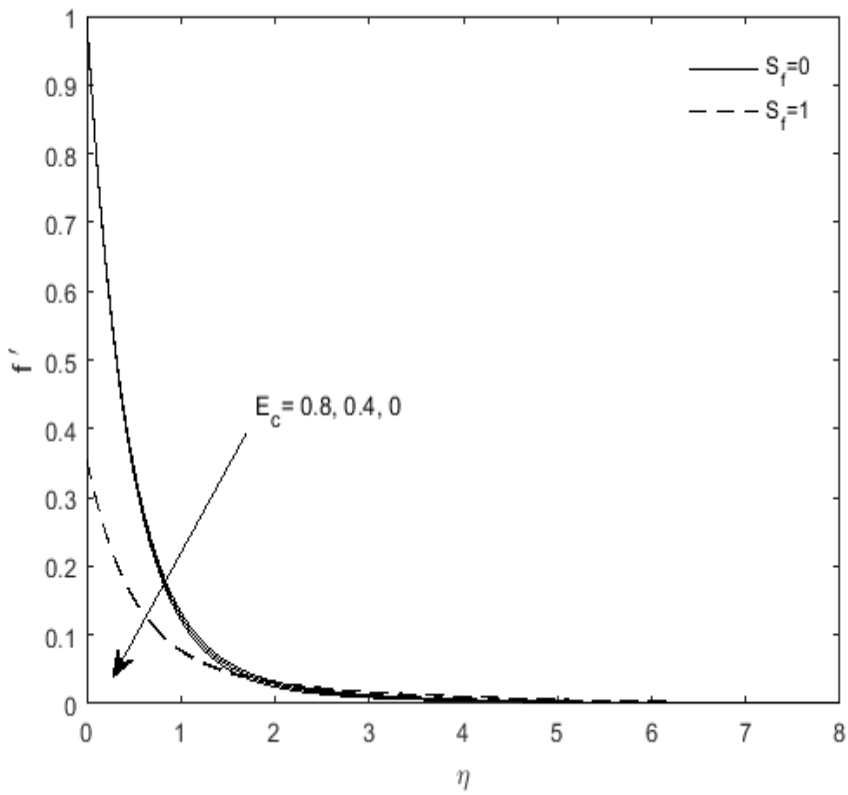


Fig.4. Velocity profile for E_c

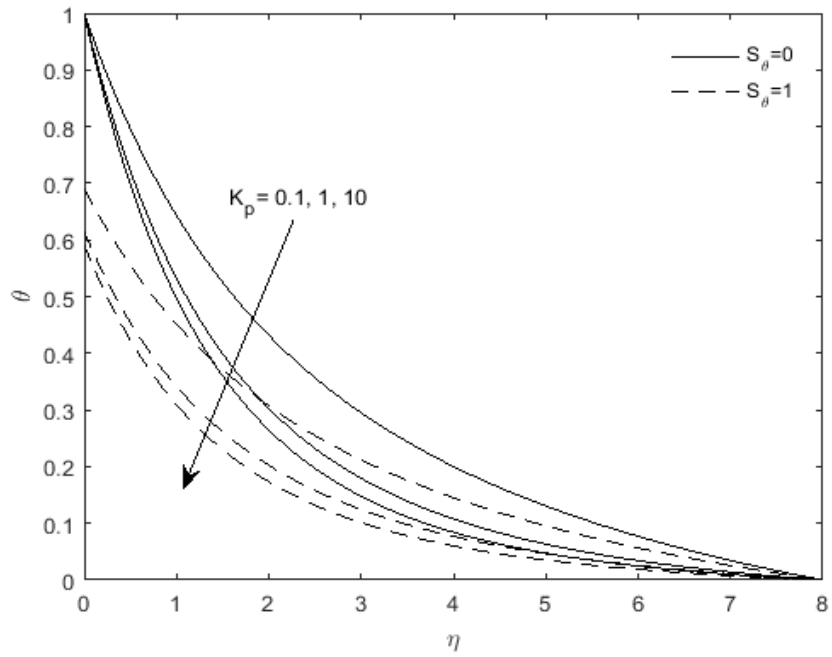


Fig.5. Temperature profile for K_p

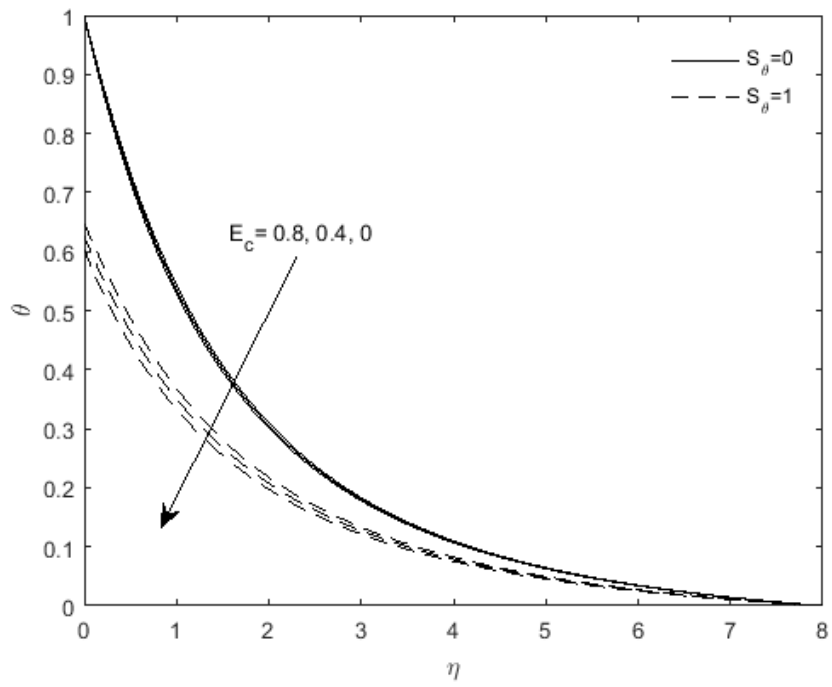


Fig.6. Temperature profile for E_c

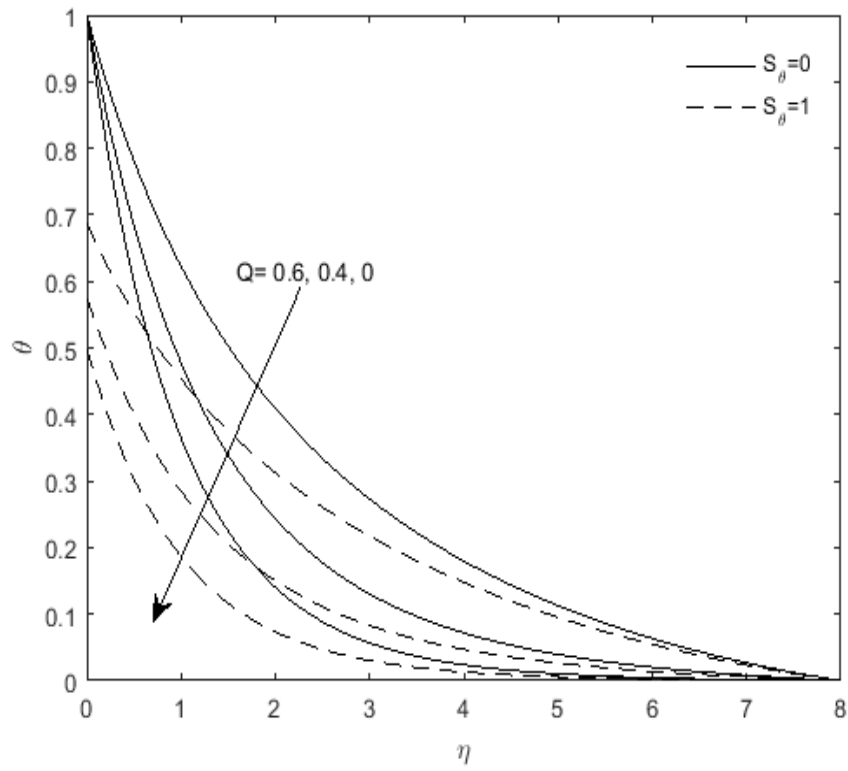


Fig.7. Temperature profile for Q

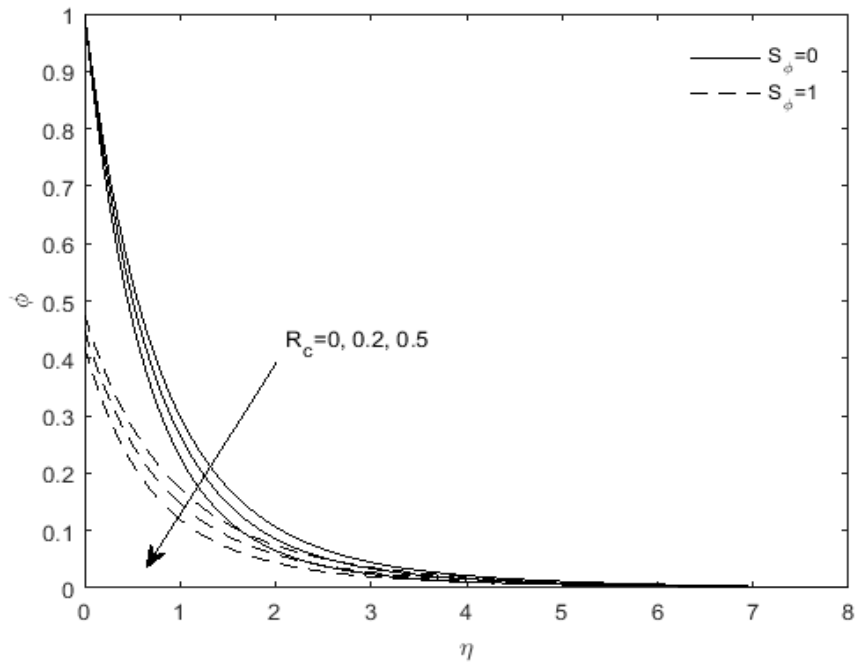


Fig.8. Concentration profile for R_c

Table 1: $-f''(0)$ for different values of δ when $M = f_w = S_f = \lambda_1 = \lambda_2 = 0, K_p \rightarrow \infty$

	$-f''(0)$		
δ	Chamkha, Aly & Mansour (2010)	Mabood and Shateyi (2019)	Present result
0.8	1.261512	1.261042	1.2610428
1.2	1.378052	1.377724	1.3777241

Table 2: $-f''(0)$ for various values of M when $\delta = f_w = S_f = \lambda_1 = \lambda_2 = 0, K_p \rightarrow \infty$

	$-f''(0)$		
M	Mabood and Das (2016)	Mabood and Shateyi (2019)	Present result
0	-1.000008	-1.0000084	1.0000624
1	1.4142135	1.41421356	1.414213606
5	2.4494897	2.44948974	2.449489744
10	3.3166247	3.31662479	3.316624790
50	7.1414284	7.14142843	7.141428429

Table 3: Values of $-\theta'(0)$ when $M = f_w = S_f = S_\theta = \delta = \lambda_1 = \lambda_2 = R = 0$

	$-\theta'(0)$		
Pr	Ali (1994)	Mabood and Shateyi (2019)	Present result
0.72	0.8058	0.8088	0.809463
1	0.9691	1.0000	1.000062
3	1.9144	1.9237	1.923652
10	3.7006	3.7207	3.720631

Table-4

λ_1	λ_2	K_p	E_c	Q	R_c	S_c	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0	0.4	1	0.2	0.5	0.5	1	-1.21798	0.54844	0.81276
0.4	0.4	1	0.2	0.5	0.5	1	-1.12895	0.56331	0.81691
0.8	0.4	1	0.2	0.5	0.5	1	-1.04472	0.57564	0.82079
0.4	1	1	0.2	0.5	0.5	1	-1.04357	0.57352	0.82050
0.4	1.5	1	0.2	0.5	0.5	1	-0.97404	0.58120	0.82341
0.4	0.4	5	0.2	0.5	0.5	1	-0.96427	0.60003	0.81996
0.4	0.4	10	0.2	0.5	0.5	1	-0.94059	0.60559	0.82040
0.4	0.4	1	0.3	0.5	0.5	1	-1.12704	0.54000	0.82336
0.4	0.4	1	0.4	0.5	0.5	1	-1.12514	0.51675	0.82978

0.4	0.4	1	0.2	1	0.5	1	-1.09941	0.36564	0.84726
0.4	0.4	1	0.2	1.5	0.5	1	-0.94533	-0.53122	0.93526
0.4	0.4	1	0.2	0.5	1	1	-1.13499	0.56215	0.87529
0.4	0.4	1	0.2	0.5	2	1	-1.14328	0.56068	0.96254
0.4	0.4	1	0.2	0.5	0.5	2	-1.14444	0.56076	0.99120
0.4	0.4	1	0.2	0.5	0.5	10	-1.16627	0.55831	1.36828