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Original Article

Hybrid quasi-ideals and hybrid A-ideals in ternary semigroups

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Abstract

We are completely aware of the fact that ternary algebraic structures in some way naturally occur in diverse fields of theoretical and mathematical physics, computer science, especially in ternary operators in Java and Python, ternary logic applied for digital signal processing, and musical systems for song composition. The primary aim of this research is to examine ternary semigroups in the context of hybrid quasi-ideals and hybrid A-ideals. Different features of hybrid quasi-ideals and hybrid A-ideals in the ternary semigroup are discussed. In addition, the idea of minimal hybrid A-ideals of ternary semigroups is defined.

Keywords: ternary semigroups, quasi-ideals, hybrid quasi-ideals, A-ideals, hybrid A-ideals

1. Introduction

Ternary algebraic structures spontaneously occur in a number of areas of theoretical and mathematical physics. Ternary semigroups have applications and implications in various areas of mathematics and beyond. Dixit and Dewan (1995) presented quasi-ideals in ternary semigroups and studied their properties. Grosek and Satko (1980) proposed the notion of an A-ideal in semigroups. Numerous academics have looked into the ideas of ternary semigroups (Dutta, Kar, & Maity, 2008; Kar & Maity, 2007; Lehmer, 1932: Los, 1955; Santigo, 1990; Sioson, 1965).

Using a parallel approach to set theory, Zadeh (1965) proposed an idea of fuzzy subsets and investigated its properties. This idea was applied in group theory and semigroup theory by Rosenfeld (1975). Fuzzy ideals were first proposed in ternary semigroups by Kar and Sarkar (2012) who examined their associated features. Kar and Sarkar (2012) presented both the fuzzy quasi-ideal and fuzzy bi-ideal of a ternary semigroup and investigated the associated features of these two ternary semigroup subsystems. Suebsung, Wattanatripop, and Chinram (2019) defined and investigated several characteristics of A-ideals and fuzzy A-ideals in ternary semigroups. Many researchers have applied the concepts in decision making problems (Ozlu, S. 2022, 2022, 2023, 2023).

Molodtsov (1999) proposed soft set theory that offers a novel method for dealing with unpredictability and is free from the issues that have demolished the traditional theoretical methods. Jun, Sang, and Muhiuddin (2018) compared the relationships among soft sets and fuzzy sets. The ideas of hybrid sub-semigroups and hybrid ideals in semigroups were presented by Anis, Khan, and Jun (2017), and a number of features were examined. Elavarasan, Porselvi, and Jun (2019) were the first to discuss hybrid generalized bi-ideals in semigroups. Modules over semi rings were examined in terms of hybrid structure by Muhiuddin, Catherine Grace John, Elavarasan, Jun, and Porselvi (2022). The concept of hybrid interior ideals and bi-ideals in ternary semigroups are proposed by Catherine Grace John, Deepika, and Elavarasan (2023) who also examined their features.

The goal of the current work is to offer hybrid quasi-ideals and hybrid A-ideals in ternary semigroups and to use these ideals to explore the characteristics of ternary semigroups.

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2. Preliminaries

A nonempty set D is called a ternary semigroup if there is a ternary operation $D \times D \times D \to D$ written as $(g,m,b) \mapsto gmb$ satisfying the following identity

$$(gmb)fe = g(mbf)e = gm(bfe), \text{ for all } g,m,b,d,e \in D.$$

D will stand for a ternary semigroup throughout this paper unless otherwise stated.

Definition 2.1. Dixit and Dewan (1995) Let $R(\neq \emptyset) \subseteq D$. Then

(i) R is a *ternary subsemigroup* of D if $RRR \subseteq R$.
(ii) R is a *left ideal* of D if $DDR \subseteq R$.
(iii) R is a *right ideal* of D if $RDD \subseteq R$.
(iv) R is a *lateral ideal* of D if $DRD \subseteq R$.
If R is a left, right and lateral ideal of D, then it is called an ideal of D.

Definition 2.2. Dixit and Dewan (1995) Let $R(\neq \emptyset) \subseteq D$. Then R is a *quasi-ideal* of D if $(RDD \cap DRD \cap DDR) \subseteq R$ and $(RDD \cap DDRDD \cap DDR) \subseteq R$ and the above can be written as follows: $(RDD \cap (DRD \cup DDRDD) \cap DDR) \subseteq R$.

Definition 2.3 Dutta, Kar, and Maity (2008) The element $g \in D$ is a *ternary idempotent* if $g^3 = g$.

Definition 2.4 Grosek and Satko, (1980) Let $R(\neq \emptyset) \subseteq D$. Then

(i) R is a left A-ideal of D if $ggR \cap R \neq \phi, \forall\, g \in D$.
(ii) R is a right A-ideal of D if $Rgg \cap R \neq \phi, \forall\, g \in D$.
(iii) R is a lateral A-ideal of D if $gRg \cap R \neq \phi, \forall\, g \in D$.
If R is a left, right, and lateral A-ideal of D, then R is an A-ideal of D.

3. Hybrid Structures in Ternary Semigroup

We give a few basic definitions of hybrid structures in this part, which are required to back up the key conclusions. In the explanation that follows, the symbol I stands for unit interval $[0,1]$, $H(D)$ stands for the gathering of all hybrid structures in D, and $P(W)$ stands for all subsets of the universal set W.

Definition 3.1. Anis, Khan, and Jun (2017) A hybrid structure in D over W is a mapping

$$\tilde{t}_\delta = (\tilde{t}, \delta) : D \to P(W) \times I, g \mapsto (\tilde{t}(g), \delta(g))$$

where $\tilde{t} : D \to P(W)$ and $\delta : D \to I$ are mappings.
To establish a partial order relation $\ll$ in $H(D)$, we state the following:

$$\left(\forall \quad \tilde{t}_\delta, \tilde{y}_\varrho \in H(D)\right)(\tilde{t}_\delta \ll \tilde{y}_\varrho \Leftrightarrow \tilde{t} \,\tilde{\subseteq}\, \tilde{y}, \delta \geq \varrho)$$

where $\tilde{t} \,\tilde{\subseteq}\, \tilde{y}$, means $\tilde{t}(g) \subseteq \tilde{y}(g)$ and $\delta \geq \varrho$ means $\delta(g) \geq \varrho(g)$ $\forall\, g \in D$. Then, the set $(H(D), \ll)$ is a partially ordered set.

Definition 3.2. Anis, Khan, and Jun (2017) Let $\tilde{y}_\varrho, \tilde{k}_\tau, \tilde{t}_\delta \in H(D)$. Then the hybrid product of $\tilde{y}_\varrho \odot \tilde{k}_\tau \odot \tilde{t}_\delta$ is defined as follows. For any $g \in D$,

$$(\tilde{y} \,\tilde{\circ}\, \tilde{k} \,\tilde{\circ}\, \tilde{t})(g) = \begin{cases} \bigcup\limits_{g=mnf} \{\tilde{y}(m) \cap \tilde{k}(n) \cap \tilde{t}(f)\} & if\ g = mnf \\ \emptyset & otherwise, \end{cases}$$

$$(\varrho \,\tilde{\circ}\, \tau \,\tilde{\circ}\, \delta)(g) = \begin{cases} \bigwedge\limits_{g=mnf} \{\varrho(m) \vee \tau(n) \vee \delta(f)\} & if\ g = mnf \\ 1 & otherwise \end{cases}$$

$for\ m,n,f \in D.$

Definition 3.3. Anis, Khan, and Jun (2017) Let $\tilde{y}_\varrho$, $\tilde{t}_\delta \in H(D)$. Then the notation $\tilde{y}_\varrho \Cap \tilde{t}_\delta$ denotes the hybrid intersection of $\tilde{y}_\varrho$ and $\tilde{t}_\delta$, its hybrid structure defined by

$$\tilde{y}_\varrho \Cap \tilde{t}_\delta : D \to P(W) \times I, g \mapsto ((\tilde{y} \,\tilde{\cap}\, \tilde{t})(g), (\varrho \vee \delta)(g))$$

where $\tilde{y} \,\tilde{\cap}\, \tilde{t} : D \to P(W), g \mapsto \tilde{y}(g) \cap \tilde{t}(g)$,
$\varrho \vee \delta : D \to I, g \mapsto \varrho(g) \vee \delta(g)$ for all $g \in D$.

Definition 3.4. Anis, Khan, and Jun (2017) Let $\tilde{y}_\varrho,\ \tilde{t}_\delta \in H(D)$. Then the notation $\tilde{y}_\varrho \Cup \tilde{t}_\delta$ denotes the hybrid union of $\tilde{y}_\varrho$ and $\tilde{t}_\delta$, its hybrid structure defined by

$$\tilde{y}_\varrho \Cup \tilde{t}_\delta : D \to P(W) \times I, g \mapsto ((\tilde{y} \,\tilde{\cup}\, \tilde{t})(g), (\varrho \wedge \delta)(g))$$

where $\tilde{y} \,\tilde{\cup}\, \tilde{t} : D \to P(W), g \mapsto \tilde{y}(g) \cup \tilde{t}(g)$,
$\varrho \wedge \delta : D \to I, g \mapsto \varrho(g) \wedge \delta(g)$ for all $g \in D$.

Definition 3.5. Anis, Khan, and Jun (2017) For $\tilde{t}_\delta \in H(D)$ and $R \in P(W)\backslash\{\phi\}$, the characteristic hybrid structure $\chi_R(\tilde{t}_\delta)$ in D over W is represented by $\chi_R(\tilde{t}_\delta) = (\chi_R(\tilde{t}), \chi_R(\delta))$, where

$$\chi_R(\tilde{t}): D \to P(W), g \mapsto \begin{cases} W & if\ g \in R \\ \phi & otherwise, \end{cases} \quad \chi_R(\delta): D \to I, g \mapsto \begin{cases} 0 & if\ g \in R \\ 1 & otherwise. \end{cases}$$

$\chi_D(\tilde{t}_\delta)$ is called the identity hybrid structure in D over W if $\chi_D(\tilde{t}_\delta) = D$, i.e., $\chi_D(\tilde{t})(g) = W,\ \chi_D(\delta)(g) = 0$.

Definition 3.6. Catherine Grace John *et al.*, (2023) A hybrid structure $\tilde{t}_\delta$ is a hybrid subsemigroup of D if

$$(\forall\, m, n, f \in D) \begin{pmatrix} \tilde{t}(mnf) \supseteq \tilde{t}(m) \cap t(n) \cap t(f) \\ \delta(mnf) \le \delta(m) \vee \delta(n) \vee \delta(f) \end{pmatrix}.$$

Definition 3.7. Catherine Grace John *et al.*, (2023) A hybrid structure $\tilde{t}_\delta$ is a hybrid ideal of D if it fulfills the following conditions:

$(A1)\ (\forall\, m, n, f \in D) \begin{pmatrix} \tilde{t}(mnf) \supseteq \tilde{t}(f) \\ \delta(mnf) \le \delta(f) \end{pmatrix}.$

$(A2)\ (\forall\, m, n, f \in D) \begin{pmatrix} \tilde{t}(mnf) \supseteq \tilde{t}(m) \\ \delta(mnf) \le \delta(m) \end{pmatrix}.$

$(A3)\ (\forall\, m, n, f \in D) \begin{pmatrix} \tilde{t}(mnf) \supseteq \tilde{t}(n) \\ \delta(mnf) \le \delta(n) \end{pmatrix}.$

Note that $\tilde{t}_\delta$ is referred to as a hybrid left ideal of D if it meets requirement (A1), $\tilde{t}_\delta$ is referred to as a hybrid right ideal of D if it meets requirement (A2), and $\tilde{t}_\delta$ is referred to as a hybrid lateral ideal of D if it meets requirement (A3).

$\tilde{t}_\delta$ is a two-sided hybrid ideal if it is meets requirements (A1) and (A2).

Definition 3.8. Anis, Khan, and Jun (2017) Let $\tilde{t}_\delta \in H(D)$. Then for any $\Gamma \in P(W)$ and $\xi \in I$, the set

$$\tilde{t}_\delta[\Gamma, \xi] := \{g \in D : \tilde{t}(g) \supseteq \Gamma \text{ and } \delta(g) \le \xi\}$$

is called the $[\Gamma, \xi] - hybrid\ cut$ in D over W.

4. Hybrid Quasi-Ideals in Ternary Semigroup

Fuzzy quasi-ideals in ternary semigroup were initially discussed by Dixit and Dewan (1995). We define hybrid quasi-ideals in D over W as follows:

Definition 4.1. Let $\tilde{y}_\varrho \in H(D)$. Then $\tilde{y}_\varrho$ of D is a hybrid quasi-ideal if it meets the conditions below:

$(i)\quad (D \odot D \odot \tilde{y}_\varrho) \Cap (D \odot \tilde{y}_\varrho \odot D) \Cap (\tilde{y}_\varrho \odot D \odot D) \ll \tilde{y}_\varrho,$

$(ii)\quad (D \odot D \odot \tilde{y}_\varrho) \Cap (D \odot D \odot \tilde{y}_\varrho \odot D \odot D) \Cap (\tilde{y}_\varrho \odot D \odot D) \ll \tilde{y}_\varrho.$

Example 4.2. Take $D = \{i, 0, -i\}$ as a ternary semigroup with common complex multiplication. Define the hybrid structure $\tilde{y}_\varrho$ in D over the set $W = \{m_1, m_2, m_3, m_4, m_5\}$ by:

$$\tilde{y}(g) = \begin{cases} \{m_1, m_4, m_5\} & if\ g = i, -i \\ \{m_1, m_2, m_4\} & if\ g = 0, \end{cases} \text{ and } \varrho(g) = \begin{cases} 0.5 & if\ g = i, -i \\ 0.3 & if\ g = 0. \end{cases}$$

Then $\tilde{y}_\varrho$ is a hybrid quasi-ideal. It is clear that every hybrid ideal is a hybrid quasi-ideal of D.

The following example demonstrate that, a hybrid quasi-ideal need not always be a hybrid ideal of D.

Example 4.3. Let $D = \{i, 0, -i\}$ be a ternary semigroup under complex multiplication and let $\tilde{y}_\varrho$ be a hybrid structure in D over the set $W = \{m_1, m_2, m_3, m_4, m_5\}$ by:

$$\tilde{y}(g) = \begin{cases} \{m_1, m_4, m_5\} & if\ g = i, -i \\ \{m_1, m_4\} & if\ g = 0, \end{cases} \text{ and } \varrho(g) = \begin{cases} 0.4 & if\ g = i, -i \\ 0.6 & if\ g = 0. \end{cases}$$

Then $\tilde{y}_\varrho$ of D is a hybrid quasi-ideal, but not a hybrid ideal of D because $\tilde{y}(-i \times 0 \times i) \nsupseteq \tilde{y}(-i)$ and $\varrho(-i \times 0 \times i) \nleq \varrho(-i)$; and $\tilde{y}(-i \times 0 \times i) \nsupseteq \tilde{y}(i)$ and $\varrho(-i \times 0 \times i) \nleq \varrho(i)$.

Theorem 4.4. For $\tilde{t}_\delta,\ \tilde{y}_\varrho \in H(D)$ and $B, F, N \in P(W)\backslash\{\emptyset\}$, the conditions below hold:

(i) $\chi_B(\tilde{t}_\delta) \Cap \chi_F(\tilde{t}_\delta) \Cap \chi_N(\tilde{t}_\delta) = \chi_{B\cap F\cap N}(\tilde{t}_\delta)$.
(ii) $\chi_B(\tilde{t}_\delta) \odot \chi_F(\tilde{t}_\delta) \odot \chi_N(\tilde{t}_\delta) = \chi_{BFN}(\tilde{t}_\delta)$.

Proof: (i) Let $g \in D$.Then $\big(\chi_B(\tilde{t}) \tilde{\cap}\, \chi_F(\tilde{t}) \tilde{\cap}\, \chi_N(\tilde{t})\big)(g) = \chi_B(\tilde{t})(g) \cap \chi_F(\tilde{t})(g) \cap \chi_N(\tilde{t})(g)$ and $\big(\chi_B(\delta) \vee \chi_F(\delta) \vee \chi_N(\delta)\big)(g) = \chi_B(\delta)(g) \vee \chi_F(\delta)(g) \vee \chi_N(\delta)(g)$.

If $g \in B \cap F \cap N$, then $\chi_{B\cap F\cap N}(\tilde{t})(G) = W = \chi_B(\tilde{t})(g) \cap \chi_F(\tilde{t})(g) \cap \chi_N(\tilde{t})(g) = \big(\chi_B(\tilde{t}) \tilde{\cap}\, \chi_F(\tilde{t}) \tilde{\cap}\, \chi_N(\tilde{t})\big)(g)$ and $\chi_{B\vee F\vee N}(\delta)(g) = 0 = \chi_B(\delta)(g) \vee \chi_F(\delta)(g) \vee \chi_N(\delta)(g) = \big(\chi_B(\delta) \vee \chi_F(\delta) \vee \chi_N(\delta)\big)(g)$.

If $g \notin B \cap F \cap N$, then $\chi_{B\cap F\cap N}(\tilde{t})(G) = \emptyset = \chi_B(\tilde{t})(g) \cap \chi_F(\tilde{t})(g) \cap \chi_N(\tilde{t})(g) = \big(\chi_B(\tilde{t}) \tilde{\cap}\, \chi_F(\tilde{t}) \tilde{\cap}\, \chi_N(\tilde{t})\big)(g)$ and $\chi_{B\vee F\vee N}(\delta)(g) = 1 = \chi_B(\delta)(g) \vee \chi_F(\delta)(g) \vee \chi_N(\delta)(g) = \big(\chi_B(\delta) \vee \chi_F(\delta) \vee \chi_N(\delta)\big)(g)$. So, $\chi_B(\tilde{t}_\delta) \Cap \chi_F(\tilde{t}_\delta) \Cap \chi_N(\tilde{t}_\delta) = \chi_{B\cap F\cap N}(\tilde{t}_\delta)$.

(ii) Let $g \in D$. If $g = xmp$ for some $x, m, p \in D$, then

$$\big(\chi_B(\tilde{t})\tilde{o}\chi_F(\tilde{t})\tilde{o}\chi_N(\tilde{t})\big)(g) = \bigcup_{a=xmp} \{\chi_B(\tilde{t})(x) \cap \chi_F(\tilde{t})(m) \cap \chi_N(\tilde{t})(p)\},$$
$$\big(\chi_B(\delta)\tilde{o}\chi_F(\delta)\tilde{o}\chi_N(\delta)\big)(g) = \bigwedge_{g=xmp} \{\chi_B(\delta)(x) \vee \chi_F(\delta)(m) \vee \chi_N(\delta)(p)\}.$$

Case 1: If there exists no $x \in B, m \in F, p \in N$ such that $g = xmp$, then $g \notin BFN$ and hence $\big(\chi_B(\tilde{t})\tilde{o}\chi_F(\tilde{t})\tilde{o}\chi_N(\tilde{t})\big)(g) = \emptyset = \chi_{BFN}(\tilde{t})(g)$ and $\big(\chi_B(\delta)\tilde{o}\chi_F(\delta)\tilde{o}\chi_N(\delta)\big)(g) = 1 = \chi_{BFN}(\delta)(g)$.

Case 2: If there exists g such that $g = nqr$, where $n \in B, q \in F, r \in N,$ then $g \in BFN$ and $\chi_B(\tilde{t})(n) = \chi_F(\tilde{t})(q) = \chi_N(\tilde{t})(r) = \chi_{BFN}(\tilde{t}) = W$ and $\chi_B(\delta) = \chi_F(\delta) = \chi_N(\delta) = \chi_{BFN}(\delta) = 0$. Therefore

$$\begin{aligned}\big(\chi_B(\tilde{t})\tilde{o}\chi_F(\tilde{t})\tilde{o}\chi_N(\tilde{t})\big)(g) &= \bigcup_{g=xmp} \{\chi_B(\tilde{t})(x) \cap \chi_F(\tilde{t})(m) \cap \chi_N(\tilde{t})(p)\}\\ &\supseteq \chi_B(\tilde{t})(n) \cap \chi_F(\tilde{t})(q) \cap \chi_N(\tilde{t})(r)\\ &= W = \chi_{BFN}(\tilde{t})(g),\\ \big(\chi_B(\delta)\tilde{o}\chi_F(\delta)\tilde{o}\chi_N(\delta)\big)(g) &= \bigwedge_{g=xmp} \{\chi_B(\delta)(x) \vee \chi_F(\delta)(m) \vee \chi_N(\delta)(p)\}\\ &\le \chi_B(\delta)(n) \vee \chi_F(\delta)(q) \vee \chi_N(\delta)(r)\\ &= 0 = \chi_{BFN}(\delta)(g).\end{aligned}$$

For $g \in D$, if $g \neq xmp$, for any $x, m, p \in D$, then $\big(\chi_B(\tilde{t})\tilde{o}\chi_F(\tilde{t})\tilde{o}\chi_N(\tilde{t})\big)(g) = \emptyset = \chi_{BFN}(\tilde{t})(g)$ and $\big(\chi_B(\delta)\tilde{o}\chi_F(\delta)\tilde{o}\chi_N(\delta)\big)(g) = 1 = \chi_{BFN}(\delta)(g)$.

From the above two cases it follows that $\chi_B(\tilde{t}_\delta) \odot \chi_F(\tilde{t}_\delta) \odot \chi_N(\tilde{t}_\delta) = \chi_{BFN}(\tilde{t}_\delta)$.

Theorem 4.5. For $\tilde{y}_\varrho \in H(D)$ and $R \in P(W)\backslash\{\emptyset\}$, the following conditions are equivalent:
(i) $\chi_R(\tilde{y}_\varrho)$ of D is a hybrid quasi-ideal,
(ii) R of D is a quasi-ideal.

Proof: $(i) \Longrightarrow (ii)$ Consider $\chi_R(\tilde{y}_\varrho)$ of D is a quasi-ideal and $g \in (DDR) \cap (DRD) \cap (RDD)$. Now

$$\begin{aligned}&\chi_R(\tilde{y})\,(g)\\ &\quad\supseteq \big[\big(\chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_R(\tilde{y})\big) \cap \big(\chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_R(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y})\big) \cap\\ &\qquad\qquad (\chi_R(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y}))\big](g)\\ &\quad\supseteq \big[\chi_{(DDR)}(\tilde{y}) \cap \chi_{(DRD)}(\tilde{y}) \cap \chi_{(RDD)}(\tilde{y})\big](g) \text{ (Using Theorem 4.4)}\\ &\quad\supseteq \big[\chi_{(DDR)\cap(DRD)\cap(RDD)}(\tilde{y})\big](g) = W,\\ &\chi_R(\varrho)\,(g)\\ &\quad\le \big[\big(\chi_D(\varrho) \,\tilde{\circ}\, \chi_D(\varrho) \,\tilde{\circ}\, \chi_R(\varrho)\big) \vee \big(\chi_D(\varrho) \,\tilde{\circ}\, \chi_R(\varrho) \,\tilde{\circ}\, \chi_D(\varrho)\big) \vee\\ &\qquad\qquad (\chi_R(\varrho) \,\tilde{\circ}\, \chi_D(\varrho) \,\tilde{\circ}\, \chi_D(\varrho))\big](g)\\ &\quad\le \big[\chi_{(DDR)}(\varrho) \vee \chi_{(DRD)}(\varrho) \vee \chi_{(RDD)}(\varrho)\big](g) \text{ (Using Theorem 4.4)}\\ &\quad\le \big[\chi_{(DDR)\vee(DRD)\vee(RDD)}(\varrho)\big](g) = 0,\end{aligned}$$

which imply $g \in R$. Thus $(DDR) \cap (DRD) \cap (RDD) \subseteq R$ and hence R is a quasi-ideal in D.

$(ii) \Longrightarrow (i)$ Let R of D be a quasi-ideal. Then $(DDR) \cap (DRD) \cap (RDD) \subseteq R$ and $(DDR) \cap (DDRDD) \cap (RDD) \subseteq R$. Now

$$\begin{aligned}
&[D \,\tilde{\circ}\, D \,\tilde{\circ}\, \chi_R(\tilde{y})] \cap [D \,\tilde{\circ}\, \chi_R(\tilde{y}) \,\tilde{\circ}\, D] \cap [\chi_R(\tilde{y}) \,\tilde{\circ}\, D \,\tilde{\circ}\, D] \\
&\quad = [\chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_R(\tilde{y})] \cap [\chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_R(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y})] \cap [\chi_R(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y}) \,\tilde{\circ}\, \chi_D(\tilde{y})] \\
&\quad = \chi_{DDR}(\tilde{y}) \cap \chi_{DRD}(\tilde{y}) \cap \chi_{RDD}(\tilde{y}) \qquad \text{(Using Theorem 4.4)} \\
&\quad = \chi_{(DDR)\cap(DRD)\cap(RDD)}(\tilde{y}) \subseteq \chi_R(\tilde{y}), \\
&[D \,\tilde{\circ}\, D \,\tilde{\circ}\, \chi_R(\varrho)] \vee [D \,\tilde{\circ}\, \chi_R(\varrho) \,\tilde{\circ}\, D] \vee [\chi_R(\varrho) \,\tilde{\circ}\, D \,\tilde{\circ}\, D] \\
&\quad = [\chi_D(\varrho) \,\tilde{\circ}\, \chi_D(\varrho) \,\tilde{\circ}\, \chi_R(\varrho)] \vee [\chi_D(\varrho) \,\tilde{\circ}\, \chi_R(\varrho) \,\tilde{\circ}\, \chi_D(\varrho)] \vee [\chi_R(\varrho) \,\tilde{\circ}\, \chi_D(\varrho) \,\tilde{\circ}\, \chi_D(\varrho)] \\
&\quad = \chi_{DDR}(\varrho) \vee \chi_{DRD}(\varrho) \vee \chi_{RDD}(\varrho) \qquad \text{(Using Theorem 4.4)} \\
&\quad = \chi_{(DDR)\vee(DRD)\vee(RDD)}(\varrho) \geq \chi_R(\varrho).
\end{aligned}$$

So, $[\chi_D(\tilde{y}_\varrho) \odot \chi_D(\tilde{y}_\varrho) \odot \chi_R(\tilde{y}_\varrho)] \Cap [\chi_D(\tilde{y}_\varrho) \odot \chi_R(\tilde{y}_\varrho) \odot \chi_D(\tilde{y}_\varrho)] \Cap [\chi_R(\tilde{y}_\varrho) \odot \chi_D(\tilde{y}_\varrho) \odot \chi_D(\tilde{y}_\varrho)] \ll \chi_D(\tilde{y}_\varrho)$.

Theorem 4.6. Let $\tilde{z}_\varpi$ be a hybrid quasi-ideal of D. Then $[\Gamma, \xi]$ −hybrid cut $\tilde{z}_\varpi[\Gamma, \xi]$ is a quasi-ideal of D for every $\Gamma \in P(W)$ and $\xi \in I$.

Proof: Let $\tilde{z}_\varpi$ be a hybrid quasi-ideal of D and $\xi \in I, \Gamma \in P(W)$ and let $g, m, n \in \tilde{z}_\varpi[\Gamma, \xi]$. Then $\tilde{z}(g), \tilde{z}(m), \tilde{z}(n) \supseteq \Gamma, \varpi(g), \varpi(m), \varpi(n) \leq \xi$. Since $\tilde{z}_\varpi$ is a hybrid quasi-ideal of D, we have (i) $\tilde{z}(gmn) \supseteq (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \tilde{z})(gmn) \cap (D \,\tilde{\circ}\, \tilde{z} \,\tilde{\circ}\, D)(gmn) \cap (\tilde{z} \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \supseteq \Gamma$ and $\varpi(gmn) \leq (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \varpi)(gmn) \vee (D \,\tilde{\circ}\, \varpi \,\tilde{\circ}\, D)(gmn) \vee (\varpi \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \leq \xi$.

(ii) $\tilde{z}(gmn) \supseteq (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \tilde{z})(gmn) \cap (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \tilde{z} \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \cap (\tilde{z} \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \supseteq \Gamma, \varpi(gmn) \leq (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \varpi)(gmn) \vee (D \,\tilde{\circ}\, D \,\tilde{\circ}\, \varpi \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \vee (\varpi \,\tilde{\circ}\, D \,\tilde{\circ}\, D)(gmn) \leq \xi$.

Thus $gmn \in \tilde{z}_\varpi[\Gamma, \xi]$. Similarly $mgn, nmg \in \tilde{z}_\varpi[\Gamma, \xi]$. Hence $\tilde{z}_\varpi[\Gamma, \xi]$ is a quasi- ideal of D.

Theorem 4.7. Let $\tilde{y}_{1\beta}, \tilde{y}_{2\varrho}, \tilde{y}_{3\delta}$ be respectively a hybrid right ideal, a hybrid lateral ideal and a hybrid left ideal of D. Then $\tilde{y}_{1\beta} \odot \tilde{y}_{2\varrho} \odot \tilde{y}_{3\delta} \ll \tilde{y}_{1\beta} \Cap \tilde{y}_{2\varrho} \Cap \tilde{y}_{3\delta}$.

Proof. Let $\tilde{y}_{1\beta}, \tilde{y}_{2\varrho}, \tilde{y}_{3\delta}$ be the hybrid right, lateral and left ideals of D respectively, and $g \in D$ be such that $g = xmp \; \forall \, x, m, p \in D$. Consider,

$$\begin{aligned}
(\tilde{y}_1 \,\tilde{\circ}\, \tilde{y}_2 \,\tilde{\circ}\, \tilde{y}_3)(g) &= \bigcup_{g=xmp} \{\tilde{y}_1(x) \cap \tilde{y}_2(m) \cap \tilde{y}_3(p)\} \\
&\subseteq \bigcup_{g=xmp} \{\tilde{y}_1(xmp) \cap \tilde{y}_2(xmp) \cap \tilde{y}_3(xmp)\} \\
&\subseteq \tilde{y}_1(g) \cap \tilde{y}_2(g) \cap \tilde{y}_3(g), \\
(\beta \,\tilde{\circ}\, \varrho \,\tilde{\circ}\, \delta)(g) &= \bigwedge_{g=xmp} \{\beta(x) \vee \varrho(m) \vee \delta(p)\} \\
&\geq \bigwedge_{g=xmp} \{\beta(xmp) \vee \varrho(xmp) \vee \delta(xmp)\} \\
&\geq \beta(g) \vee \varrho(g) \vee \delta(g).
\end{aligned}$$

If $g \neq xmp$, then $(\tilde{y}_1 \,\tilde{\circ}\, \tilde{y}_2 \,\tilde{\circ}\, \tilde{y}_3)(g) = \emptyset \subseteq (\tilde{y}_1 \cap \tilde{y}_2 \cap \tilde{y}_3)(g)$ and $(\beta \,\tilde{\circ}\, \varrho \,\tilde{\circ}\, \delta)(g) = 1 \geq (\beta \vee \varrho \vee \delta)(g)$. Hence $\tilde{y}_{1\beta} \odot \tilde{y}_{2\varrho} \odot \tilde{y}_{3\delta} \ll \tilde{y}_{1\beta} \Cap \tilde{y}_{2\varrho} \Cap \tilde{y}_{3\delta}$.

Theorem 4.8. Let $\tilde{y}_{1\varrho}, \tilde{y}_{2\beta}$ be respectively a hybrid right ideal and a hybrid left ideal of D. Then $\tilde{y}_{1\varrho} \odot D \odot \tilde{y}_{2\beta} \ll \tilde{y}_{1\varrho} \Cap \tilde{y}_{2\beta}$.

Proof. Let $\tilde{y}_{1\varrho}, \tilde{y}_{2\beta}$ be a hybrid right ideal and a hybrid left ideal of D respectively. Consider $g \in D$ such that $g = xmp \; \forall x, m, p \in D$. Now

$$\begin{aligned}
(\tilde{y}_1 \,\tilde{\circ}\, D \,\tilde{\circ}\, \tilde{y}_3)(g) &= \bigcup_{g=xmp} \{\tilde{y}_1(x) \cap D(m) \cap \tilde{y}_2(p)\} \\
&= \bigcup_{g=xmp} \{\tilde{y}_1(x) \cap W \cap \tilde{y}_2(p)\} \\
&\subseteq \bigcup_{g=xmp} \{\tilde{y}_1(xmp) \cap \tilde{y}_2(xmp)\} \\
&= \tilde{y}_1(g) \cap \tilde{y}_2(g),
\end{aligned}$$

$$(\varrho \tilde{\circ} D \tilde{\circ} \beta)(g) = \bigwedge_{g=xmp} \{\varrho(x) \vee D(m) \vee \beta(p)\}$$
$$= \bigwedge_{g=xmp} \{\rho(xmp) \vee 0 \vee \beta(xmp)\}$$
$$= \bigwedge_{g=xmp} \{\rho(x) \vee \beta(p)\}$$
$$= \bigwedge_{g=xmp} \{\rho(xmp) \vee \beta(xmp)\}$$
$$\geq \rho(g) \vee \beta(g).$$

If $g \neq xmp$, then $(\tilde{y}_1 \tilde{\circ} D \tilde{\circ} \tilde{y}_3)(g) = \emptyset \subseteq (\tilde{y}_1 \cap \tilde{y}_2)(g)$ and $(\varrho \tilde{\circ} D \tilde{\circ} \beta)(g) = 1 \geq (\varrho \vee \beta)(g)$. Hence $\tilde{y}_{1\varrho} \odot D \odot \tilde{y}_{2\beta} \ll \tilde{y}_{1\varrho} \Cap \tilde{y}_{2\beta}$.

5. Hybrid A-Ideals in Ternary Semigroup

Fuzzy A-ideals in ternary semigroups were first discussed by Suebsung, Wattanatripop, and Chinram (2019). Now we describe the hybrid A-ideals in ternary semigroup as follows.

Definition 5.1. Let $\tilde{k}_\tau,\ \tilde{y}_\varrho \in H(D)$. Then $\tilde{k}_\tau$ is a hybrid left (resp., right, lateral) A-ideal of D if $\forall\ g \in D$,

(i) $\left(\chi_g(\tilde{y}) \tilde{\circ} \chi_g(\tilde{y}) \tilde{\circ} \tilde{k}\right) \cap \tilde{k} \neq \emptyset$ $(resp.,$ $\left(\tilde{k} \tilde{\circ} \chi_g(\tilde{y}) \tilde{\circ} \chi_g(\tilde{y})\right) \cap \tilde{k} \neq \emptyset,$ $\left(\chi_g(\tilde{y}) \tilde{\circ} \tilde{k} \tilde{\circ} \chi_g(\tilde{y})\right) \cap \tilde{k} \neq \emptyset$);

(ii) $\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau \neq 1$ $(resp.,$ $\left(\tau \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \chi_g(\varrho)\right) \vee \tau \neq 1,$ $\left(\chi_g(\varrho) \tilde{\circ} \tau \tilde{\circ} \chi_g(\varrho)\right) \vee \tau \neq 1$).

If $\tilde{k}_\tau$ is a hybrid left, a hybrid right and a hybrid lateral A-ideal of D, then it is a hybrid A-ideal of D.

Example 5.2. Consider that $D = \{\mathbb{Z}_6\} = \{0, 1, 2, 3, 4, 5\}$ is a ternary semigroup with usual addition and define a hybrid structure $\tilde{k}_\tau$ in D over $W = \{w_1, w_2, w_3, w_4\}$ with $\tilde{k}$ denoting any constant mapping from D to I, and $\tau(0) = 0, \tau(1) = 0.8, \tau(2) = 0, \tau(3) = 0.2, \tau(4) = 0.3, \tau(5) = 0$. For any $\tilde{y}_\varrho \in H(D)$, we have

If $g = 0$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](1) = 0.8 \neq 1.$
If $g = 1$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](3) = 0.2 \neq 1.$
If $g = 2$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](3) = 0.2 \neq 1.$
If $g = 3$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](1) = 0.8 \neq 1.$
If $g = 4$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](3) = 0.2 \neq 1.$
If $g = 5$, we have $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](1) = 0.8 \neq 1.$

Then**,** if $g = 0$, we have $[\left(\chi_g(\tilde{y}) \tilde{\circ} \chi_g(\tilde{y}) \tilde{\circ} \tilde{k}\right) \cap \tilde{k}](g) \neq \emptyset\ and$ $\left[\left(\chi_g(\varrho) \tilde{\circ} \chi_g(\varrho) \tilde{\circ} \tau\right) \vee \tau\right](g) \neq 1$ for all $g \in D$. Hence $\tilde{k}_\tau$ is a hybrid left A-ideal of D.

Definition 5.3. Let $\tilde{t}_\delta \in H(D)$. Then support of $\tilde{t}_\delta$ is defined by

$$supp(\tilde{t}_\delta) = \{g \in D : \tilde{t}(g) \neq \emptyset\ and\ \ \varrho(g) \neq 1\}.$$

Theorem 5.4. For any $\tilde{y}_\varrho, \tilde{k}_\tau \in H(D)$ and $R(\neq \emptyset) \subseteq D$, we have

(i) R of D is a left A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a hybrid left A-ideal.
(ii) R of D is a lateral A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a hybrid lateral A-ideal.
(iii) R of D is a right A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a hybrid right A-ideal.
(iv) R of D is a A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a hybrid A-ideal.

Proof: (i) Suppose that R of D is a left A-ideal. Then for each $g \in D$, there exists $m \in ggR \cap R$ such that $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} \chi_R(\tilde{y})\right) \cap \chi_R(\tilde{y})\right](m) \neq \emptyset$ and $\left[\left(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} \chi_R(\varrho)\right) \vee \chi_R(\varrho)\right](g) \neq 1$, so $\chi_R(\tilde{y}_\varrho)$ is a hybrid left A-ideal of D. Conversely, if $\chi_R(\tilde{y}_\varrho)$ of D is a hybrid left A-ideal, then for all $m \in D$, we have $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} \chi_R(\tilde{y})\right) \cap \chi_R(\tilde{y})\right](m) \neq \emptyset$ and $\left[\left(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} \chi_R(\varrho)\right) \vee \chi_R(\varrho)\right](m) \neq 1$, which imply $m \in ggR \cap R$. So $ggR \cap R \neq \emptyset$. Hence R of D is a left A-ideal.

The proofs of (ii) as well as (iii) are similar.

Combining (i), (ii) and (iii) yields (iv).

Theorem 5.5. Let $\tilde{y}_\varrho,\ \tilde{k}_\tau \in H(D)$. Then

(i) $\tilde{y}_\varrho$ of D is a hybrid left A-ideal $\Leftrightarrow supp(\tilde{y}_\varrho)$ of D is a left A-ideal.
(ii) $\tilde{y}_\varrho$ of D is a hybrid lateral A-ideal $\Leftrightarrow supp(\tilde{y}_\varrho)$ of D is a lateral A-ideal.

(iii) $\tilde{y}_\varrho$ of D is a hybrid right A-ideal $\Leftrightarrow supp(\tilde{y}_\varrho)$ of D is a right A-ideal.
(iv) $\tilde{y}_\varrho$ of D is a hybrid A-ideal $\Leftrightarrow supp(\tilde{y}_\varrho)$ of D is a A-ideal.

Proof: (i) Consider $\tilde{y}_\varrho$ of D that is a hybrid left A-ideal and $\tilde{k}_\tau \in H(D)$. Then $[(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} \tilde{y}) \cap \tilde{y}](g) \neq \emptyset$ and $[(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} \varrho) \vee \varrho](g) \neq 1 \ \forall g \in D$. For each $g \in D$, there exist $m, b \in D$ such that $m = ggb, \tilde{y}(m) \neq \emptyset$, $\varrho(m) \neq 1$ and $\tilde{y}(b) \neq \emptyset$, $\varrho(b) \neq 1$. So $m, b \in supp(\tilde{y}_\varrho)$. This implies $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} \chi_{supp(\tilde{y}_\varrho)}(\tilde{y})\right) \cap \chi_{supp(\tilde{y}_\varrho)}(\tilde{y})\right](m) \neq \emptyset$ and $\left[\left(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} \chi_{supp(\tilde{y}_\varrho)}(\varrho)\right) \cap \chi_{supp(\tilde{y}_\varrho)}(\varrho)\right](m) \neq 1$. So $\chi_{supp(\tilde{y}_\varrho)}(\tilde{y}_\varrho)$ is a hybrid left A-ideal of D. Then by Theorem 5.4(i), $supp(\tilde{y}_\varrho)$ is a left A-ideal of D. Conversely, suppose that $supp(\tilde{y}_\varrho)$ of D is a left A-ideal. Then by Theorem 5.4(i), $\chi_{supp(\tilde{y}_\varrho)}(\tilde{y}_\varrho)$ of D is a left A-ideal. This gives that for any $\tilde{k}_\tau \in H(D)$, $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} \chi_{supp(\tilde{y}_\varrho)}(\tilde{y})\right) \cap \chi_{supp(\tilde{y}_\varrho)}(\tilde{y})\right] \neq \emptyset$ and $\left[\left(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} \chi_{supp(\tilde{y}_\varrho)}(\varrho)\right) \vee \chi_{supp(\tilde{y}_\varrho)}(\varrho)\right] \neq 1 \ \forall g \in D$. Then there exist $m, b \in D$ such that $m = ggb$ and $m, b \in supp(\tilde{y}_\varrho)$, so $\tilde{y}(m) \neq \emptyset, \varrho(m) \neq 1$ and $\tilde{y}(b) \neq \emptyset, \varrho(b) \neq 1$. So $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} (\tilde{y})\right) \cap (\tilde{y})\right](m) \neq \emptyset$ and $[(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} (\varrho)) \vee (\varrho)](m) \neq 1$. Thus $\left[\left(\chi_g(\tilde{k}) \tilde{\circ} \chi_g(\tilde{k}) \tilde{\circ} (\tilde{y})\right) \cap (\tilde{y})\right] \neq \emptyset$ and $[(\chi_g(\tau) \tilde{\circ} \chi_g(\tau) \tilde{\circ} (\varrho)) \vee (\varrho)] \neq 1 \ \forall \ g \in D$. Hence $\tilde{y}_\varrho$ is a hybrid left A-ideal of D.

The proofs of (ii) and (iii) are similar.

Combining (i), (ii) and (iii) yields (iv).

Definition 5.6. A hybrid left A-ideal $\tilde{y}_\varrho$ is minimal if for all hybrid left A-ideals $\tilde{t}_\delta$ of D such that $\tilde{t}_\delta \ll \tilde{y}_\varrho$, we have $supp(\tilde{t}_\delta) = supp(\tilde{y}_\varrho)$.

Theorem 5.7. Let $\tilde{y}_\varrho \in H(D)$ and $R(\neq \emptyset) \subseteq D$. Then

(i) R of D is a minimal left A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a minimal hybrid left A-ideal.
(ii) R of D is a minimal lateral A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a minimal hybrid lateral A- ideal.
(iii) R of D is a minimal right A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a minimal hybrid right A- ideal.
(iv) R of D is a minimal A-ideal $\Leftrightarrow \chi_R(\tilde{y}_\varrho)$ of D is a minimal hybrid A-ideal.

Proof: (i) Let R of D be a minimal left A-ideal. From Theorem 5.4(i), $\chi_R(\tilde{y}_\varrho)$ is a hybrid left A-ideal of D. Consider $\tilde{t}_\delta$ that is a hybrid left A-ideal of D such that $\tilde{t}_\delta \ll \chi_R(\tilde{y}_\varrho)$. Using Theorem 5.5$(i)$, $supp(\tilde{t}_\delta)$ is a left A-ideal of D. Then $supp(\tilde{t}_\delta) \ll supp(\chi_R(\tilde{y}_\varrho)) = R$. By our assumption R is minimal, so $R \subseteq supp(\tilde{t}_\delta)$. Hence $supp(\tilde{t}_\delta) = R = supp\left(\chi_R(\tilde{y}_\varrho)\right)$. Therefore, $\chi_R(\tilde{y}_\varrho)$ is minimal. Conversely, assume that $\chi_R(\tilde{y}_\varrho)$ of D is a minimal hybrid left A-ideal. Then R of D is a left A-ideal using Theorem 5.4(i). Let G of D be a left A-ideal such that $G \subseteq D$. By Theorem 5.4(i), $\chi_G(\tilde{y}_\varrho)$ of D is a hybrid left A-ideal such that $\chi_G(\tilde{y}_\varrho) \ll \chi_R(\tilde{y}_\varrho)$. Hence $G = supp\left(\chi_G(\tilde{y}_\varrho)\right) = supp\left(\chi_R(\tilde{y}_\varrho)\right) = R$. Therefore R is minimal.

Theorem 5.8. Let $R(\neq \emptyset) \subseteq D$ and $\tilde{y}_\varrho \in H(D)$. Then

(i) R has no proper left A-ideal of D $\Leftrightarrow$ for all hybrid left A-ideals $\tilde{y}_\varrho$ of D, $supp(\tilde{y}_\varrho) = R$.
(ii) R has no proper right A-ideal of D $\Leftrightarrow$ for all hybrid right A-ideals $\tilde{y}_\varrho$ of D, $supp(\tilde{y}_\varrho) = R$.
(iii) R has no proper lateral A-ideal of D $\Leftrightarrow$ for all hybrid lateral A-ideals $\tilde{y}_\varrho$ of D, $supp(\tilde{y}_\varrho) = R$.
(iv) R has no proper A-ideal of D $\Leftrightarrow$ for all hybrid A-ideals $\tilde{y}_\varrho$ of D, $supp(\tilde{y}_\varrho) = R$.

Proof: (i) Suppose that $\tilde{y}_\varrho$ of D is a hybrid left A-ideal. From Theorem 5.4 (i), $supp(\tilde{y}_\varrho)$ of D is a left A-ideal. As R has no proper left A-ideal, $supp(\tilde{y}_\varrho) = R$. Conversely, let $\tilde{y}_\varrho$ of D be a hybrid left A-ideal and $supp(\tilde{y}_\varrho) = R$. Take G of D to be a proper left A-ideal. Then $\chi_G(\tilde{y}_\varrho)$ is a hybrid left A-ideal using Theorem5.4. (i) and $supp\ (\chi_G(\tilde{y}_\varrho)) = G \neq R$, a contradiction. Hence R of D has no proper left A-ideal.

The proofs of (ii), (iii) and (iv) are similar.

6. Conclusions

In algebraic structures, the study of ternary semigroups has led to advances in the understanding of ideal theory, regularity theory, congruence, and Green's relations. In computer science, ternary semigroups have been utilized in the design and analysis of algorithms, particularly in areas such as data compression and error correction. The concept of A-ideals provides a generalization of the notions of right ideals, lateral ideals, and left ideals. The applicability of hybrid structures has been assessed in different algebraic structures, including rings, semirings, and lattices, providing insights into their adaptability and efficiency. Furthermore, comparative studies between the existing models and hybrid structures could be considered to understand their respective strengths and limitations in various decision-making scenarios. In this paper, we proposed a notion of hybrid quasi-ideals and examined the relations between quasi-ideals and hybrid quasi-ideals in ternary semigroups.

Also, we introduced the concept of A-hybrid ideals and obtained equivalent conditions for a set to be a minimal A-ideal. Additionally, we found a relationship between A-ideal and its hybrid characteristic A-ideal. We characterized minimal A-ideals in ternary semigroups using minimal hybrid A-ideals.

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