ORIGINAL ARTICLE

A method to analyze effects of surface variational model on positional geometric variability

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Abstract

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After machining, a processed workpiece is left with irregular topographies which may affect functional applications of its surface. Particularly when the workpiece is restrained again for a consequent machining operation and the surface is selected as a locating reference, any surface deviations from its perfect geometry will lead to inaccuracy of a machined feature. This study proposes a method to analyze effects of distributions chosen to explain irregularities of locating surfaces on feature positional tolerancing. Three types of distribution, a normal, a beta and a uniform distributions, were used in the analysis. From the simulation, it is found that the variability of the machined feature is obviously dependent on the distribution selected. Aside from mathematical complexity, this affected feature variability is another factor to be considered in choosing an appropriate surface distribution. The method developed herein will help a designer to impose more efficient tolerancing system, reduce production cost eventually.

Key words : datum establishment, tolerance, feature variability, surface distribution

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ระเบียบวิธีในการวิเคราะห์ผลกระทบของตัวแบบความไม่แน่นอนของพื้นผิวอ้างอิง ที่มีต่อการกำหนดตำแหน่งของการตัด

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โดยทั่วไปแล้ว พื้นผิวของชิ้นงานที่ผ่านการตัดมักจะมีรูปแบบเฉพาะตัวที่มีลักษณะไม่สม่ำเสมอ (irregular) และเมื่อนำชิ้นงานมาทำการจับยึดเพื่อที่จะสร้างฟีทเจอร์ (feature) โดยใช้พื้นผิวที่ผ่านการตัดมาแล้วนั้นเป็นพื้นผิวอ้างอิง ก็จะทำให้การกำหนดตำแหน่งของฟีทเจอร์นั้นผิดพลาดไป งานวิจัยนี้เป็นการนำเสนอระเบียบวิธีในการวิเคราะห์ผลกระทบของรูปแบบการกระจาย (distribution) ของพื้นผิวอ้างอิงที่มีต่อความแม่นยำในการวางตำแหน่งของฟีทเจอร์ โดยเปรียบเทียบระหว่างการกระจายแบบปกติ แบบเบต้า และแบบยูนิฟอร์ม ผลสรุปที่ได้ระบุว่าความไม่แน่นอนของตำแหน่งฟีทเจอร์มีความสัมพันธ์กับรูปแบบการกระจายของพื้นผิวอ้างอิงนั้น จากวิธีการดังกล่าวนี้จะช่วยให้นักออกแบบสามารถที่จะตัดสินใจในการกำหนดระยะพิกัดเผื่อของฟีทเจอร์ที่มีประสิทธิภาพได้

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Irregularities of a machined surface topography can be broken down to surface roughness, waviness, and form error. For example, a structure of turned surfaces is a combination of a waviness component generated by a feeding process and a random roughness component caused by a built-up edge on a cutting tool (Stout *et al*., 1990). Unlike turned surfaces, ground surfaces under microscope show gouge marks which result from sharp grits attached to a grinding wheel periphery. It appears that type of process and process parameters can differentiate manufactured surface characteristics as shown in Figure 1. After machining, a workpiece is left with irregular topographies which may affect functional applications of the surface. Particularly when the surface is selected as a locating reference, any deviations from its perfect geometry will lead to consequent workpiece inaccuracy. Eliminating such deviations to improve the workpiece accuracy is not always an economical solution. However, their undesirable effects on the workpiece quality can be controlled if we know how the deviations are distributed across the entire surface.

Because of the uncountable parameters involved, the actual distribution of a machined surface is almost impracticable to determine. Once derived; however, the calculation would be too complicated giving inapplicable results. Variational models widely found in published work are usually based on a Gaussian or normal distribution. Nassef and ElMaraghy (1995) assumed variability of manufactured surfaces was distributed as multinormal. Multinormal random surface points were generated and estimated surfaces were constructed by interpolating to the points. The authors then investigated the simulated surfaces to determine geometric deviation of the surfaces before calculating optimal tolerance zones that would result in desired probability of rejection. The drawback is that systematic errors may produce an asymmetric distribution, which is not suitable to explain with the Gaussian or normal function. However, as a sample size increases, the distribution can be assumed Gaussian following the central limit theorem. Besides a Gaussian distribution, a beta distribution is also suggested because of its advantageous applications in representing manufacturing variability as following:

1. The beta distribution is more flexible. It has the capability to conform various types of variability, when the Gaussian distribution is applied only to a symmetric function.

2. The beta distribution has finite upper and lower limits, which is favorable in computation.

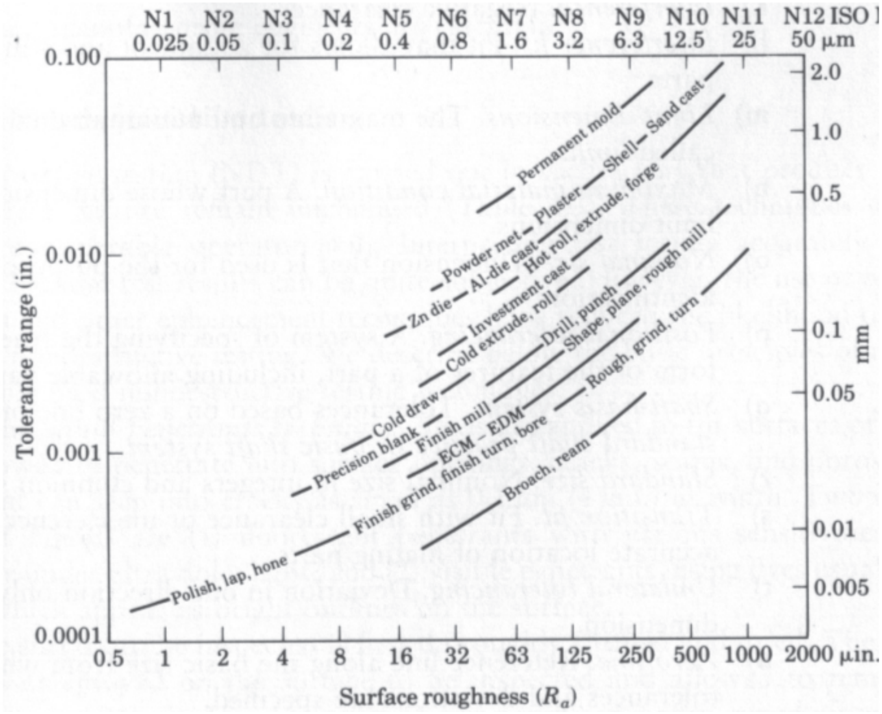

Figure 1. Tolerances and surface roughness obtained in various manufacturing processes. These tolerances apply to 25 mm workpiece dimension. (Kalpakjian, 1991).

The beta distribution is also chosen over a normal or Gaussian distribution because of its capability of supporting the calculation of an inverse probability distribution function (Lin *et al.*, 1997). He (1991) used the beta distribution to explain variability of dimension in manufacturing processes. Since the variability is distributed varying from process to process and application to application, the author then proposed a beta distribution as it has the capability to cover all kinds of distributions from rectangular to normal distributions as well as asymmetrical distributions. Treacy *et al.* (1991) carried out an analysis on tolerance distribution of an assembly of mechanical parts. For the sake of its flexibility, the authors decided on a beta distribution to represent variability of resultant tolerance chain of the assembly though the model may be associated with calculation complexity. Because practically every sum of interacting dimensions gives an expected variability that falls between the results from a worst case and root sum of square approaches, Di Stefano (2003) developed a mean shift model modified from Mansoor (1964) where a factor "f" was imposed to determine a process bias. The factor is a function of a mean shift ratio, a confidence level, and a number of dimensions in an assembly. This model is applicable to Gaussian (f = 0) to non-Gaussian (f = 1) distributed dimensions. Unlike Gaussian and beta distributions, a uniform distribution is far less popular. ElMaraghy *et al.* (1995) employed the distribution to represent the variation explained by different tolerance types. Choudhuri and De Meter (1999) carried out a simulation to analyze the impact of locator tolerance scheme on the potential datum related, geometric errors of linear and machined features. They used a set of equally spaced numbers to simulate locator variability. The results revealed a linear relationship between some variables, which is not very surprising when the analysis was done over a small data range.

There are also works on datum establishment which left the controversial issue of a distribution function unspecified. Nassef and ElMaraghy (1997) proposed a new criterion, mismatch probability, to determine appropriate tolerancing scheme where production cost was kept minimum. Because the procedure used with dimensional tolerancing is not applicable to geometric tolerancing, a new algorithm based on genetic algorithm was invented to evaluate each tolerancing candidate. Though computational time was relatively long, the benefits of choosing the appropriate tolerances paid off later on. Roy and Li (1998) developed mathematical models representing real world form variations. They proposed two strategies used to construct the approximated form of surfaces. The first technique was for simulating curved surfaces from randomly generated surface points. In order to create approximated planar surfaces, the second technique employed a linear regression analysis from the random points to estimate plane parameters. Besides the form variation, the authors also presented a computational scheme to represent size, orientation and position tolerance zones. Variational models of these tolerances were established relying on random data points. Bhat and De Meter (2000) conducted a simulation analysis to evaluate efficiency in accurate locating among three different datum establishment methods. The sample workpiece and its features were constrained by geometric tolerances. Random variables were generated to simulate variation according to each tolerance value generating variant feature position and orientation. The datum establishment method giving the least error which means that it is unlikely to be affected by such erratic geometry of the feature was considered the most functional method.

It can be seen that these research were more intensified on development of methods of reference establishment without taking into account an actual pattern of surface variability. The reasons involved in choosing an appropriate surface distribution should be more than computational complexity. This study proposes another aspect of tolerancing analysis. A method to analyze effects of chosen surface distributions on feature positional tolerancing is discussed in the following sections.

Analysis

In this section, the effects of variational models representing datum feature variability on positional errors of a feature to be produced are discussed. As shown in Figure 2, a datum feature surface is allowed to deviate from its perfect geometry by d_i where $|d_i| \leq \frac{T}{2}$; T is a tolerance constraining the datum feature. A prismatic workpiece with a hole and toleranced datum features (Figure 3) is used as an example in the analysis. Problems arise when the hole location is specified with respect to a misplaced reference frame, which is established from the deviated datum feature.

1. Tolerance Distributions

Three types of distributions usually found used to explain machined surface variability, d_i, are a normal, a beta and a uniform distributions. In this study, it is assumed that the distribution curve will span over the entire range of the tolerance width.

1.1 Gaussian or normal distribution

A unit normal probability density function is given as follows:

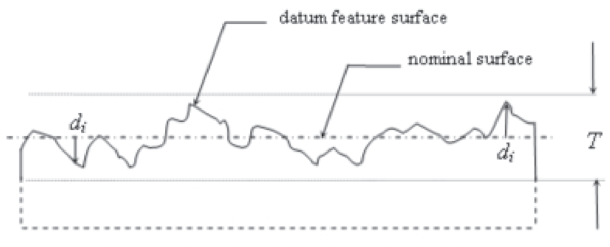

Figure 2. Datum feature and tolerance width, *T*

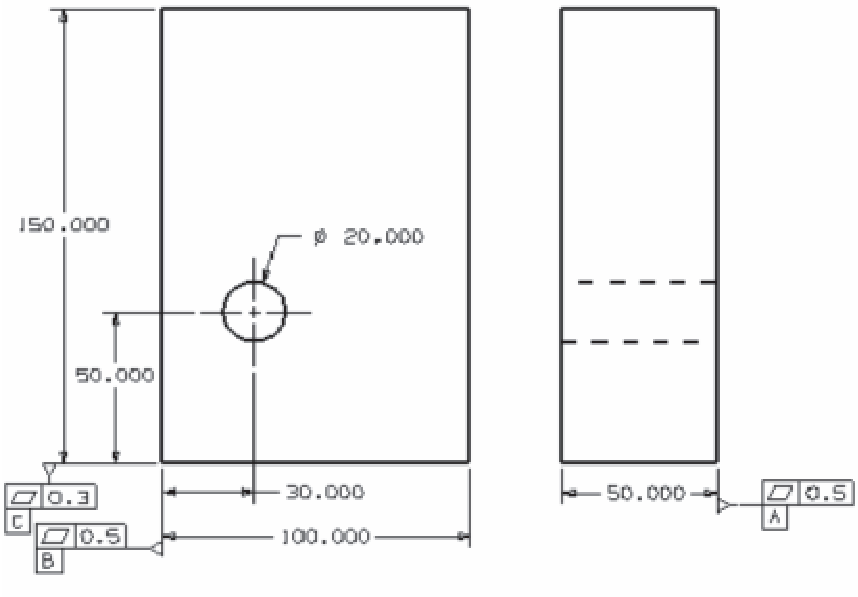

Figure 3. A sample workpiece used in simulations.

$$f(u) = \frac{1}{\sqrt{2\Pi}} e^{-x^2/2} \quad -\infty < u < \infty \tag{1}$$

The normal distribution curve spreads infinitely and symmetry about its mean as shown in Figure 4. Since it is impossible to define the end of its coverage, a 99.73% containment is assumed. When $N(0,1)$ represents a unit normal distribution which has its mean at zero and standard deviation of one, the surface points assimilating each datum feature are distributed as

$$d \sim (\frac{T}{6}) \bullet N(0,1) \tag{2}$$

1.2 Uniform Distribution

A unit uniform probability density function is given as follows:

$$f(u) = 1,\ 0 \le u \le 1 \tag{3}$$

When $U(0,1)$ represents a unit beta distribution which spans over the range from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \bullet U(0,1) - \frac{T}{2} \tag{4}$$

1.3 Beta Distribution

A unit beta probability density function is given as follows:

$$f(u;\alpha,\beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1}(1-u)^{\beta-1}, 0 \le u \le 1, \alpha \ge 0, \beta \ge 0 \\ 0, elsewhere \end{cases} \tag{5}$$

Since the study assumes a symmetric surface variability pattern over the tolerance zone, the values of shape controlling parameters, α and β, used in the simulations therefore are chosen to be equal. When $B(\alpha,\beta)$ represents a unit beta distribution covering the region from zero to one, the surface points assimilating each datum feature are distributed as

$$d \sim T \bullet B(\alpha,\beta) - \frac{T}{2} \tag{6}$$

2. Datum Establishment Methods

In this study, the 3-2-1 locating method is used to establish a reference frame. Following ASME Y14.5M, the primary datum plane is imposed from three highest points on the primary datum feature. Orthogonal with the primary plane, the secondary plane lies on two extreme points.

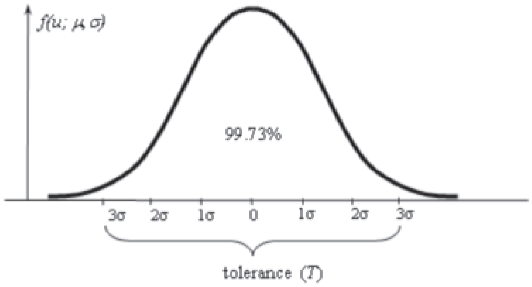

Figure 4. Tolerance determination when a normal distribution is assumed.

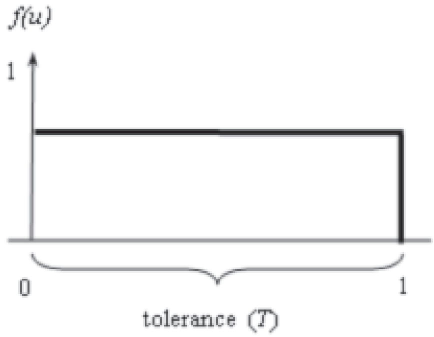

Figure 5. Tolerance determination when a uniform distribution is assumed.

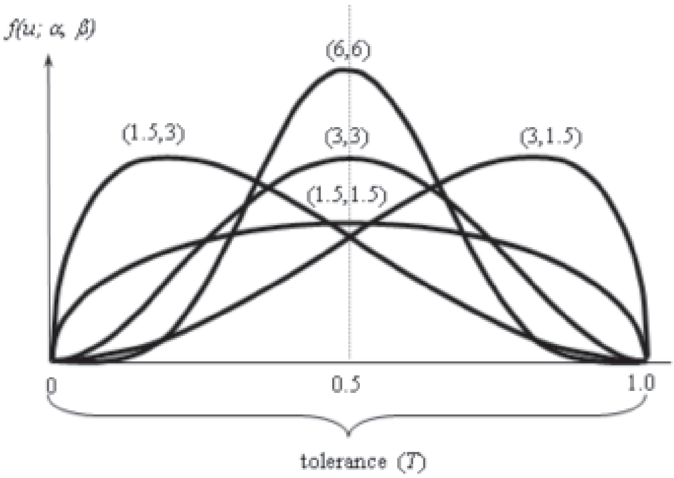

Figure 6. Tolerance determination when a beta distribution is assumed.

The last plane, the tertiary plane, is established from the outermost point while maintaining perpendicular to the preceding two planes as shown in Figure 7. By implementing this method, it is inevitable the frame constructed would be sensitive to the pattern the datum feature irregularities are distributed.

3. Datum Planes Definitions

Planes as elements of the reference frame must be calculated according to the datum establishment method described in section 2.2. The hole which we use as an example will be located with respect to these planes.

An algebraic equation of a plane is

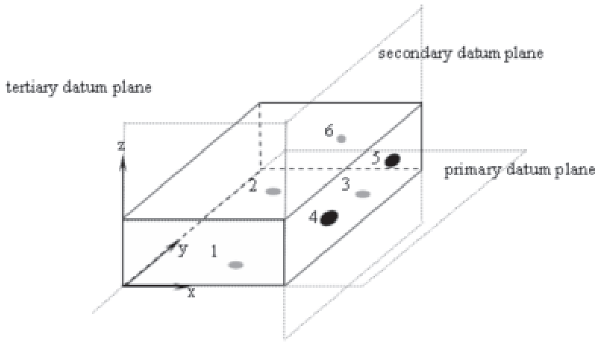

Figure 7. The system of datum planes.

$$\frac{Ax + By + Cz - D}{\sqrt{(A^2 + B^2 + C^2)}} = 0 \tag{7}$$

where [A B C] is a vector normal to the plane
D is an orthogonal distance from the origin to the plane

To locate an arbitrary plane, its normal vector and a distance from the plane to the origin must be defined.

Primary datum feature

The normal vector of the primary datum feature ($\vec{N}_p$) can be obtained from

$$\vec{N}_p = (\vec{P}_{pm1} - \vec{P}_{pm2}) \times (\vec{P}_{pm3} - \vec{P}_{pm2}) \tag{8}$$

and the orthogonal distance is

$$D_p = (\vec{n}_p \bullet \vec{P}_p) \tag{9}$$

where $\vec{P}_{pmi}$ are the extremities of the primary datum feature
$\vec{n}_p$ is a unit vector of $\vec{N}_p$
$\vec{P}_p$ is a point on the primary datum feature

Secondary datum feature

To comply with the principle of 3-2-1 method, the secondary datum feature must be perpendicular to the primary one while passing through 2 highest surface points. Therefore,

$$\vec{N}_s = \vec{N}_p \times (\vec{P}_{sm2} - \vec{P}_{smi}) \tag{10}$$

where $\vec{P}_{smi}$ are the extremities of the secondary datum feature

Tertiary datum feature

The vector normal to the tertiary datum feature is defined by

$$\vec{N}_t = \vec{N}_p \times \vec{N}_s \tag{11}$$

The determination of location and orientation of the planes are also shown in Figure 8.

The distances from the origin to the secondary and tertiary datum feature, D_s and D_t, are calculated the same way as shown in Eq. 9. After the reference system imposed from the workpiece datum feature is established, coordinates of the hole relative to the nominal and actual systems are compared. The effects of variational surface model will then be analyzed. Please note that the nominal reference system is obtained when datum features are devoid in absence of any irregularities.

4. Positional Error Determination

Coordinates of an arbitrary point in a rectangular or Cartesian system is a Euclidean perpendicular distance from the point to a corresponding axis. By means of geometric transformation, a coordinate of a point P in a reference system F is related to its coordinate with respect to another reference system, M, as shown in Eq. 12 and Figure 9.

$$[P]^F = A[P]^M + L \tag{12}$$

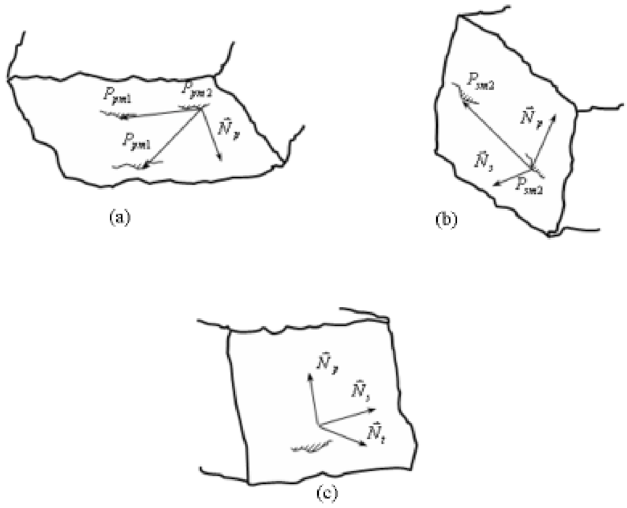

Figure 8. Datum planes location and orientation determination (a) primary datum plane (b) secondary datum plane (c) tertiary datum plane.

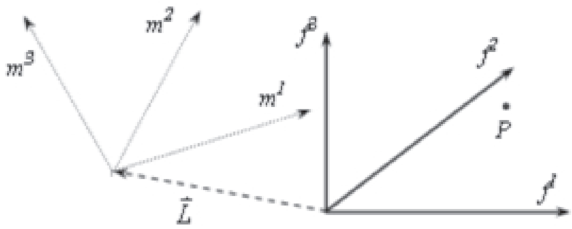

Figure 9. *F* ref. system relative to *M* ref. system.

where $[P]^F$ = a point coordinate in *F* reference system

$[P]^M$ = a point coordinate in *M* reference system

L is a distance vector in which *M* system located away from *F* system

and

$$A = \begin{vmatrix} f^1 \bullet m^1 & f^1 \bullet m^2 & f^1 \bullet m^3 \\ f^2 \bullet m^1 & f^2 \bullet m^2 & f^2 \bullet m^3 \\ f^3 \bullet m^1 & f^3 \bullet m^2 & f^3 \bullet m^3 \end{vmatrix} \quad (13)$$

where f^i is a direction cosine of ith axis of *F* ref erence system

m^i is a direction cosine of ith axis of *M* reference system

From Figure 7 and Eq. 13, we obtain

$$A = \begin{vmatrix} i \bullet \vec{n}_s & i \bullet \vec{n}_t & i \bullet \vec{n}_p \\ j \bullet \vec{n}_s & j \bullet \vec{n}_t & j \bullet \vec{n}_p \\ k \bullet \vec{n}_s & k \bullet \vec{n}_t & k \bullet \vec{n}_p \end{vmatrix} \quad (14)$$

In the current model, the system established from imperfect datum features is represented by the *M* reference system which is deviated from the nominal or *F* reference system. When the hole is produced, its location is specified relative to the machine or *F* system. Unfortunately, once the workpiece is removed from the machine, its accuracy is determined based on its actual or *M* system. The errors, *e*, of feature position, *P*, as resulted from such discrepancy between both systems are calculated as follows.

$$e = P - (A \bullet P + L) \quad (15)$$

5. Positional Geometric Variation Region

According to ASME Y14.5M, a positional tolerance is the permissible variation in the location of a feature about its exact true position. For a cylindrical feature like a hole, the positional tolerance is a diameter of the tolerance zone within which the axis of the feature must lie. In this study, the positional geometric variability is defined in a similar way except that it circumscribes actual locations as affected by undesirable factors. Following the algorithm explained in section 2.1-2.4, the feature's center is located. Because of irregular locating datum surface, inconsistency of the feature location is anticipated and can be illustrated as a cloud of points after a number of trials carried out (Figure 10). A circle bounding a specified proportion of possible feature locations (positional geometric variation region) is then calculated (See Sangnui (2002) for details). How big the circle is indicates how much the feature is varied in its position, and it should not exceed the tolerance associated with the hole.

Results and Discussion

The objective of this study is to present the impact of surface variational model on feature positional tolerancing. Three different distributions; a uniform, a normal, and a beta distribution are chosen to represent surface irregularities. Certain parameter values (α, β) of a beta distribution are selected to create various shapes of the distribution. Shown in Figure 6, the distribution is leaning towards uniformity when the parameters are approaching 1, and as the parameters are greater the distribution will become more centrally concentrated. Details of the analysis conducted are described in the following steps.

1. Random numbers representing workpiece surface points are generated according to the tolerance distributions of interest (normal, uniform and beta distributions). See section 2.1.

2. Datum feature surfaces are partitioned into a number of small rectangle patches.

3. To simulate a deviated surface, generated random numbers are assigned to the appropriate direction (i.e. for the primary datum feature, the surface errors will be in the *z* direction) at grid points connecting the patches as shown in Figure 11.

4. Following the algorithm explained in section 2, the actual feature locations are calculated.

5. Tolerance regions associated with a desired probability of acceptance are calculated for each surface distribution pattern based on 300 simulation runs. When the number is too large the resultant distribution tends to follow the normal distribution pattern, according to the central limit theorem, and the true information will be masked. Too small number of sample runs, on the other hand, is also not recommended since the results will be unclear.

6. An average of 50 radiuses of circular positional region is calculated and shown in Table 1.

7. Results from each distribution are compared.

From the data in Table 1, it can be seen that the radii of positional geometric variation obtained from all eight simulation runs exhibit the same

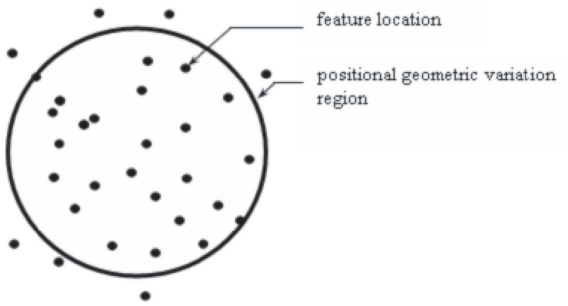

Figure 10. Positional geometric variation region.

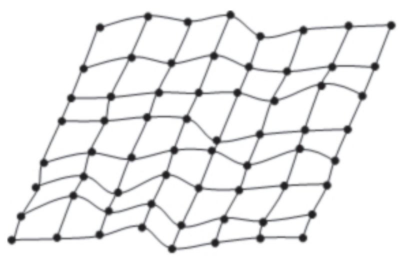

Figure 11. A simulated datum feature.

Table 1. The averaged radiuses of positional geometric variation region at 95% containment

Data	Datum Feature Tolerance Specifications	Radiuses of Feature Positional Geometric Variation Region (mm.)				
		Uniform	Normal	Beta		
				$\alpha, \beta = 1.5$	$\alpha, \beta = 4$	$\alpha, \beta = 7$
1	(Tp = 3, Ts = 3, Tt = 3)	0.0202	0.2993	0.0694	0.2568	0.2894
2	(Tp = 3, Ts = 5, Tt = 3)	0.0196	0.2864	0.0706	0.2521	0.2780
3	(Tp = 3, Ts = 5, Tt = 5)	0.0295	0.4125	0.1037	0.3660	0.4000
4	(Tp = 3, Ts = 3, Tt = 5)	0.0288	0.3888	0.0980	0.3421	0.4099
5	(Tp = 5, Ts = 5, Tt = 5)	0.0323	0.4982	0.1163	0.4057	0.5062
6	(Tp = 5, Ts = 3, Tt = 5)	0.0336	0.4965	0.1137	0.4345	0.4946
7	(Tp = 5, Ts = 3, Tt = 3)	0.0233	0.3693	0.0858	0.3291	0.3753
8	(Tp = 5, Ts = 5, Tt = 3)	0.0246	0.3665	0.0825	0.3470	0.4066

pattern across different surface distributions. As the surface variability uniformly spreads over the allowable range (in the case of the uniform distribution), the resultant positional variation is expectedly smaller compared to one derived from more centrally concentrated distributions like the normal or the beta distributions ($\alpha = \beta$). It is clearly shown that feature positional geometric variation is dependent on the variational model chosen to explain datum feature variability. The difference in positional tolerance radii is getting clearer as the surface tolerances becomes wider. Noted that, this conclusion is regarded only when the datum establishment method is 3-2-1.

Conclusion

When the workpiece is subject to a machining operation, a feature to be produced is located with respect to a machine reference system. The system must have a certain relationship with a workpiece reference system which is established from the workpiece datum feature. In that way, the feature will be precisely located. However, with the existence of uncertainties in manufacturing processes, the relationship is usually hard or impossible to identify. If the pattern of irregularities is characterized, such discrepancy will be quantified, the undesirable displacement will be resolved, and ultimately the accuracy of the workpiece will be improved.

There are several variational models proposed to explain the pattern of irregularities of machined surfaces. Only the models' advantages and drawbacks in computation handling were reported. This study has investigated another interesting aspect, which is their effect over resultant positional tolerancing of a feature when referencing features are characterized by different variational models. From the analysis, the results indicated a distinct arrangement from one distribution to another. The area circumscribing possible feature locations is growing bigger as the surface variability is distributed apart from uniformity and when surface irregularities are evenly distributed, smaller feature variability is obtained. As a results, the variability of the feature is obviously dependent on the distribution selected to explain irregular referencing surfaces. This source of variability should not be disregarded and left out of a system of tolerancing. It is well understood that the system is a critical element in a manufacturing process. An efficient tolerancing system not only bridges a gap between a design and a manufacturing department, but it does also help in production cost reduction.

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