



Original Article

An integer programming for airplane routing in the U.S. Center-TRACON

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Abstract

Air travel has been a major transportation for commerce and tour in many countries. As the demand of air traffic has been increasing, air traffic management has confronted with poverty of handling the increase of the demand of runway facilities where congestion often takes place. In order to cope with such problems, runway efficiency enhancement or capacity increasing are taken into account. In air traffic management, the effective air space utilization and air control workload management can be improved by the use of many up-to-date technologies in forms of decision support tools. This study developed a computer-aided decision support model in the form of integer programming. The purpose of the model was to allocate airplanes arrival at U.S. Center-TRACON airspace to enter feeder gates and to design optimal routes along the track to runway. Results of optimal path of the airplanes throughout the TRACON air space system which yield a minimum delay were presented.

Keywords: airplane allocation, airplane routing, integer programming, air traffic management, The Center-TRACON Automation System

1. Introduction

In the past several years, many organizations collaborated with air traffic have developed various kinds of automated decision support tools to assist air traffic management. One of the famous decision support tools proposed by NASA and Federal Aviation Administration (FAA) is the Center-TRACON Automation System (CTAS) which is developed to improve airport capacity, productivity, and reduce delay while maintaining controller workload at a reasonable level. The main objective of this paper is to use a mathematical optimization to allocate the airplanes and design their routes from entering the border of the Center-TRACON and landing

at the feeder gate by minimizing delay or minimizing the time the airplane spend in the system. Most of the time, a significant amount of delay occurs in preparation for landing during the flight. Persistently, it is necessary for the plane to encircle or slowdown in order to wait for the sequence of arrival schedule designed by air traffic controllers. Such problem could be considered as a network problem and the potential time saving could be analyzed by using different scenarios to outline a traffic merge near the airport.

This study is the extension work of Supsomboon and Zabinsky (2003) by considering input automation and increasing more tracks. In the Center-TRACON system, airplanes will arrive at the boarder of the center and enter one of the four quadrants of four different diagonal directions (i.e., north, south, east, and west). The Center-TRACON and its four quadrants are shown in Figure 1. Each quadrant has three tracks. The controller would use some tools to assign the airplane to enter the best track in the quadrant expecting

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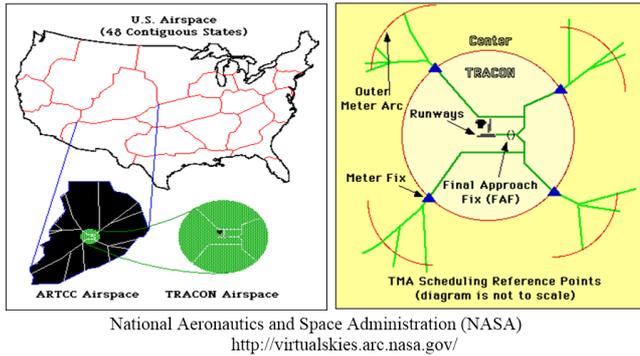


Figure 1. Center-TRACON

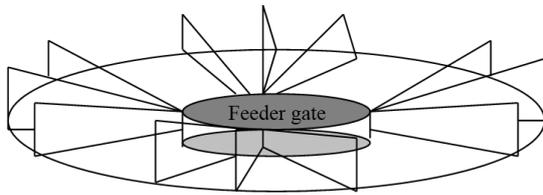


Figure 2. Center-TRACON feeder gate and its 12 tracks in four quadrants

to minimize delay. The airplanes need to stick with same quadrant corresponded to the direction prior to entering the center. Due to the limitation of the available software, this study extends the work of previous study by considering two tracks in the same quadrant. The planes route in the track would be assigned by using minimum-cost-of-delay criteria. The automatic input method will also be presented. Due to this basic concept, the model can be enlarged to cope with all tracks in all four quadrants. Figure 2 presents the three tracks in four quadrants. Airplanes will traverse from the border of the center to the feeder gate. To avoid conflicts and minimize delay, an optimal plane allocation and routing are needed.

2. Literature Review

Expanding airport or runway is not always a good idea in order to reduce air traffic congestion. A fair solution in a short term period can be achieved by reducing the impact of delays produced by congestion. Therefore, managing the demand of air traffic flows to prevent capacity shortage is a potential alternative answer. Nowadays, air traffic flow management has been widely studied. Many studies focus on reducing congestion range from changing routes of flight (re-routing). Some studies concentrate on controlling aircraft speed from monitoring the rate of traffic flows (metering), and some on imposing a delay prior to flight departures (ground holding).

Jackson and *et al.* (2005) proposed the distributed air traffic management by using the capability of airborne electronic systems to relieve workload of controller. They

also presented an overview of avionics capabilities and a detailed description of five specific examples of airborne capability which can be used to increase airspace capacity. Lecchini-Visintini (2006) presented a framework for conflict resolution that taking account the levels of uncertainty by using a stochastic simulator. The conflict resolution task was created as problem of optimizing an expected value criterion. Markov chain Monte Carlo was used to carry out optimization of the expected value resolution criterion through the procedures. Simulation examples were shown to illustrate the proposed conflict resolution strategy. Bayen and *et al.* (2007) presented a method for the numerical computation of reachable sets for hybrid systems which is applicable to specific phases of landing: descent, flare, and touchdown. Oishi and *et al.* (2008) proposed an invariance-preserving abstraction that generates a discrete event system. This system can be used to analyze, to verify, and to design user-interfaces for hybrid manual-automation systems. Margellos and Lygeros (2009) presented the applications of reachability methods and computational tools based on game theory to solve problem in air traffic management. An original concept of operations was developed in the CATS project based on the Target Windows. Lymperopoulos and Lygeros (2010) showed the formulation of multi-aircraft sensor fusion problem as a high-dimensional state estimation problem. The inefficiency of several sequential Monte Carlo algorithms on feasibility studies involving multiple aircraft were also demonstrated in the study. Sun and *et al.* (2010) used a linear time varying aggregate traffic flow model to develop traffic flow management strategies by using optimization algorithms. Castelli and *et al.* (2011) proposed a mathematical formulation to identify the critical flights that may be liable to produce undesired downstream effects if subjected to delay by defining a set of time window which must be met during the flight execution. Hawley and *et al.* (2013) presented the collaborative approach in Air Traffic Management (ATM) to security management in order to improve situational awareness, quantitative risk assessments and the governance of security. Simaiakis and Balakrishnan (2014) proposed a stochastic model of runway departures and algorithm of dynamic programming for the control at airports. The runway system was modeled as a semi-Markov process by using a multi-variable state description that includes the capacity forecast. Zillies and *et al.* (2014) investigated whether wind optimal routes has a positive impact on travel time and fuel consumption of inner-European flights. Fuel consumptions achieved from wind and distance optimized trajectories were compared in order to evaluate the savings potential.

3. The Model

The model was started at the moment when the airplane reaches the center until it reaches the gate according to the model of traffic destined for TRACON feeder gate. Typically, the distance from the border of the center to the TRACON feeder gate is 330 nautical miles (n.m.). However,

the distance of 35 n.m. is studied, in order to simplify model due to the limitation of the computer program. Each particular track consists of waypoints at five altitudes. In our model, only three altitudes, FL 390, 370 and 350 are considered. The small circles in the network represent the waypoints the airplane travel along the way on the track and they are named by the capital letter $A1, B1, \dots, R1$ for track 1 and $A2, B2, \dots, R2$ for track 2. A^1 and A^2 represent the areas of the two tracks. Figure 3 illustrates waypoints in a track. For safety reasons, the minimum vertical separation of the airplane is 5 n.m. Therefore, the distance between the node in the network is assigned to be 5 n.m.. The consecutive waypoints are connected by the imaginary arcs. During the way to feeder gate, the aircraft are allowed to change one altitude to the lower level. In order to land, the planes at the higher level must drop to the lowest level. When more than two planes trying to seize the same waypoint, only one can accomplish while the others wait until that waypoint is available. The waiting planes are held on imaginary holders. These holders represent a holding pattern in the air. They are treated as any other intermediate nodes and are expressed by the $A1'', B1'', C1''$ and so on for the regular nodes $A1, B1$, and $C1$, respectively. There is one holding node for each particular regular node except for the nodes at the end of the track E, K and R , because the airplanes are not allowed to delay or wait at the end of the track for safety reasons. As soon as the waypoint is available for the next waiting airplane, it can get out of the holding node and continue traversing the next waypoint. Totally, there are four different kinds of nodes in the network: source, waypoints, destination, and holders. Figure 4 presents the network in each particular track.

The sources of aircraft to enter the Center are identified by $\omega_i^E(t)$ and $\beta_i^E(t)$, and leave by $\omega_i^L(t)$ and $\beta_i^L(t)$ for track 1 and 2, respectively. The arcs are denoted by their initial node (i) and terminal node (j) at any particular time step (t). For instance, the number of planes traversed from node A to node B is identified by the variable $X_{ij}(t) = X_{A1B1}(t)$ and $Y_{ij}(t) = Y_{A2B2}(t)$. In this model, we study only one type of aircraft with the same speed at the same level of altitude, therefore passing will never happen and FIFO discipline is applied. As the higher the altitude the faster the airplane can go, we assume that a particular airplane needs one time step to traverse between two adjacent waypoints at the highest altitude and two time steps, then three time steps for the lower altitude levels, respectively. The average amounts of time step required to traverse arc (i, j) are represented by τ_{ij} and γ_{ij} . All airplanes from both tracks share the destination node S , as there is only one entrance of runway for each quadrant. Once the airplane chooses a particular track, it cannot change but has to travel along that track until it descends to the runway.

The network problem of minimizing the summation of time steps with associated costs was formulated as an integer programming. Parameters, variables and formulation are presented in the tables below.

Notations

Parameters

- T = Number of time steps in the total time
- C_{ij}^1 = Cost associated to number of time steps used at each level of altitude of Track 1
- C_{ij}^2 = Cost associated to number of time steps used at each level of altitude of Track 2
- τ_{ij} = Average amount of time required to traverse arc (i, j) by an airplane in Track 1
- γ_{ij} = Average amount of time required to traverse arc (i, j) by an airplane in Track 2
- $\omega_i^E(t)$ = Number of airplane entering the system at Track 1 at node i , at time t
- $\beta_i^E(t)$ = Number of airplane entering the system at Track 2 at node i , at time t
- $S(i)$ = Successors of node i . The set $S(i)$ contains all the nodes that can be reached from node i by traversing only one arc.
- $P(i)$ = Predecessors of node i . The set $P(i)$ contains all the nodes that can reach from node i by traversing only one arc.

Decision variables

- $X_{ij}(t)$ = Number of airplanes leaving node i at time t , going to node j in Track 1
- $Y_{ij}(t)$ = Number of airplanes leaving node i at time t , going to node j in Track 2

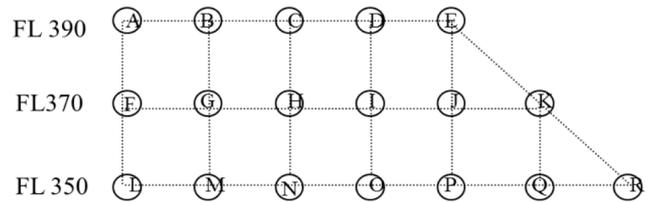


Figure 3. Waypoints in a track

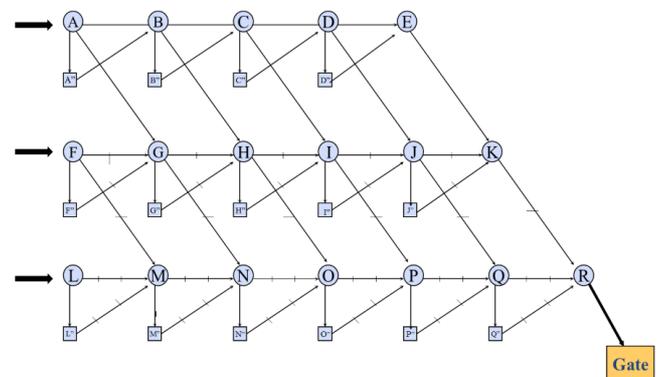


Figure 4. Network in each particular track.

$\omega_i^L(t)$ = Number of airplanes leaving the system at destination i at time t of Track 1
 $\beta_i^L(t)$ = Number of airplanes leaving the system at destination i at time t of Track 2

Recall that this modeled is formulated for two tracks in one quadrant of feeder gates. Each pair of similar constraints is for Tract 1 and Tract 2, respectively. The objective function in the model, constraint (1), is to minimize the summation of the time steps the airplanes traverse in the system since they arrive until they leave with the associated costs. The costs C_{ij}^1 and C_{ij}^2 are associated with the amounts of time required for each altitude in each section which are τ_{ij} and γ_{ij} . Constraints (2) and (3) are flow continuity. Every airplane enter the node must leave, accept the destination node S . These constraints also describe that the numbers of airplanes inter system and node equals the numbers leave. Constraint (4) assigns the airplane to enter destination node S one plane at one time step regardless of track they come from. As regard constraints (5) and (6), the minimum separation is taken into account. They describe that maximum rate-flow of each arc is one airplane. One arc can never be occupied by more than one plane. Consider the network diagram, one imaginary arc is represented by two arrows, one is the traverse from regular node to regular node (e.g., A to B) and the other is from the holding node to regular node (e.g., A'' to B). Each imaginary node can be occupied by one airplane from one of the two cases (i.e., arc capacity), which are presented by equation (7) and (8). Constraints (9) and (10) limit the capacity of any intermediate node not to exceed one plane at any point in time. The last constraint (11) includes the natural integrity and non-negativity constraints.

Formulation

Objective

$$\text{Minimize } \sum_{t=1}^T \sum_{(i,j) \in A^1} C_{ij}^1 X_{ij}(t) + \sum_{t=1}^T \sum_{(i,j) \in A^2} C_{ij}^2 Y_{ij}(t) \quad (1)$$

Subject to

$$\sum_{j \in S(i)} X_{ij}(t) - \sum_{j \in P(i)} X_{ji}(t - \tau_{ij}) = \begin{cases} \omega_i^E(t) & , i = A1, F1, L1 \\ -\omega_i^L(t) & , i = S \\ 0 & , i = \text{intermediate nodes} \end{cases} \quad \forall(i,t) \quad (2)$$

$$\sum_{j \in S(i)} Y_{ij}(t) - \sum_{j \in P(i)} Y_{ji}(t - \gamma_{ij}) = \begin{cases} \beta_i^E(t) & , i = A2, F2, L2 \\ -\beta_i^L(t) & , i = S \\ 0 & , i = \text{intermediate nodes} \end{cases} \quad \forall(i,t) \quad (3)$$

$$\omega_i^L(t) + \beta_i^L(t) \leq 1 \quad (4)$$

$$\sum_{u=t+1}^{t+\tau_{ij}-1} X_{ij}(u) \leq 1 - X_{ij}(t) \quad \forall(i,t) \quad (5)$$

$$\sum_{u=t+1}^{t+\gamma_{ij}-1} Y_{ij}(u) \leq 1 - Y_{ij}(t) \quad \forall(i,t) \quad (6)$$

$$X_{ij}(t) + X_{i''j}(t) \leq 1 \quad \forall(i,t) \quad (7)$$

$$Y_{ij}(t) + Y_{i''j}(t) \leq 1 \quad \forall(i,t) \quad (8)$$

$$\sum_{\substack{j \in S(i) \\ i \neq j}} X_{ij}(t) \leq 1 \quad \forall(i,t) \quad (9)$$

$$\sum_{\substack{j \in S(i) \\ i \neq j}} Y_{ij}(t) \leq 1 \quad \forall(i,t) \quad (10)$$

$$0 \leq X_{ij}(t) \leq 1, 0 \leq Y_{ij}(t) \leq 1, 0 \leq \omega_i^L(t) \leq 1, 0 \leq \beta_i^L(t) \leq 1, \text{ all are non-negative integer} \quad (11)$$

Automatic Input

$$Z^E(t) = \sum_{t=1}^T \sum_{i \in S(i)} \omega_{ij}^E(t) + \beta_{ij}^E(t)$$

As the sources of aircraft to enter the Center are identified by two parameters, $\omega_i^E(t)$ and $\beta_i^E(t)$, it means that each airplane need to be assigned specifically to a particular entering node (A, F , and L) at time (t) according to the exact direction it reaches the gate as parameters of the model. It will be more effective for air traffic control management if the airplanes can be reallocated to enter the entering nodes based on minimizing the total delay time in the

$$Z^E(t) = \sum_{t=1}^T \sum_{i \in S(i)} \omega_{ij}^E(t) + \beta_{ij}^E(t)$$

system. This can be formulated by adding one more constraint to the above model. $Z^E(t)$ is defined to be a new parameter which represents the total number of airplanes entering the system at any time t . Then, parameters $\omega_i^E(t)$ and $\beta_i^E(t)$ are converted to be decision variables. The optimization model will automatically determine which entering node the plane should seize when they enter the gate with the minimum system cost.

4. Case Study

In the case study, the simple values of costs associated with the number of time steps used of altitudes FL 390, 370 and 350 are assigned to be 1, 2, and 3, respectively, for both tracks (C_{ij}^1 and C_{ij}^2) and for all nodes. We propose two methods of assigning airplanes to enter the TRACON: First, the airplanes will enter the Center-TRACON at the altitude to where they traverse prior to approaching the TRACON. In

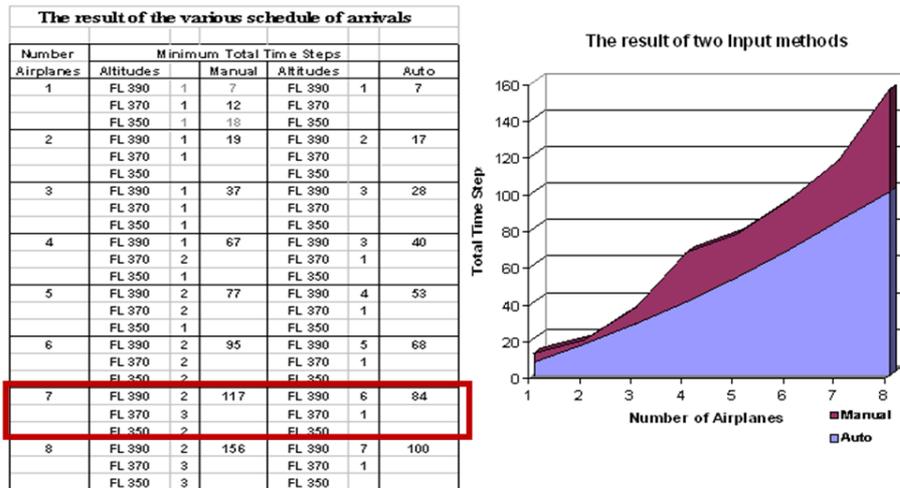


Figure 5. Results of various arrivals for eight scenarios of manual and auto input.

this case, the controller would assign airplane to seize the altitude manually. It is called manual input method (i.e., manually input the airplane into the TRACON). Second, the airplanes will enter the Center-TRACON by optimization model which yield optimal routes to the system. The solution of the optimization model would provide the optimal track each particular airplane should seize the boarder of the center, so called automatic input. For both input methods, the optimization model will provide the optimal routes for each airplane.

As mentioned earlier, one assumption is that only one type of airplanes is considered. That means each plane have same speed while traversing in the same altitude. To simplify the network, it is assumed that airplanes are required one time step to traverse from one node to the adjacent node at the highest altitude, two and three time steps at the lower altitudes, respectively. It means that the higher altitude the airplanes traverse, the faster the airplanes could reach the feeder gate. In case of congestion where the airplanes cannot reside in the highest altitude or maintain in the higher altitudes, they can choose either to hold on to the current altitude (re-circle) or to descend to the lower level. The queue discipline assumed in the study is first in first out (FIFO). In general, it is very seldom that more than one airplane would enter the same track at the same time. However, to investigate the worst case scenarios and to test whether the model can handle the complicated situation, we would study eight scenarios by assigning from 1 to 8 numbers of airplanes to approach the same track simultaneously. Figure 5 presents the results of various arrival schedules for eight scenarios along with the comparison of manual input versus automatic input. Figure 6 shows an example of airplane routes of the scenario where seven airplanes entering the Center at the same time.

The routes of the planes were also tracked by each time steps shown in Figure 6. Minimum total time step of

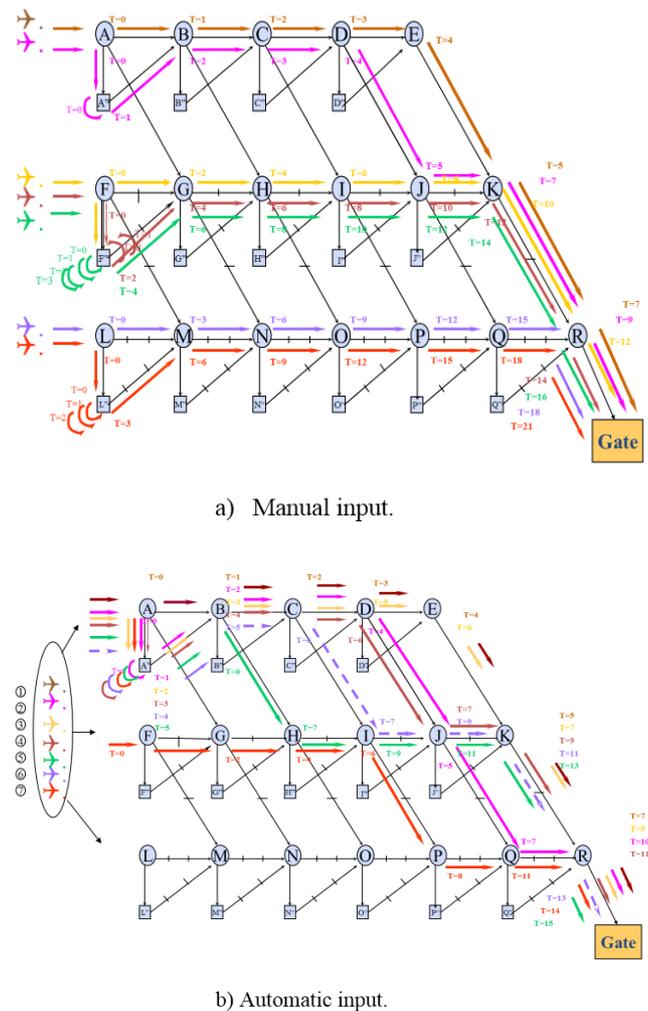


Figure 6. Airplanes' routes: scenario of the seven airplanes entering the center at the same time: a) manual input, and b) automatic input.

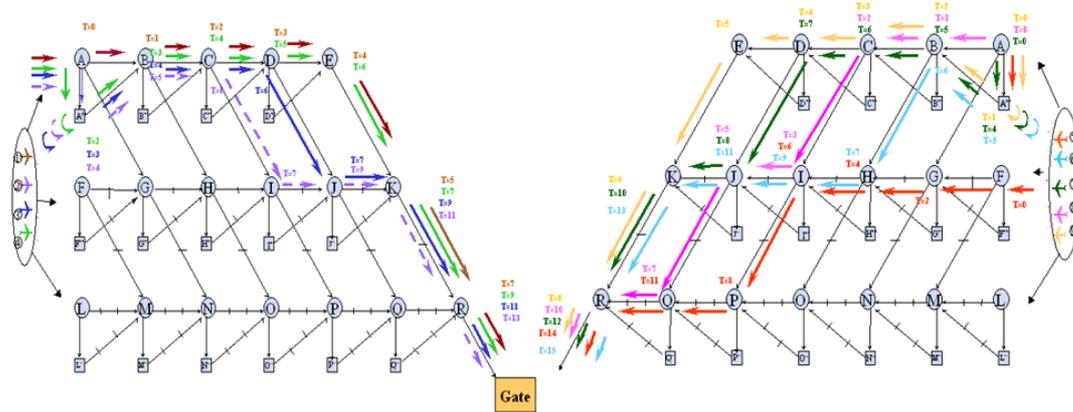


Figure 7. Nine airplanes entering two tracks simultaneously

manual input in a) is 117, while automatic input in b) is 84. For further example of two tracks, Figure 7 represents the scenario of nine airplanes entering both tracks simultaneously. The minimum total time step is 108.

5. Result Analysis

The results of computer runs show that automatic input method gives better solutions than the manual one. It is because the manual input takes only advantage of optimal route, while the automatic input takes two benefits of optimal allocation and optimal route. Due to the assumption that the higher the altitude, the higher the speed of the aircraft allowed, the airplanes seize the higher level of altitude rather than descend for both input methods. To avoid crashing when overcrowding, some planes hold on the holding node and some descend to the lower level which yield the minimum total delay of the entire system.

6. Conclusions

This study developed a computer-aided decision support tool based on an integer programming for air traffic management. The results of the optimum solutions which yielded the minimum delay provided allocation of airplanes entering the U.S. Center-TRACON airspace and assigned the optimal routes along the track to feeder gate. The results of automatic input method were more desirable than the manual input because the automatic input method provides the solutions of optimal airplane allocation and routing, while manual input gives only optimal route. These two input methods are applicable in diverse situations. With the concepts of original integer programming proposed in this study, larger model coping with all tracks in all four quadrants could be simply created.

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