Songklanakarin J. Sci. Technol.
47 (6), 430–437, Nov. – Dec. 2025

http://www.sjst.psu.ac.th

Original Article

Soft intersection almost quasi-ideals of semigroups

Aslıhan Sezgin[1*], and Fatıma Zehra Kocakaya[2]

[1] *Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Turkey*

[2] *Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Turkey*

Received: 4 April 2024; Revised: 18 June 2025; Accepted: 20 August 2025

Abstract

The notion of soft intersection quasi-ideal is a generalization of soft intersection left (right) ideal of semigroups. In this paper, we introduce the concept of soft intersection almost quasi-ideal, and its generalization, weakly almost quasi-ideal of a semigroup. We thoroughly examine their fundamental characteristics. Contrary to the soft intersection ideal theory, we show that every soft intersection almost quasi-ideal is a soft intersection almost ideal. It is also illustrated that an idempotent soft intersection almost quasi-ideal is both a soft intersection almost weak interior ideal and a soft intersection almost subsemigroup. Moreover, by obtaining that when a nonempty set A is almost quasi-ideal, then its soft characteristic function is soft intersection almost quasi-ideal, and vice versa, we acquire numerous intriguing connections in terms of minimality, primeness, semiprimeness, and strongly primeness between almost quasi-ideals and soft intersection almost quasi-ideals.

Keywords: soft set, quasi-ideal, soft intersection (almost) quasi-ideal

1. Introduction

Semigroups were first formally studied in the early 1900s. Since finite automata and finite semigroups are naturally related, the theory of finite semigroups has been particularly significant in theoretical computer science. This relationship dates back to the 1950s.

To study algebraic structures and their applications, ideals are essential. Ideals were first introduced by Dedekind to help with the study of algebraic numbers, and Noether expanded on them to include associative rings. Good and Hughes (1952) introduced the concept of bi-ideal for semigroups. Steinfeld (1956) first proposed the notion of quasi-ideal for semigroups, and then for rings. The quasi-ideals are a generalization of left and right ideals, whereas the bi-ideals are a generalization of quasi-ideals.

*Corresponding author
Email address: aslihan.sezgin@amasya.edu.tr

In addition, the notion of almost left, right, and two-sided ideals of semigroups was introduced by Grosek and Satko (1980). Later in 1981, Bogdanovic (1981) introduced the idea of almost bi-ideals in semigroups as an extension of bi-ideals. Wattanatripop, Chinram, and Changphas (2018a) introduced the idea of almost quasi-ideals of semigroups. Kaopusek, Kaewnoi, and Chinram (2020) proposed the concepts of almost interior ideals and weakly almost interior ideals of semigroups and examined their characteristics by utilizing the idea of almost ideals and interior ideals of semigroups. Researchers have focused a great deal of emphasis on semigroups' almost ideals. Iampan, Chinram, and Petchkaew (2021), Chinram and Nakkhasen (2022), Gaketem (2022), and Gaketem and Chinram (2023) proposed the idea of almost subsemigroups; almost bi-quasi-interior ideals; almost bi-interior ideals and almost bi-quasi ideals of semigroups, respectively. Furthermore, Wattanatripop *et al*. (2018a), Iampan *et al*. (2021), Chinram and Nakkhasen (2022), Gaketem (2022), Gaketem and Chinram (2023), Wattanatripop, Chinram, and Changphas (2018b), Krailoet, Simuen, Chinram, and Petchkaew (2021) examined various forms of fuzzy almost ideals.

In an attempt to model uncertainty, Molodtsov (1999) introduced the concept of a soft set. This is defined as a function from the parameter set E to the power set of U. Soft set operations, which form the foundation of the theory, were examined in detail by Maji, Biswas, and Roy (2003), Pei and Miao (2005), Ali, Feng, Liu, Min, and Shabir (2009), Sezgin and Atagün (2011), Ali, Shabir, and Naz (2011), Sezgin, Ahmad, and Mehmood (2019), Stojanovic (2021), Sezgin, Aybek, and Atagün (2023), Sezgin, Aybek, and Güngör (2023), Sezgin, Çağman, Atagün, and Aybek (2023), Sezgin and Dagtoros (2023), Sezgin and Yavuz (2023), Sezgin and Aybek (2024), Sezgin and Çalışıcı (2024), Sezgin and Sarıalioğlu (2024), Sezgin and Yavuz (2024), Sezgin, Atagün, and Çağman (2025), and Sezgin and Şenyiğit (2025). The definition of a soft set and its operations were modified by Çağman and Enginoğlu (2010). Furthermore, Çağman, Çıtak, and Aktaş (2012) established the concept of soft intersection groups, which has been used to study a variety of soft algebraic systems. Soft sets were first used in semigroup theory by Sezer, Çağman, and Atagün (2014) and Sezer, Çağman, Atagün, Ali, and Türkmen (2015). Semigroups with soft intersections left (right/sided) ideals, (generalized) bi-ideals, interior ideals, and quasi-ideals were all examined by Sezer *et al*. (2014) and Sezer *et al*. (2015). In terms of soft intersection substructures of semigroups, Sezgin and Orbay (2022) characterized some types of semigroups. A variety of algebraic structures, including soft sets, were also examined by Feng, Jun, and Zhao (2008), Sezer, Atagün, and Çağman (2013, 2014), Atagün and Sezer (2015), Sezgin, Çağman, and Atagün (2017), Khan, Izhar, and Sezgin (2017), Atagün and Sezgin (2017, 2018, 2022), Gulistan, Feng, Khan, and Sezgin, (2018), Sezgin (2018), Jana, Pal, Karaaslan, and Sezgin (2019), Atagün, Kamacı, Taştekin, and Sezgin (2019), Özlü and Sezgin (2020), Sezgin, Atagün, Çağman, and Demir (2022), Riaz *et al*. (2023), and Manikantan, Ramasamy, and Sezgin (2023).

Rao (2018a, 2018b, 2020a, 2020b) introduced a few novel forms of semigroup ideals, including bi-interior ideals, bi-quasi-interior ideals, bi-quasi ideals, quasi-interior ideals and weak interior ideals.

Soft intersection quasi-ideal of semigroups proposed by Sezer *et al*. (2014) is a generalization of soft intersection left (right) ideal. In this paper, we propose the concept of "soft intersection almost quasi-ideals" and its generalization, "soft intersection weakly almost quasi-ideals." We show that every soft intersection almost quasi-ideal of a semigroup is a soft intersection weakly almost quasi-ideal; and that every soft intersection almost quasi-ideal is a soft intersection almost ideal; nevertheless, the converses do not hold with counterexamples. Also, we illustrate that an idempotent soft intersection almost quasi-ideal is both a soft intersection almost weak interior ideal and a soft intersection almost subsemigroup. We observe that, under the binary operation of soft union, a semigroup can be constructed by soft intersection almost quasi-ideals of a semigroup, but not under the soft intersection operation. Additionally, we establish the connection between a semigroup's soft intersection almost quasi-ideal and almost quasi-ideal as regards minimality, primeness, semiprimeness, and strongly primeness by obtaining that if a nonempty set A is almost quasi-ideal, then its soft characteristic function is soft intersection almost quasi-ideal, and vice versa.

2. Preliminary Topics

In this section, we review several fundamental notions related to semigroups and soft sets.

Definition 2.1 Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E \rightarrow P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x)): x \in E, f_K(x) \in P(U)\}$$

(Molodtsov, 1999; Çağman and Enginoğlu, 2010). Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.2 Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$ (Çağman and Enginoğlu, 2010).

Definition 2.3 Let f_A, $f_B \in S_E(U)$. If $f_A(x) \subseteq f_B(x)$ for all $x \in E$, then f_A is called a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called a soft equal to f_B and denoted by $f_A = f_B$ (Çağman and Enginoğlu, 2010).

Definition 2.4 Let $f_A, f_B \in \mathrm{S_E(U)}$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ (Çağman and Enginoğlu, 2010).

Definition 2.5 For a soft set f_A, the support of f_A is defined by

$supp(f_A)$={$x \in A : f_A(x) \neq \emptyset$} (Feng *et al*., 2008)

It is obvious that a soft set with an empty support is a null soft set; otherwise, the soft set is nonnull.

Note 2.6 If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$ (Sezgin and İlgin, 2024a)

A semigroup S is a nonempty set with an associative binary operation and throughout this paper, S stands for a semigroup, and all the soft sets are the elements of $S_S(U)$ unless otherwise specified.

Definition 2.7 Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by (Sezer *et al*., 2015)

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup_{x=yz} \{f_S(y) \cap g_S(z)\}, & if\ \exists y, z \in S\ such\ that\ x = yz \\ \emptyset, & otherwise \end{cases}$$

Theorem 2.8 Let f_S, g_S, $h_S \in S_S(U)$. Then,

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

ii) If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$ (Sezer *et al*., 2015)

It is obvious that $f_S \circ g_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$ or $g_S = \emptyset_S$ (Sezgin and İlgin, 2024b)

Definition 2.9 Let A be a subset of S. We denote by S_A the soft characteristic function of A and define it as

$$S_A(x) = \begin{cases} U, & if\ x \in A \\ \emptyset, & if\ x \in S \backslash A \end{cases}$$

(Sezer *et al*., 2015). If $f_S(x) = U$ for all $x \in S$, then we denote such a kind of soft set by $\widetilde{\mathbb{S}}$ throughout this paper. It is obvious that $\widetilde{\mathbb{S}} = S_S$, that is, $\widetilde{\mathbb{S}}(x) = U$ for all $x \in S$ (Sezer *et al*., 2015)

Corollary 2.10 $supp(S_A) = A$ (Sezgin and İlgin, 2024a).

Theorem 2.11 Let X and Y be nonempty subsets of S. Then,
i) $X \subseteq Y$ if and only if $S_X \widetilde{\subseteq} S_Y$
ii) $S_X \widetilde{\cap} S_Y = S_{X\cap Y}$ and $S_X \widetilde{\cup} S_Y = S_{X\cup Y}$
iii) $S_X \circ S_Y = S_{XY}$ (Sezer *et al*., 2015, Sezgin and İlgin, 2024a)

Definition 2.12 Let x be an element in S. We denote by S_x the soft characteristic function of x and define as

$$S_x(y) = \begin{cases} U, & if\ \ y = x \\ \emptyset, & if\ \ y \neq x \end{cases}$$

(Sezgin and İlgin, 2024b).

Definition 2.13 A soft set over U is called a soft intersection quasi-ideal of S over U if

$$(f_S \circ \widetilde{\mathbb{S}})\ \widetilde{\cap}\ (\widetilde{\mathbb{S}} \circ f_S) \widetilde{\subseteq} f_S$$

(Sezer *et al*., 2014). For the sake of brevity, soft intersection quasi-ideal is abbreviated by SI-Q-ideal in what follows.

Definition 2.14 Let f_S be a soft set over U. Then, f_S is called a soft intersection almost subsemigroup of S if $(f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$ (Sezgin and İlgin, 2024a); is called a left (right) ideal of S if for all $x \in S$, $(S_x \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$ $\left((f_S \circ S_x) \widetilde{\cap} f_S \neq \emptyset_S\right)$; is called a soft intersection almost two-sided ideal (or briefly soft intersection almost ideal) of S if f_S is both soft intersection almost left and soft intersection almost right ideal of S (Sezgin and İlgin, 2024b), is called a soft intersection almost weak interior ideal of S, if for all $x \in S$, $(S_x \circ f_S \circ f_S) \widetilde{\cap} f_S \neq \emptyset_S$ and $(f_S \circ f_S \circ S_x) \widetilde{\cap} f_S \neq \emptyset_S$ (Sezgin and İlgin, 2024c).

For more about A-ideals (almost-ideals) and fuzzy A-ideals of ternary semigroups, we refer to Suebsung, Wattanatripop, and Chinram (2019); for hybrid quasi-ideals and hybrid A-ideals in ternary semigroups, we refer to Deepika, Elavarasan, and Catherine Grace John (2024).

3. Soft Intersection almost Quasi-Ideal of Semigroups

Definition 3.1 Let f_s be a soft set over U.

(i) f_s is called a soft intersection almost quasi-ideal of S if for all x, y $\in S$,

$$[(f_s \circ S_x)\ \widetilde{\cap}\ (S_y \circ f_s)]\ \widetilde{\cap}\ f_s \neq \emptyset_s$$

(ii) f_s is called a soft intersection weakly almost quasi-ideal of S if for all x, y $\in S$,

$$[(f_s \circ S_x)\ \widetilde{\cap}\ (S_x \circ f_s)]\ \widetilde{\cap}\ f_s \neq \emptyset_s$$

Hereafter, soft intersection almost quasi-ideal of S and soft intersection weakly almost quasi-ideal of S are denoted by SI-almost Q-ideal and SI-weakly almost Q-ideal, respectively.

Example 3.2 Let S={m, n} be the semigroup with the following Cayley Table.

	m	n
m	m	n
n	n	n

Let f_s, h_s, and g_s be soft sets over $U=\mathbb{Z}^+$ as follows:

$$f_s = \{(m,\{2,3\}),(n,\{4,5\})\}$$
$$h_s = \{(m,\{6,7\}),(n,\{1,8\})\}$$
$$g_s = \{(m,\{9,12,15\}),(n,\emptyset)\}$$

Here, f_s and h_s are both SI-almost Q-ideals. It is obvious that

$$[(f_s \circ S_m)\ \widetilde{\cap}\ (S_m \circ f_s)]\ \widetilde{\cap}\ f_s = \{(m,\{2,3\}),(n,\{4{,}5\})\} \neq \emptyset_s$$
$$[(f_s \circ S_n)\ \widetilde{\cap}\ (S_n \circ f_s)]\ \widetilde{\cap}\ f_s = \{(m,\emptyset),(n,\{4{,}5\})\} \neq \emptyset_s$$

Therefore, f_s is an SI-weakly almost Q-ideal. Similarly,

$$[(f_s \circ S_m)\ \widetilde{\cap}\ (S_n \circ f_s)]\ \widetilde{\cap}\ f_s = \{(m,\emptyset),(n,\{4{,}5\})\} \neq \emptyset_s$$
$$[(f_s \circ S_n)\ \widetilde{\cap}\ (S_m \circ f_s)]\ \widetilde{\cap}\ f_s = \{(m,\emptyset),(n,\{4{,}5\})\} \neq \emptyset_s$$

Therefore, f_s is an SI-almost Q-ideal. Similarly, h_s is an SI-almost Q-ideal. Indeed;

$$[(h_s \circ S_m)\ \widetilde{\cap}\ (S_m \circ h_s)]\ \widetilde{\cap}\ h_s = \{(m,\{6,7\}),(n,\{1,8\})\} \neq \emptyset_s$$
$$[(h_s \circ S_n)\ \widetilde{\cap}\ (S_n \circ h_s)]\ \widetilde{\cap}\ h_s = \{(m,\emptyset),(n,\{1,8\})\} \neq \emptyset_s$$

And also,

$$[(h_s \circ S_m)\ \widetilde{\cap}\ (S_n \circ h_s)]\ \widetilde{\cap}\ h_s = \{(m,\emptyset),(n,\{1,8\})\} \neq \emptyset_s$$
$$[(h_s \circ S_n)\ \widetilde{\cap}\ (S_m \circ h_s)]\ \widetilde{\cap}\ h_s = \{(m,\emptyset),(n,\{1,8\})\} \neq \emptyset_s$$

Thus, h_s is an SI-almost Q-ideal. One can also show that g_s is not an SI-almost Q-ideal as

$$[(g_s \circ S_n)\ \widetilde{\cap}\ (S_n \circ g_s)]\ \widetilde{\cap}\ g_s = \emptyset_s$$

Proposition 3.3 Every SI-almost Q-ideal is an SI-weakly almost Q-ideal.

Proof: Let f_s be an SI-almost Q-ideal. Then, for all $x, y \in S$,

$$[(f_s \circ S_x)\ \widetilde{\cap}\ (S_y \circ f_s)]\ \widetilde{\cap}\ f_s \neq \emptyset_s$$

Hence,

$$[(f_s \circ S_x)\ \widetilde{\cap}\ (S_x \circ f_s)]\ \widetilde{\cap}\ f_s \neq \emptyset_s$$

for all $x \in S$. So, f_s is an SI-weakly almost Q-ideal.

Since SI-weakly almost-Q ideal is a generalization of SI-almost Q-ideal, from now on all the theorems and proofs are given for SI-almost Q-ideal instead of SI-weakly almost Q-ideal.

The following example shows that the converse of Proposition 3.3 is not true in general:

Example 3.4 Let $\mathcal{S}=\{k,c,y\}$ be the semigroup with the following Cayley Table.

	k	c	y
k	k	c	y
c	c	y	k
y	y	k	c

f_s be soft sets over $U=\mathbb{Z}^+$ as follows:

$$f_s = \{(k,\{3,7,9\}),(c,\{7,8\}),(y,\{3,5\})\}$$

Since

$$[(f_s \circ S_k) \tilde{\cap} (S_k \circ f_s)] \tilde{\cap} f_s = \{(k,\{3,7,9\}),(c,\{7,8\}),(y,\{3,5\})\} \neq \emptyset_s$$
$$[(f_s \circ S_c) \tilde{\cap} (S_c \circ f_s)] \tilde{\cap} f_s = \{(k,\{3\}),(c,\{7\}),(y,\emptyset)\} \neq \emptyset_s$$
$$[(f_s \circ S_y) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s = \{(k,\{7\}),(c,\emptyset),(y,\{3\})\} \neq \emptyset_s$$

Therefore, f_s is an SI-weakly almost Q-ideal. However, f_s is not an SI-almost Q-ideal since

$$[(f_s \circ S_c) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s = \{(k,\emptyset),(c,\emptyset),(y,\emptyset)\} = \emptyset_s$$

Proposition 3.5 Let f_s be an SI-Q-ideal. f_s is either $(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) = \emptyset_s$ for some $x,y \in S$ or SI-almost Q-ideal.

Proof: Let f_s be an SI-Q ideal, thus $(f_s \circ \tilde{\mathbb{S}}) \tilde{\cap} (\tilde{\mathbb{S}} \circ f_s) \tilde{\subseteq} f_s$ and let $(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) \neq \emptyset_s$. We need to show that for all $x,y \in S$,

$$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \neq \emptyset_s$$

Since $(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) \tilde{\subseteq} (f_s \circ \tilde{\mathbb{S}}) \tilde{\cap} (\tilde{\mathbb{S}} \circ f_s) \tilde{\subseteq} f_s$,
it follows that $(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) \tilde{\subseteq} f_s$.
From assumption $(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) \neq \emptyset_s$ is obvious. Then,

$$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s = (f_s \circ S_x) \tilde{\cap} (S_y \circ f_s) \neq \emptyset_s$$

implying that f_s is an SI-almost Q-ideal.
Here it is obvious that $\emptyset_s$ is an SI-Q-ideal, since

$$(\emptyset_s \circ \tilde{\mathbb{S}}) \tilde{\cap} (\tilde{\mathbb{S}} \circ \emptyset_s) = \emptyset_s \tilde{\cap} \emptyset_s = \emptyset_s \tilde{\subseteq} \emptyset_s$$

but $\emptyset_s$ is not an SI-almost Q-ideal as

$$[(\emptyset_s \circ S_x) \tilde{\cap} (S_y \circ \emptyset_s)] \tilde{\cap} \emptyset_s = [\emptyset_s \tilde{\cap} \emptyset_s] \tilde{\cap} \emptyset_s = \emptyset_s$$

Corollary 3.6 If f_s is an SI-almost Q-ideal, then f_s needs not be an SI-Q ideal.

Example 3.7 In Example 3.2, it is shown that f_s and h_s are SI-almost Q-ideal; however, f_s and h_s are not SI-Q ideals. In fact,

$$[(f_s \circ \tilde{\mathbb{S}}) \tilde{\cap} (\tilde{\mathbb{S}} \circ f_s)](n) = [f_s(m) \cup f_s(n)] \nsubseteq f_s(n)$$

thus, f_s is not an SI-Q ideal. Similarly,

$$[(h_s \circ \tilde{\mathbb{S}}) \tilde{\cap} (\tilde{\mathbb{S}} \circ h_s)](n) = [h_s(m) \cup h_s(n)] \nsubseteq h_s(n)$$

thus, h_s is not an SI-Q ideal.

Theorem 3.8 Every SI-almost Q-ideal is an SI-almost left (right/two-sided) ideal.

Proof: Assume that f_s is an SI-almost Q-ideal. Hence, for all $x,y \in S$, $[(S_x \circ f_s) \tilde{\cap} (f_s \circ S_y)] \tilde{\cap} f_s \neq \emptyset_s$. We need to show that $(S_x \circ f_s) \tilde{\cap} f_s \neq \emptyset_s$ and $(f_s \circ S_x) \tilde{\cap} f_s \neq \emptyset_s$. In fact,

$$\emptyset_s \neq [(S_x \circ f_s) \tilde{\cap} (f_s \circ S_y)] \tilde{\cap} f_s \tilde{\subseteq} (S_x \circ f_s) \tilde{\cap} f_s$$

and

$$\emptyset_s \neq [(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \tilde{\subseteq} (f_s \circ S_x) \tilde{\cap} f_s$$

Since $[(S_x \circ f_s) \tilde{\cap} (f_s \circ S_y)] \tilde{\cap} f_s \neq \emptyset_s$, it is obvious that $(S_x \circ f_s) \tilde{\cap} f_s \neq \emptyset_s$ and $(f_s \circ S_x) \tilde{\cap} f_s \neq \emptyset_s$. Thus, f_s is an SI-almost left and SI-almost right ideal. Hence, f_s is an SI-almost two-sided ideal.

The converse of Theorem 3.8 is not true as shown by the following example:

Example 3.9 We know that f_s is not an SI-almost Q-ideal in Example 3.4; however, f_s is an SI-almost two-sided ideal. Now let's first show that f_s is an SI-almost left ideal. Indeed,

$$[(S_k \circ f_s) \tilde{\cap} f_s] = \{(k,\{3,7,9\}),(c,\{7,8\}),(y,\{3,5\})\} \neq \emptyset_s$$
$$[(S_c \circ f_s) \tilde{\cap} f_s] = \{(k,\{3\}),(c,\{7\}),(y,\emptyset)\} \neq \emptyset_s$$
$$[(S_y \circ f_s) \tilde{\cap} f_s] = \{(k,\{7\}),(c,\emptyset),(y,\{3\})\} \neq \emptyset_s$$

Therefore, f_s is an SI-almost left ideal. Now let's show that f_s is an SI-almost right ideal. Since

$$[(f_s \circ S_k) \tilde{\cap} f_s] = \{(k,\{3,7,9\}),(c,\{7,8\}),(y,\{3,5\})\} \neq \emptyset_s$$
$$[(f_s \circ S_c) \tilde{\cap} f_s] = \{(k,\{3\}),(c,\{7\}),(y,\emptyset)\} \neq \emptyset_s$$
$$[(f_s \circ S_y) \tilde{\cap} f_s] = \{(k,\{7\}),(c,\emptyset),(y,\{3\})\} \neq \emptyset_s$$

f_s is an SI-almost right ideal. Hence, f_s is an SI-almost two-sided ideal.

Proposition 3.10 Let f_S be an idempotent SI-almost Q-ideal. Then, f_S is an SI-almost subsemigroup.

Proof: Assume that f_S is an idempotent SI-almost Q-ideal.
Then, $f_S \circ f_S = f_S$ and for all $x,y \in S$,
$(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_S \neq \emptyset_S$.
We need to show that

$$(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$$

Since,

$$\begin{aligned}\emptyset_S &\neq [(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_S \\ &= [(f_S \circ S_x) \tilde{\cap} (S_y \circ f_S) \tilde{\cap} f_S] \tilde{\cap} f_S \\ &= [(f_S \circ S_x) \tilde{\cap} (S_y \circ f_S) \tilde{\cap} (f_S \circ f_S)] \tilde{\cap} f_S \\ &\tilde{\subseteq} (f_S \circ f_S) \tilde{\cap} f_S\end{aligned}$$

hence $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, so f_S is an SI-almost subsemigroup.

Proposition 3.11 Let f_S be an idempotent SI-almost Q-ideal. Then, f_S is an SI-almost weak interior ideal.

Proof: Assume that f_S is an idempotent SI-almost Q-ideal. Then, $f_S \circ f_S = f_S$ and for all $x, y \in S$,
$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \neq \emptyset_S$.
We need to show that for all $x \in S$

$$(S_x \circ f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S \text{ and } (f_S \circ f_S \circ S_x) \tilde{\cap} f_S \neq \emptyset_S$$

Since, for all $x \in S$,

$$\begin{aligned}\emptyset_S \neq [(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_S &\tilde{\subseteq} (f_S \circ S_x) \tilde{\cap} f_S \\ &= (f_S \circ f_S \circ S_x) \tilde{\cap} f_S\end{aligned}$$

and

$$\begin{aligned}\emptyset_S \neq [(f_s \circ S_y) \tilde{\cap} (S_x \circ f_s)] \tilde{\cap} f_S &\tilde{\subseteq} (S_x \circ f_S) \tilde{\cap} f_S \\ &= (S_x \circ f_S \circ f_S) \tilde{\cap} f_S,\end{aligned}$$

f_S is an SI-almost weak interior ideal.

Theorem 3.12 Let $f_s \tilde{\subseteq} h_s$ such that f_s is an SI-almost Q-ideal, then h_s is an SI-almost Q-ideal.

Proof: Assume that f_s is an SI-almost Q-ideal. Hence, for all $x, y \in S$, $[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \neq \emptyset_s$. We need to show that $[(h_s \circ S_x) \tilde{\cap} (S_y \circ h_s)] \tilde{\cap} h_s \neq \emptyset_s$. Indeed,

$$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \tilde{\subseteq} [(h_s \circ S_x) \tilde{\cap} (S_y \circ h_s)] \tilde{\cap} h_s$$

Since

$$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \neq \emptyset_s,$$

thus, $[(h_s \circ S_x) \tilde{\cap} (S_y \circ h_s)] \tilde{\cap} h_s \neq \emptyset_s$ implying that h_s is an SI-almost Q-ideal.

Theorem 3.13 Let f_s and h_s be SI-almost Q-ideals. Then, $f_s \tilde{\cup} h_s$ is an SI-almost Q-ideal.

Proof: Since f_s is an SI-almost Q-ideal and $f_s \tilde{\subseteq} f_s \tilde{\cup} h_s$, $f_s \tilde{\cup} h_s$ is an SI-almost Q-ideal by Theorem 3.12.

Corollary 3.14 The finite union of SI-almost Q-ideals is an SI-almost Q-ideal.

Corollary 3.15 Let f_s or h_s be SI-almost Q-ideal. Then, $f_s \tilde{\cup} h_s$ is an SI-almost Q-ideal. Here note that if f_s and h_s are SI-almost Q-ideals, then $f_s \tilde{\cap} h_s$ needs not to be an SI-almost Q-ideal.

Example 3.16 Consider the SI-almost Q-ideals f_s and h_s in Example 3.2. Since,

$$f_s \tilde{\cap} h_s = \{(m, \emptyset), (n, \emptyset)\} = \emptyset_s$$

$f_s \tilde{\cap} h_s$ is not an SI-almost Q-ideal.

Lemma 3.17 Let $x \in S$ and Y be a nonempty subset of S. Then, $S_x \circ S_Y = S_{xY}$. If X is a nonempty subset of S and $y \in S$, then it is obvious that $S_X \circ S_y = S_{Xy}$ (Sezgin and İlgin, 2024b)

Theorem 3.18 Let A be a subset of S. Then, A is an almost Q-ideal if and only if S_A, the soft characteristic function of A, is an SI-almost Q-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that $\emptyset \neq A$ is an almost Q-ideal. Then, $(Ax \cap yA) \cap A \neq \emptyset$ for all $x, y \in S$, and so there exists $k \in S$ such that $k \in (Ax \cap yA) \cap A$. Since,

$$\begin{aligned}([(S_A \circ S_x) \tilde{\cap} (S_y \circ S_A)] \tilde{\cap} S_A)(k) &= (S_{Ax} \tilde{\cap} S_{yA})(k) \cap S_A(k) \\ &= (S_{Ax \cap yA})(k) \cap S_A(k) \\ &= (S_{(Ax \cap yA) \cap A})(k) \\ &= U \neq \emptyset\end{aligned}$$

It follows that $[(S_A \circ S_x) \tilde{\cap} (S_y \circ S_A)] \tilde{\cap} S_A \neq \emptyset_s$. Thus, S_A is an SI-almost Q-ideal.

Conversely assume that S_A is an SI-almost Q-ideal. Hence, we have $[(S_A \circ S_x) \tilde{\cap} (S_y \circ S_A)] \tilde{\cap} S_A \neq \emptyset_s$ for all $x, y \in S$. In order to show that A is an almost Q-ideal, we should prove that

$$A \neq \emptyset \text{ and } (Ax \cap yA) \cap A \neq \emptyset, \text{ for all } x, y \in S.$$

$A \neq \emptyset$ is obvious from assumption. Now,

$$\begin{aligned}&\emptyset_s \neq [(S_A \circ S_x) \tilde{\cap} (S_y \circ S_A)] \tilde{\cap} S_A \Rightarrow \exists k \in \\ &S; ([(S_A \circ S_x) \tilde{\cap} (S_y \circ S_A)] \tilde{\cap} S_A)(k) \neq \emptyset \\ &\Rightarrow \exists k \in S;\ (S_{Ax} \tilde{\cap} S_{yA})(k) \cap S_A(k) \neq \emptyset \\ &\Rightarrow \exists k \in S;\ (S_{Ax \cap yA})(k) \cap S_A(k) \neq \emptyset \\ &\Rightarrow \exists k \in S;\ (S_{(Ax \cap yA) \cap A})(k) \neq \emptyset \\ &\Rightarrow \exists k \in S;\ (S_{(Ax \cap yA) \cap A})(k) = U \\ &\Rightarrow k \in (Ax \cap yA) \cap A\end{aligned}$$

Hence, $(Ax \cap yA) \cap A \neq \emptyset$. Consequently, A is an almost Q-ideal.

Lemma 3.19 Let f_s be a soft set over U. Then, $f_s \tilde{\subseteq} S_{supp(f_s)}$ (Sezgin and İlgin, 2024a)

Theorem 3.20 If f_s is an SI-almost Q-ideal, then $supp(f_s)$ is an almost Q-ideal.

Proof: Assume that f_s is an SI-almost Q-ideal. Thus, $[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \neq \emptyset_s$ for all $x, y \in S$. In order to show that $supp(f_s)$ is an almost Q-ideal, by Theorem 3.18, it is enough to show that $S_{supp(f_s)}$ is an SI-almost Q-ideal. By Lemma 3.19,

$$[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \tilde{\subseteq} [(S_{supp(f_s)} \circ S_x) \tilde{\cap} (S_y \circ S_{supp(f_s)})] \tilde{\cap} S_{supp(f_s)}$$

and $[(f_s \circ S_x) \tilde{\cap} (S_y \circ f_s)] \tilde{\cap} f_s \neq \emptyset_S$, it implies that $[(S_{supp(f_s)} \circ S_x) \tilde{\cap} (S_y \circ S_{supp(f_s)})] \tilde{\cap} S_{supp(f_s)} \neq \emptyset_S$.

Consequently, $S_{supp(f_s)}$ is an SI-almost Q-ideal and by Theorem 3.18, $supp(f_s)$ is an almost Q-ideal. Here note that the converse of Theorem 3.20 is not true in general as shown in the following example.

Example 3.21 We know that f_s is not an SI-almost Q-ideal in Example 3.4 and it is obvious that $supp(f_s) = \{k, c, y\} = S$. It is obvious that $[supp(f_s)\{x\} \cap \{y\}supp(f_s)] \cap supp(f_s) \neq \emptyset$ for all $x, y \in S$. That is to say, $supp(f_s)$ is an almost Q-ideal; although f_s is not an SI-almost Q-ideal.

Definition 3.22 An SI-almost Q-ideal f_S is called a minimal if for any SI-almost Q-ideal h_S whenever $h_S \tilde{\subseteq} f_S$, then $supp(h_S) = supp(f_S)$.

Theorem 3.23 A is a minimal almost Q-ideal if and only if S_A, the soft characteristic function of A, is a minimal SI-almost Q-ideal, where $\emptyset \neq A \subseteq S$.

Proof: Assume that A is a minimal almost Q-ideal. Thus, A is an almost Q-ideal, and so S_A is an SI-almost Q-ideal by Theorem 3.18. Let f_s be an SI-almost Q-ideal such that $f_s \tilde{\subseteq} S_A$. By Theorem 3.20, $supp(f_s)$ is an almost Q-ideal, and by Note 2.6, and Corollary 2.10,

$$supp(f_s) \subseteq supp(S_A) = A.$$

Since A is a minimal almost Q-ideal, $supp(f_s) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost Q-ideal by Definition 3.22.

Conversely, let S_A be a minimal SI-almost Q-ideal. Thus, S_A is an SI-almost Q-ideal, and A is an almost Q-ideal by Theorem 3.18. Let B be an almost Q-ideal such that $B \subseteq A$. By Theorem 3.18, S_B is an SI-almost Q-ideal, and by Theorem 2.11. (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost Q-ideal,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.10. Thus, A is a minimal almost Q-ideal.

Definition 3.24 Let f_s, g_s, and h_s be any SI-almost Q-ideals. If $h_s \circ g_s \tilde{\subseteq} f_s$ implies that $h_s \tilde{\subseteq} f_s$ or $g_s \tilde{\subseteq} f_s$, then f_s is called an SI-prime almost Q-ideal.

Definition 3.25 Let f_s and h_s be any SI-almost Q-ideals. If $h_s \circ h_s \tilde{\subseteq} f_s$ implies that $h_s \tilde{\subseteq} f_s$, then f_s is called an SI-semiprime almost Q-ideal.

Definition 3.26 Let f_s, g_s, and h_s be any SI-almost Q-ideals. If $(h_s \circ g_s) \tilde{\cap} (g_s \circ h_s) \tilde{\subseteq} f_s$ implies that $h_s \tilde{\subseteq} f_s$ or $g_s \tilde{\subseteq} f_s$, then f_s is called an SI-strongly prime almost Q-ideal.

It is obvious that every SI-strongly prime almost Q-ideal is an SI-prime almost Q-ideal, and every SI-prime almost Q-ideal is a soft semiprime almost Q-ideal.

Theorem 3.27 If S_P, the soft characteristic function of P, is an SI-prime almost Q-ideal, then P is a prime almost Q-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost Q-ideal. Thus, S_P is an SI-almost Q-ideal and thus, P is an almost Q-ideal by Theorem 3.18. Let A and B be almost Q-ideals such that $AB \subseteq P$. Thus, by Theorem 3.18, S_A and S_B are SI-almost Q-ideals, and by Theorem 2.11. (i) and (iii), $S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P$. Since S_P is an SI-prime almost Q-ideal and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.11. (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost Q-ideal.

Theorem 3.28 If S_P, the soft characteristic function of P, is an SI-semiprime almost Q-ideal, then P is a semiprime almost Q-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Similar to the proof of Theorem 3.27.

Theorem 3.29 If S_P, the soft characteristic function of P, is an SI-strongly prime almost Q-ideal, then P is a strongly prime almost Q-ideal, where $\emptyset \neq P \subseteq S$.

Proof: Similar to the proof of Theorem 3.27.

4. Conclusions

In this study, we introduced the concept of soft intersection almost quasi-ideal and its generalization "soft intersection weakly almost quasi-ideal" and studied their basic properties. We illustrate that every soft intersection almost quasi-ideal is a soft intersection weakly almost quasi-ideal; and that every soft intersection almost quasi-ideal is a soft intersection almost ideal of semigroup; nevertheless, the converses do not hold as demonstrated with counterexamples. Also, it was shown that an idempotent soft intersection almost quasi-ideal is both a soft intersection almost weak interior ideal and a soft intersection almost subsemigroup. We obtained the relations between soft intersection almost quasi-ideal of a semigroup and almost quasi-ideal of a semigroup according to minimality, primeness, semiprimeness, and strongly primeness with the theorem that if a nonempty set A is almost quasi-ideal, then its soft characteristic function is soft intersection almost quasi-ideal and vice versa. In the following studies, many types of soft intersection almost ideals, consisting of interior ideal, bi-ideal, bi-interior ideal, bi-quasi ideal, quasi-interior ideal, and bi-quasi-interior ideal of semigroups may be examined.

Acknowledgements

This paper is a part of the second author's MSc. Thesis at Amasya University, Amasya, Türkiye.

References

Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers Mathematics with Applications, 57*(9), 1547-1553.

Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers and Mathematics with Applications, 61*(9), 2647–2654.

Atagün, A. O., Kamacı, Hüseyin, Taştekin, İ., & Sezgin, A. (2019). P-properties in Near-rings. *Journal of Mathematical and Fundamental Sciences, 51*(2), 152-167.

Atagün, A. O., & Sezer, A. S. (2015). Soft sets, soft semimodules and soft substructures of semimodules. *Mathematical Sciences Letters, 4*(3), 235-242.

Atagün, A. O., & Sezgin, A. (2017). Int-soft substructures of groups and semirings with applications. *Applied Mathematics and Information Sciences, 11*(1), 105-113.

Atagün, A. O., & Sezgin, A. (2018). A new view to near-ring theory: Soft near-rings. *South East Asian Journal of Mathematics and Mathematical Sciences, 14*(3), 1-14.

Atagün, A. O., & Sezgin, A. (2022). More on prime, maximal and principal soft ideals of soft rings. *New Mathematics and Natural Computation*, *18*(01), 195-207.

Bogdanovic, S. (1981). Semigroups in which some bi-ideal is a group. *Zbornik radova PMF Novi Sad, 11*(81), 261–266.

Çağman, N., Çıtak, F., & Aktaş, H. (2012). Soft int-group and its applications to group theory. *Neural Computing and Applications, 2*, 151–158.

Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni-int decision making. *European Journal of Operational Research*, *207*(2), 848-855.

Chinram, R., & Nakkhasen, W. (2022). Almost bi-quasi-interior ideals and fuzzy almost bi-quasi-interior ideals of semigroups. *Journal of Mathematics and Computer Science, 26*, 128–136.

Deepika, M., Elavarasan, B., & Catherine Grace John, J. (2024). Hybrid quasi-ideals and hybrid A-ideals in ternary semigroups A-ideals and fuzzy A-ideals of ternary semigroups. *Songklanakarin Journal of Science and Technology, 46*(1), 16-23.

Feng, F., Jun, Y. B., & Zhao, X. (2008). Soft semirings. *Computers and Mathematics with Applications, 56*(10), 2621-2628.

Gaketem, T. (2022). Almost bi-interior ideal in semigroups and their fuzzifications. *European Journal of Pure and Applied Mathematics*, *15*(1), 281-289.

Gaketem, T., & Chinram, R. (2023). Almost bi-quasi ideals and their fuzzifcations in semigroups. *Annals of the University of Craiova, Mathematics and Computer Science Series*, *50*(2), 342-352.

Good, R. A., & Hughes, D. R. (1952). Associated groups for a semigroup. *Bulletin of the American Mathematical Society, 58*, 624-625.

Grosek, O., & Satko, L. (1980). A new notion in the theory of semigroup. *Semigroup Forum, 20*, 233–240.

Gulistan, M., Feng, F., Khan, M., & Sezgin, A. (2018). Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. *Mathematics, 6*(12), 293.

Iampan, A., Chinram, R., & Petchkaew, P. (2021). A note on almost subsemigroups of semigroups. *International Journal of Mathematics and Computer Science, 16*(4), 1623–1629.

Jana, C., Pal, M., Karaaslan, F., & Sezgin, A. (2019). (α, β)-soft intersectional rings and ideals with their applications. *New Mathematics and Natural Computation, 15*(2), 333–350.

Kaopusek, N., Kaewnoi, T., & Chinram, R. (2020). On almost interior ideals and weakly almost interior ideals of semigroups. *Journal of Discrete Mathematical Sciences and Cryptography, 23*, 773–778.

Khan, A., Izhar, M., & Sezgin, A. (2017). Characterizations of Abel Grassmann's groupoids by the properties of their double-framed soft ideals. *International Journal of Analysis and Applications, 15*(1), 62-74.

Krailoet, W., Simuen, A., Chinram, R., & Petchkaew, P. (2021). A note on fuzzy almost interior ideals in semigroups. *International Journal of Mathematics and Computer Science, 16*, 803–808.

Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and Mathematics with Applications, 45*(4-5), 555-562.

Manikantan, T., Ramasamy, P., & Sezgin, A. (2023). Soft quasi-ideals of soft near-rings. *Sigma Journal of Engineering and Natural Sciences, 41*(3), 565-574.

Molodtsov, D. (1999). Soft set theory-first results. *Computers and Mathematics with Applications, 37*(4-5), 19–31.

Özlü, Ş., & Sezgin, A. (2020). Soft covered ideals in semigroups. *Acta Universitatis Sapientiae Mathematica, 12*(2), 317-346.

Pei, D., & Miao, D. (2005). From soft sets to information systems. *IEEE International Conference on Granular Computing, 2*, 617-621.

Rao, M. M. K. (2018a). Bi-interior ideals of semigroups. *Discussiones Mathematicae General Algebra and Applications, 38,* 69–78.

Rao, M. M. K. (2018b). A study of a generalization of bi-ideal, quasi-ideal and interior ideal of semigroup. *Mathematica Moravica, 22*, 103–115.

Rao, M. M. K. (2020a). Left bi-quasi ideals of semigroups. *Southeast Asian Bulletin of Mathematics, 44*, 369–376.

Rao, M. M. K. (2020b). Quasi-interior ideals and weak-interior ideals. *Asia Pacific Journal of Mathematics, 7*(21), 1-20.

Riaz, M., Hashmi, M. R., Karaaslan, F., Sezgin, A., Shamiri, M. M. A. A., & Khalaf, M. M. (2023). Emerging trends in social networking systems and generation gap with neutrosophic crisp soft mapping. *CMES-computer Modeling in Engineering and Sciences*, *136*(2), 1759-1783.

Sezer, A. S., Atagün, A. O., & Çağman, N. (2013). A new view to N-group theory: Soft N-groups. *Fasciculi Mathematici*, *51*, 123-140.

Sezer, A. S., Atagün, A. O., & Çağman, N. (2014). N-group SI-action and its applications to N-Group theory. *Fasciculi Mathematici, 52*, 139-153.

Sezer, A. S., Çağman, N., & Atagün, A. O. (2014). Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; a new approach to semigroup theory II. *Journal of Multiple-valued Logic and Soft Computing, 23*(1-2), 161-207.

Sezer, A. S., Çağman, N., Atagün, A. O., Ali, M. I., & Türkmen, E. (2015). Soft intersection semigroups, ideals and bi-ideals; a new application on semigroup theory I. *Filomat, 29*(5), 917-946.

Sezgin, A. (2018). A new view on AG-groupoid theory via soft sets for uncertainty modeling. *Filomat, 32*(8), 2995–3030.

Sezgin, A., Ahmad, S., & Mehmood, A. (2019). A new operation on soft sets: Extended difference of soft sets. *Journal of New Theory, 27*, 33-42.

Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. *Computers and Mathematics with Applications*, *61*(5), 1457-1467.

Sezgin, A., Atagün, A. O., & Çağman, N. (2025). A complete study on and-product of soft sets, *Sigma Journal of Engineering and Natural Sciences, 43*(1), 1-14.

Sezgin, A., Atagün, A. O., Çağman, N., & Demir, H. (2022). On near-rings with soft union ideals and applications. *New Mathematics and Natural Computation, 18*(2), 495-511.

Sezgin, A., & Aybek, F. N. (2024). New restricted and extended soft set operations: Restricted gamma and extended gamma operations. *Big Data and Computing Visions, 4*(4), 272-306.

Sezgin, A., Aybek, F. N., & Atagün, A. O. (2023). New soft set operation: Complementary soft binary piecewise intersection operation. *Black Sea Journal of Engineering and Science, 6*(4), 330-346.

Sezgin, A., Aybek, F. N., & Güngör, N. B. (2023). New soft set operation: Complementary soft binary piecewise union operation. *Acta Informatica Malaysia, 7*(1), 38-53.

Sezgin, A., Çağman, N., & Atagün, A. O. (2017). A completely new view to soft intersection rings via soft uni-int product. *Applied Soft Computing, 54*, 366-392.

Sezgin, A., Çağman, N., Atagün, A. O., & Aybek, F. N. (2023). Complemental binary operations of sets and their application to group theory. *Matrix Science Mathematic, 7*(2), 114-121.

Sezgin, A., & Çalışıcı, H. (2024). A comprehensive study on soft binary piecewise difference operation. *Eskişehir Teknik Üniversitesi Bilim ve Teknoloji Dergisi B-Teorik Bilimler, 12*(1), 32-54.

Sezgin, A., & Dagtoros, K. (2023). Complementary soft binary piecewise symmetric difference operation: A novel soft set operation. *Scientific Journal of Mehmet Akif Ersoy University*, *6*(2), 31-45.

Sezgin, A., & İlgin, A. (2024a). Soft intersection almost subsemigroups of semigroups. *International Journal of Mathematics and Physics, 15*(1), 13-20.

Sezgin, A., & İlgin, A. (2024b). Soft intersection almost ideals of semigroups. *Journal of Innovative Engineering and Natural Science, 4*(2), 466-481.

Sezgin, A., & İlgin, A. (2024c). Soft intersection almost weak-interior ideals of semigroups: A theoretical study. *JNSM Journal of Natural Sciences and Mathematics of UT, 9*(17-18), 372-385.

Sezgin, A., & Orbay, M. (2022). Analysis of semigroups with soft intersection ideals. *Acta Universitatis Sapientiae Mathematica, 14*(1), 166-210.

Sezgin, A., & Sarıalioğlu, M. (2024). Complementary extended gamma operation: A new soft set operation. *Natural and Applied Sciences Journal, 7*(1), 15-44.

Sezgin, A., & Şenyiğit, E. (2025). A new product for soft sets with its decision-making: Soft star-product. *Big Data and Computing Visions, 5*(1), 52-73.

Sezgin, A., & Yavuz, E. (2023). A new soft set operation: Soft binary piecewise symmetric difference operation. *Necmettin Erbakan University Journal of Science and Engineering, 5*(2), 189-208.

Sezgin, A., & Yavuz, E. (2024). Soft binary piecewise plus operation: A new type of operation for soft sets. *Uncertainty Discourse and Applications, 1*(1), 79-100.

Steinfeld, O. (1956). Uher die quasi ideals. *Von halbgruppend Publication Mathematical Debrecen, 4,* 262-275.

Stojanovic, N. S. (2021). A new operation on soft sets: Extended symmetric difference of soft sets. *Military Technical Courier, 69*(4), 779-791.

Suebsung, S., Wattanatripop, K., & Chinram, R. (2019). A-ideals and fuzzy A-ideals of ternary semigroups. *Songklanakarin Journal of Science and Technology, 41*(2), 299-304.

Wattanatripop, K., Chinram, R., & Changphas, T. (2018a). Quasi-A-ideals and fuzzy A-ideals in semigroups. *Journal of Discrete Mathematical Sciences and Cryptography, 21*(5), 1131–1138.

Wattanatripop, K., Chinram, R., & Changphas, T. (2018b). Fuzzy almost bi-ideals in semigroups. *International Journal of Mathematics and Computer Science, 13*(1), 51–58.