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*Original Article*

# Network reliability analysis for 3G cellular topology design

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**Abstract**

Network reliability is an important issue in the topological design of the 3G cellular networks in order to guarantee consistent services. This paper aims to present network reliability analysis of various network topologies, including spanning tree, single ring, multiring, and two versions of hybrid ring-and-tree. We derived reliability analysis models for the hybrid ring-and-tree topologies in terms of the operational probability of the network links. The preliminary results show that the network reliability of the hybrid multiring-and-tree topologies is 116.78% and 13.78% higher than that of the spanning tree and the single ring topology, respectively, while it is only about 37.3% less than that of the pure multiring topologies.

**Keywords:** network reliability, topological network design, 3G cellular networks

## 1. introduction

Third generation (3G) cellular networks are being deployed throughout the world to support voice, data and multimedia services at high communication speed. Network designers need well-designed network topologies to carry various kinds of traffic and guarantee the quality of the services in the 3G cellular networks. To provide uninterrupted services, network designers must pay special attention to network reliability issues, especially the reliability of network topology.

The network reliability[1] is defined as the probability that the nodes in the network can communicate with others and the network topology is defined as the physical interconnections between nodes in the network. Optical fiber cables, copper wired cables or microwave links may be used to interconnect the nodes. Considering a 3G cellular network, it consists of a set of radio access network subsystems (RNS) connecting to the core network (CN), which consists of serving GPRS (General Packet Radio Service) support node (SGSN), and gateway GPRS support node (GGSN). Each RNS consists of a radio network controller (RNC), which controls the radio resources and services and one or more access points called Node B or radio base stations (RBS). Each RBS transmits/receives data traffic to/from mobile stations (MS) in the cell site. These traffic demands go through links between RBS and RNC and are aggregated into higher capacity links to the SGSN/GGSN. The network designers aim to interconnect these components in the 3G cellular networks and ensure that they can communicate in case any

[1] Network reliability can be computed by using the operational probability of the network component. The operational probability is defined as the percentage of time that the network component maintains its functions. It is the ratio of the mean-time-between-failure (MTBF) to the sum of the MTBF and the mean-time-to-repair (MTTR). The MTBF and MTTR can be measured by the network monitoring system.

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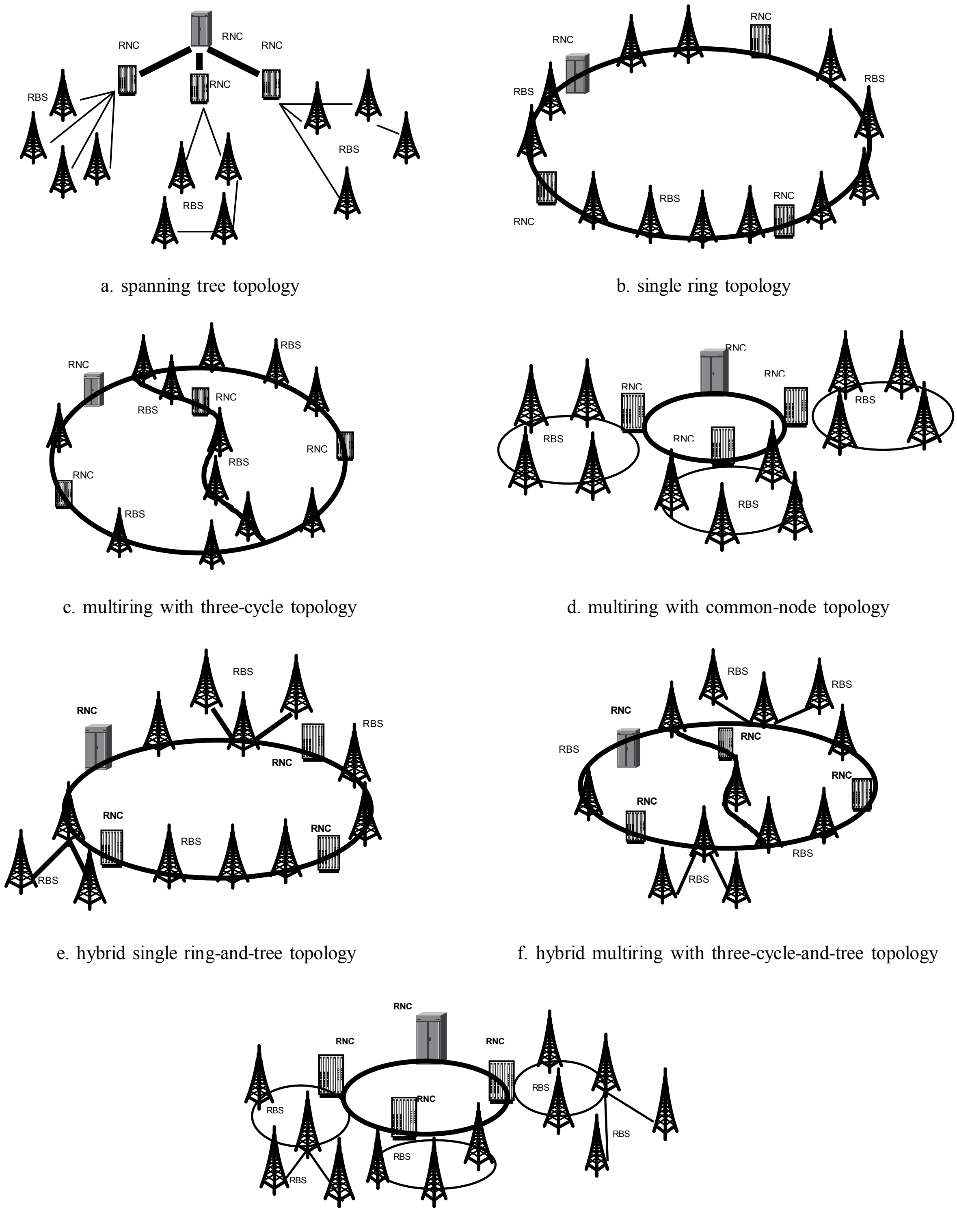


a. spanning tree topology

b. single ring topology

c. multiring with three-cycle topology

d. multiring with common-node topology

e. hybrid single ring-and-tree topology

f. hybrid multiring with three-cycle-and-tree topology

g. hybrid multiring with common-node-and-tree topology

Figure 1. Network topologies.

single link failure occurs (Harmatos 2002).

Several studies existing in the literature addressing the network reliability issues to protect the network in case of a single link failure. Jan (1992) derived reliability models for three different network topologies, including the spanning tree, the single ring, and the multiring with three cycles (Figure 1a-c). The author showed that the multiring with three-cycle topology yields the highest network reliability among the three topologies. The reason is that in case of failure more spare links in the three-cycle topology can be formed than that of the spanning tree and the single ring topology. Charnsripinyo *et al.* (2004) proposed the use of multiring with common-node topology (Figure 1d) for the wireless access networks and presented reliability models for such topology. Although the multiple-ring topology could result in high network reliability, it may be impossible to always build the network with this topology because the geographical restriction such as mountains or lakes could require higher cost in constructing the ring topology. Billionet *et al.* (2005) proposed the use of single ring topology for radio mobile access networks. In fact, the reliability of the single ring topology is less than that of the pure multiring topology[2]. Devaraj *et al.* (2004) proposed the spanning tree topology for data communication network design. However, the spanning tree topology is vulnerable to failure and has minimum network reliability because there is no back-up path in the spanning tree topology. In case any link failure, the communication between nodes in the network could be lost.

In this paper, we present the network reliability analysis of the hybrid topologies for 3G wireless access networks. We consider two types of the hybrid topologies, the hybrid single-ring-and-tree and the hybrid multiring-and-tree, because these topologies are more practical in some network environment where the geographical obstruction may exist. The hybrid single-ring-and-tree is a topology consisting of one ring and the remaining nodes form a spanning-tree topology. The hybrid multiring-and-tree is a topology consisting of several rings and the remaining nodes form a spanning-tree topology.

Using graph theory, we derived the network reliability model for the hybrid topology and compared with other topologies, including the spanning tree, the single ring, the multiring with three-cycle, the multiring with common-node. The hybrid topology is an alternative approach that allows relaxation on forming the network topology where in practical network installation one might face geographical restrictions or budget limitations on network construction. To the best of the authors' knowledge, the study on the network reliability of the hybrid topology has not been presented.

The second part of the paper is organized as follows. In Section 2, we mathematically derive network reliability models for the hybrid ring-and-tree topology. In Section 3, our preliminary analysis results, which compare the reliability of various networks, are presented. Finally, Section 4 provides conclusion and describes on-going work.

## 2. Network Reliability Model

Let a graph $G(V, E)$ represents cellular networks, in which $V$ is the set of nodes, which are the components in the 3G cellular network and $E$ is the set of edges. We assume that all nodes are perfectly reliable but edges may work with the probability $p$ or fail with the probability $q=(1-p)$. All edge failures are assumed to occur independently of each other. The network reliability of $G$ is denoted by $R(G)$, which is the probability that all nodes in $G$ can communicate with each other. Let $n = |V|$ is the number of nodes in the network.

Let us first review the existing network reliability models. Jan (1992) presented reliability models for the spanning tree ($G_{st}$), the single ring ($G_r$), and the multiring with three-cycle ($G_{mr\text{-}3c}$) topology as follows (details in appendix):

$$R(G_{st}) = p^{n-1} \tag{1}$$

$$R(G_r) = p^n + np^{n-1}(1-p) \tag{2}$$

$$R(G_{mr\text{-}3c}) = p^{n+1} + (n+1)p^n(1-p) + \left(\frac{n+1}{3}\right)^2 p^{n-1}(1-p)^2 \tag{3}$$

Note that Equation (3) is the reliability model for the optimal multiring with three-cycle topology in which the main ring is separated into two sub-rings by a path ($C_h$, the common-edges of the two sub-rings). In the optimal configuration, the path $C_h$ contains $(n+1)/3$ edges and the two sub-rings consist of an equal number of nodes (Jan 1992) (see appendix for a detailed derivation). Charnsripinyo *et al.* (2004) presented the reliability model for the multiring with common-node ($G_{mr\text{-}cn}$) topology. It is written as

$$R(G_{mr\text{-}cn}) = \prod_{i=1}^{m} \{p^{n_i} + n_i p^{n_i-1}(1-p)\} \tag{4}$$

where $m$ denotes the number of sub-rings excluding the main ring, $n_i$ denotes the number of nodes in $i^{th}$ sub-ring, and $n_1$ denotes the number of nodes in the main ring. In this paper, we propose new network reliability models for the hybrid single ring-and-tree and the hybrid multiring-and-tree topology as follows:

### 2.1 Hybrid Single Ring-and-Tree

The hybrid single ring-and-tree topology ($G_{h\text{-}sr\text{-}t}$) consists of the main ring connecting the number of nodes in the network and the spanning trees connecting the remaining nodes to the main ring as shown in Figure1e. Note that the remaining nodes that are not in the main ring can form more than one spanning trees. Let $m$ denotes the number of nodes in the main ring and $k$ denotes the number of edges in the spanning trees. We derive the network reliability for the hybrid single ring-and-tree as follows:

$R(G_{h\text{-}sr\text{-}t})$ = (Pr[all $m$ edges in a ring are operational] + Pr[$m$-1 edges in a ring are operational and one edge fails]) × Pr[$k$ edges in spanning trees are operational]

[2] Unlike the single ring topology in which all nodes in the network form one ring, the pure multiring topology is the topology consisting of several interconnected rings of nodes.

We obtain

$$R(G_{h\text{-}sr\text{-}t}) = (p^m + mp^{m-1}(1-p))p^k \quad (5)$$

## 2.2 Hybrid Multiring-and-Tree

The hybrid multiring-and-tree topology consists of a number of rings and the spanning trees connecting nodes that are not in the rings. The rings in the topology can form in several ways. Here we consider two kinds of the multirings that appear in literature; the multiring with three-cycle and the multiring with common-node.

### 2.2.1 Hybrid Multiring with Three-Cycle-and-Tree

In the hybrid multiring with three-cycle-and-tree topology ($G_{h\text{-}mr3c\text{-}t}$) (Figure 1f), the rings are in the form of three cycles in which the main ring consists of $n_1$ nodes and the path adding to the ring contain $\lfloor (n_1+1)/3 \rfloor$ edges. The spanning tree consists of $k$ edges. We derive the network reliability for the hybrid multiring with three-cycle-and-tree topology as follows:

$R(G_{h\text{-}mr3c\text{-}t})$ = Pr[the multiring with three cycles are operational] × Pr[$k$ edges in spanning trees are operational]

We obtain

$$R(G_{h\text{-}mr3c\text{-}t}) = (p^{n_1+1} + (n_1+1)p^{n_1}(1-p) + (\frac{n_1+1}{3})^2 p^{n_1-1}(1-p)^2)p^k \quad (6)$$

### 2.2.2 Hybrid Multiring with Common-Node-and-Tree

In the hybrid multiring with common-node-and-tree topology ($G_{h\text{-}mrcn\text{-}t}$) (Figure 1g), the rings are in the form of the main ring connect with $m$ sub-rings and other nodes that are not in the rings form spanning trees. Let $n_i$ denotes the number of nodes in $i^{th}$ sub-ring, where $n_1$ denotes the number of nodes in the main ring and $k$ is the number of edges in the spanning trees. We derive the network reliability for the hybrid multiring with common-node-and-tree topology as follows:

$R(G_{h\text{-}mrcn\text{-}t})$ = Pr[the multiring with common-node are operational] × Pr[$k$ edges in spanning trees are operational]

We obtain

$$R(G_{h\text{-}mrcn\text{-}t}) = \left( \prod_{i=1}^{m} \{p^{n_i} + n_i p^{n_i-1}(1-p)\} \right) p^k \quad (7)$$

## 3. Preliminary Results and Discussion

The preliminary numerical experiments were conducted considering a network consisting of 16 nodes ($n$ = 16). The probability of edge working properly, $p$, is 0.9. We evaluate the network reliability of seven topologies as illustrated in Figure 1[3]. In the multiring with three-cycle topology, the two sub-rings consists of 11 nodes in which 6 nodes are in the common path separating the main ring into two sub-rings (i.e., $c_1 = c_2 = 5$, $h = 6$). In the multiring with common-node topology, the main ring consists of four nodes and there are three sub-rings; each consists of four nodes ($n_1 = n_2 = n_3 = n_4 = 4$). In the hybrid single ring-and-tree topology, there are four nodes in the spanning tree and the remaining nodes are in the ring (i.e., $m$ = 12, $k$ = 4). In the hybrid multiring with three-cycle-and-tree topology, the main ring consists of 12 nodes and the spanning trees consists of four nodes (i.e., $c_1 = c_2 = 4$, $h = 4$, $k = 4$). Lastly, in the hybrid multiring with common-node-and-tree topology, the main ring consist of three nodes and there are three sub-rings; each consists of three nodes and the spanning trees consists of four nodes (i.e., $n_1 = n_2 = n_3 = n_4 = 3$, $k = 4$).

Table 1 shows network reliabilities of those seven topologies. It appears that the spanning tree topology has the lowest network reliability whereas the multiring with common-node topology has the highest one. The reason is

Table 1. Network reliability characteristics of different topologies.

| NetworkTopology | NetworkCharacteristics | Network Reliability |
|---|---|---|
| Spanning Tree | Spanning tree of 16 nodes | 0.205891 |
| Single Ring | Single ring of 16 nodes | 0.514728 |
| Multiring with three-cycle | $c_1 = c_2 = 5$, $h = 6$ | 0.547899 |
| Multiring with common-node | $n_1 = n_2 = n_3 = n_4 = 4$ | 0.806647 |
| Hybrid single ring-and-tree | $m = 12$, $k = 4$ | 0.432371 |
| Hybrid multiring with three-cycle-and-tree | $c_1 = c_2 = 4$, $h = 4$, $k = 4$ | 0.446326 |
| Hybrid multiring with common-node-and-tree | $n_1 = n_2 = n_3 = n_4 = 3$ , $k = 4$ | 0.585646 |

[3] These topologies exist in practical networks and are generally used for the comparison in several studies (Jan 1992; Charnsripinyo *et al.*, 2004; Devaraj *et al.*, 2004; Billionnet *et al.*, 2005).

that no backup path can be formed in the spanning tree topology in which a single link failure can interrupt communication services in the network whereas in the multiring with common-node topology, the backup paths can be formed in the multiple rings.

It can be observed that the single ring topology has the lowest network reliability compared to other forms of ring topologies. The network reliability of the multiring with three-cycle topology is higher than that of the single ring topology but lower than that of the multiring with common-node topology. The reason is that the single ring topology can handle only a single link failure whereas the multiring topologies can survive more than one link failure, given that only one link failure occurs in each sub-ring.

The network reliabilities of the hybrid multiring-and-tree topologies are much higher than those of the spanning tree and the network reliabilities of the single ring topology are only slightly lower than those of the multiring topologies. Among the network topologies considered in this example, the network reliability of the hybrid multiring-and-tree topologies is 116.78% and 13.76% higher than that of the spanning tree and the single ring topology, respectively, whereas it is only about 37.73% less than that of the multiring topologies. We can see that the hybrid multiring-and-tree topologies result in high network reliabilities and they can be applicable to the practical networks in case it may not be possible to install communication links to form a ring of nodes in some areas. Therefore, the hybrid topology is an applicable solution, providing high network reliability and relaxing the ring connection requirements.

## 4. Conclusion

In this paper, we propose network reliability analysis of various networks in terms of the operational probability for 3G cellular topological network. We first showed the reliability analysis for network topologies existing in current networks. And we then derived a new network reliability models for the hybrid single ring-and-tree and the hybrid multiring-and-tree topology. Next we showed the results of hybrid topology comparing with other topologies. which have higher reliability. We found that the hybrid topologies result in high network reliabilities while relaxing the ring connection requirements. Hence, the hybrid topology is a novel solution while there are limitations in building the practical networks.

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## Appendix

The following provides derivation of the network reliability for the spanning tree, the single ring and the multiring with three-cycle topology (Jan 1992).

Let $G(V, E)$ is a graph representing a network of nodes $V$ that are interconnected by a set of edges $E$. It is assume that all nodes are perfectly reliable whereas the edges have operational probability of $p$ (i.e. the probability of not functioning of $q=1-p$). It is assumed that the failure of each edge occurs independently. The network reliability of $G$ is denoted by $R(G)$, which is the probability that all nodes in $G$ can communicate with each other. Let $n = |V|$ is the number of nodes in the network.

- **A spanning tree topology ($G_{st}$)**

A spanning tree is a topology consisting of $n$ nodes interconnected by $n$-$1$ edges without forming any loop (or ring). The network reliability of $R(G_{st})$ is written as follows:

$R(G_{st})$ = Probability that $n$-1 links are in operational state

$$R(G_{st}) = p^{n-1}$$

- **A single ring topology ($G_r$)**

A single ring is a topology consisting of $n$ nodes interconnected by $n$ edges to form a ring configuration. The network reliability of $R(G_r)$ is written as follows:

$R(G_r)$ = Probability that $n$ links are in operational state + Probability that $n$-1 links are in operational state

$$= p^n + {}^nC_1 p^{n-1} q$$

$$= p^n + \frac{n!}{1 \cdot (n-1)!} p^{n-1} q$$

$$R(G_r) = p^n + np^{n-1} q$$

- **A multiring with three-cycle topology ($G_{mr\text{-}3c}$)**

A multiring with three-cycle is a topology in which the main ring is separated into two sub-rings by a common path which is a series of connecting edges as shown in Figure 1c. First, let us consider a simple form of a multiring with three-cycle topology as shown in Figure 2. It consists of a ring of nodes in which there is one interconnected edge, called a common edge, across the ring. We call this a single ring with a common edge ($G_{r+}$). The edge $i$-$j$ is added in the topology so that the single ring is divided into two parts; the first part consisting of $n_1$ nodes and the second part consisting of $n_2$ nodes where $n_1+n_2 = n$. The network reliability of $G_{r+}$ can be derived as follows:

$R(G_{r+})$ = Probability that all nodes can communicate to each other while the $i$-$j$ edge is in operational state + Probability that all nodes can communicate to each other while the $i$-$j$ edge fails

$$= pR(G_{r_{n_1}})R(G_{r_{n_2}}) + (1-p)R(G_{r_n})$$

$$= p\{p^{n_1} + n_1 p^{n_1-1} q\}\{p^{n_2} + n_2 p^{n_2-1}\} + q\ \{p^n + np^{n-1} q\}$$

$$= p\{p^n + n_2 p^{n-1} q + n_1 p^{n-1} q + n_1 n_2 p^{n-2} q^2\} + q\ \{p^n + np^{n-1} q\}$$

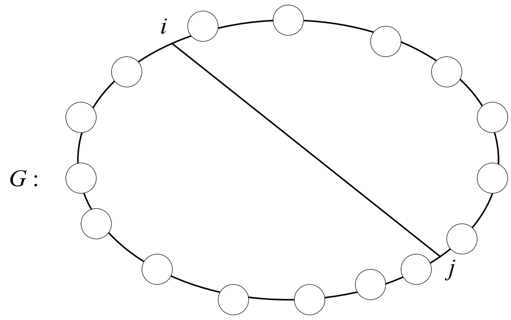


Figure 2. A single ring with a common edge.

$= p^n + np^{n-1}q + n_1 n_2 p^{n-1} q^2$

since, $n_2 = n - n_1$

Thus, $R(G_{r+}) = p^n + np^{n-1}q + n_1(n-n_1)p^{n-1}q^2$

To obtain the maximum network reliability, the function $R(G_{r+})$ is differentiated and set to zero. Then, it can be derived that when $n_1 = n_2 = n/2$, the network topology results in the highest network reliability.

Now let us consider the multiring with three-cycle topology ($G_{mr-3c}$). The following notation is defined:

$k$ = the number of edges in the common path

$h$ = the number of nodes that are in the main ring (i.e., not in the common path)

For maximum network reliability, $n_1 = n_2 = h/2$

Thus, the network reliability of the multiring with three-cycle topology, $R(G_{mr-3c})$, can be derived as follows:

$R(G_{mr-3c})$ = Probability that the nodes in the first part and the second path can communicate to each other while the common path is in operational state + Probability that all nodes in the main ring can communicate to each other while the common path fails

$$= R(G_{n1})R(G_{n2})p^k + R(G_h)(kp^{k-1}q)$$

$$= R(G_{\frac{h}{2}})R(G_{\frac{h}{2}})p^k + R(G_h)(kp^{k-1}q)$$

$$= p^k\{p^{\frac{h}{2}} + \frac{h}{2}p^{\frac{h}{2}-1}q\}^2 + \{p^h + hp^{h-1}q\}\{kp^{k-1}q\}$$

$$= p^k\{p^h + hp^{h-1}q + \frac{h^2}{4}p^{h-2}q^2\} + \{p^h + hp^{h-1}q\}\{kp^{k-1}q\}$$

$$= p^h\{p^k + kp^{k-1}q\} + hp^{h-1}q\{p^k + kp^{k-1}q\} + \frac{h^2}{4}p^{h-2}q^2p^k$$

$$= \{p^{n+1} + kp^nq\} + \{(n+1-k)[p^nq + kp^{n-1}q^2]\} + \frac{(n+1-k)^2}{4}p^{n-1}q^2$$

$$= p^{n+1} + (n+1)p^nq + \{(n+1-k)k + \frac{(n+1-k)^2}{4}\}p^{n-1}q^2$$

$$= p^{n+1} + (n+1)p^nq + \{\frac{(n+1)^2 + 2k(n+1) - 3k)^2}{4}\}p^{n-1}q^2$$

To obtain the maximum network reliability, the function $R(G_{mr-3c})$ is differentiated and set to zero. Then, it can be derived that when $k = (n+1)/3$, the network topology results in the highest network reliability. Substituting $k$ in the equation above we obtain

$$R(G_{mr-3c}) = p^{n+1} + (n+1)p^nq + \frac{(n+1)^2}{3}p^{n-1}q^2 .$$