



Original Article

Analysis and design of nomographs for minimum headway calculation

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Abstract

This study proposes the design and analysis of nomographs for minimum headway calculations using critical blocks to reduce the complexity of mathematical equations. For trains with different speeds, block overlapping should be checked forward or backward in a looping manner, while nomographs simplify the analysis and eliminate iterations. Nomographs for headway calculation are constructed using Python scripts in PyNomo software. The associated variables include train speed, train length, and block length. Three nomographs are designed for three types of operation: Model 1 for cases wherein two trains have equal speeds, Model 2 for those wherein the leading train is faster, and Model 3 for those wherein the leading train is slower. The validation reveals that the nomographs yield minimum headways that are close to the result obtained by mathematical derivation. Data can be used to create a train schedule for safe operation without conflict.

Keywords: block length, minimum deadway, nomographs, critical block, train schedule

1. Introduction

To maximize efficiency of train scheduling, the headways between the trains should be minimized (Fransoo & Bertrand, 2000; Li *et al.*, 2014). Currently, the minimum headway can be determined using the block time model (de Fabris *et al.*, 2014; Fumasoli *et al.*, 2015; Landex & Kaas, 2005; Medeossi *et al.*, 2011; Parkinson, 1996). This model can be used for trains with equal (Lindner, 2011) and unequal (Harrod, 2009; Huisman & Boucherie, 2001) speeds. The key variables affecting the headway are train lengths (Banks, 2002; Mao *et al.*, 2006), block length (Abril *et al.*, 2008; Dicembre & Ricci, 2011; LEE, 1997; Landex & Kaas, 2005; Liu *et al.*, 2011; Union Internationale des Chemins de fer [UIC], 2004), and speed (Mitra *et al.*, 2010).

The determination of the minimum headway should consider the critical block, which is the block that defines a safe minimum headway (Goverde *et al.*, 2013). A time-distance diagram may be used to classify train operation and determine the effects of speed, train length, and block length upon the critical block. The analysis should be divided into two cases: equal and unequal block lengths. For unequal block lengths, when the train speeds are equal, the longest block will be the critical one. If the train speeds are different, minimum headway determination becomes more complex as and looping processes are required to check key conditions.

Nomograms or nomographs are designed using a graphical form to analyze and present the results (Cantinotti *et al.*, 2016; Gluchoff, 2012; Lu *et al.*, 2016). A nomograph is normally constructed to determine solutions under various cases (Auerswald *et al.*, 2014) and forecast results. They have been widely used, particularly in the medical field (Kawai *et al.*, 2015; Lee *et al.*, 2015; Morris *et al.*, 1993; Samplaski *et al.*, 2014) and constitute an extremely useful tool for solving repetitive problems that might otherwise require complex mathematical equations (Bandyopadhyay, 1983; Thananiyaudom, 1997) they are flexible for various applications.

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This research constructs graphical nomography tools as a prototype solution for reducing complexity and determining minimum headway assuming key relevant factors, including train length, block length, and speed. This research focuses partially on unidirectional operation with equal and unequal block lengths.

2. Blocking Time

Time Headway is a key measure in determining line capacity and establishing the timetable. Time Headway has defined the difference between the time when the front of a train arrives at a point on the track and the time the front of the next train arrives at the same referenced points on both trains.

The analysis of time headway can be classified into equal and unequal block length scenarios. This analysis should consider the critical block length that defines minimum

headway and maximum capacity without conflict at any location. The time spent in the critical block comprises running time, signal-watching time (*wt*), clearing time in signal (*tfc*), and release time (*rt*). The combination of these components is known as blocking time (de Fabris *et al.*, 2014; Hansen & Pachl, 2014; Medeossi *et al.*, 2011; Pachl, 2002).

The headway analysis starts with assigning a speed V_i for the first train and V_j for the second on a route with n blocks. Only one train can enter a block at a given time. The analysis comprises two cases: trains with same speeds or $V_i = V_j$ (Lindner, 2011) and trains with different speeds or $V_i > V_j$ and $V_i < V_j$ (Hernando *et al.*, 2010; Huisman & Boucherie, 2001; Kanai *et al.*, 2011; Mussone & Wolfler Calvo, 2013; Vromans *et al.*, 2006). The time-distance diagram in Table 1 shows the operation under equal block length conditions and the effects of the number of trains, order, block length, and speed difference on the critical block (Sangphong *et al.*, 2017).

Table 1. Determination of headway under equal block length operation.

Type	Formula	Time-Distance Diagram
$V_i = V_j$	$\frac{BL + l_i}{V_i} + wt + tfc + ct + rt$	
$V_i > V_j$	$\frac{BL + l_i}{V_i} + wt + tfc + ct + rt$	
$V_i < V_j$	$\frac{nBL + l_i}{V_i} + T_{FB} - \frac{(n-1)BL}{V_j}$ $T_{FB} = wt + tfc + ct + rt$	

Where

V_i	= speed of train i	V_j	= speed of train j
BL	= Block length	n	= number of blocks
l	= Train length	ct	= Clearing time in the block
rt	= Release time	wt	= Signal watching time
tfc	= Signal clearing time		

When the block lengths are different, headway analysis becomes more complicated. A hierarchical check is required to prevent conflicts. Complex mathematical models

take into account train speed, train length, and block length in determining minimum headway. Figure 1 shows a diagram explaining the steps for identifying the critical block and determining the safe minimum headway.

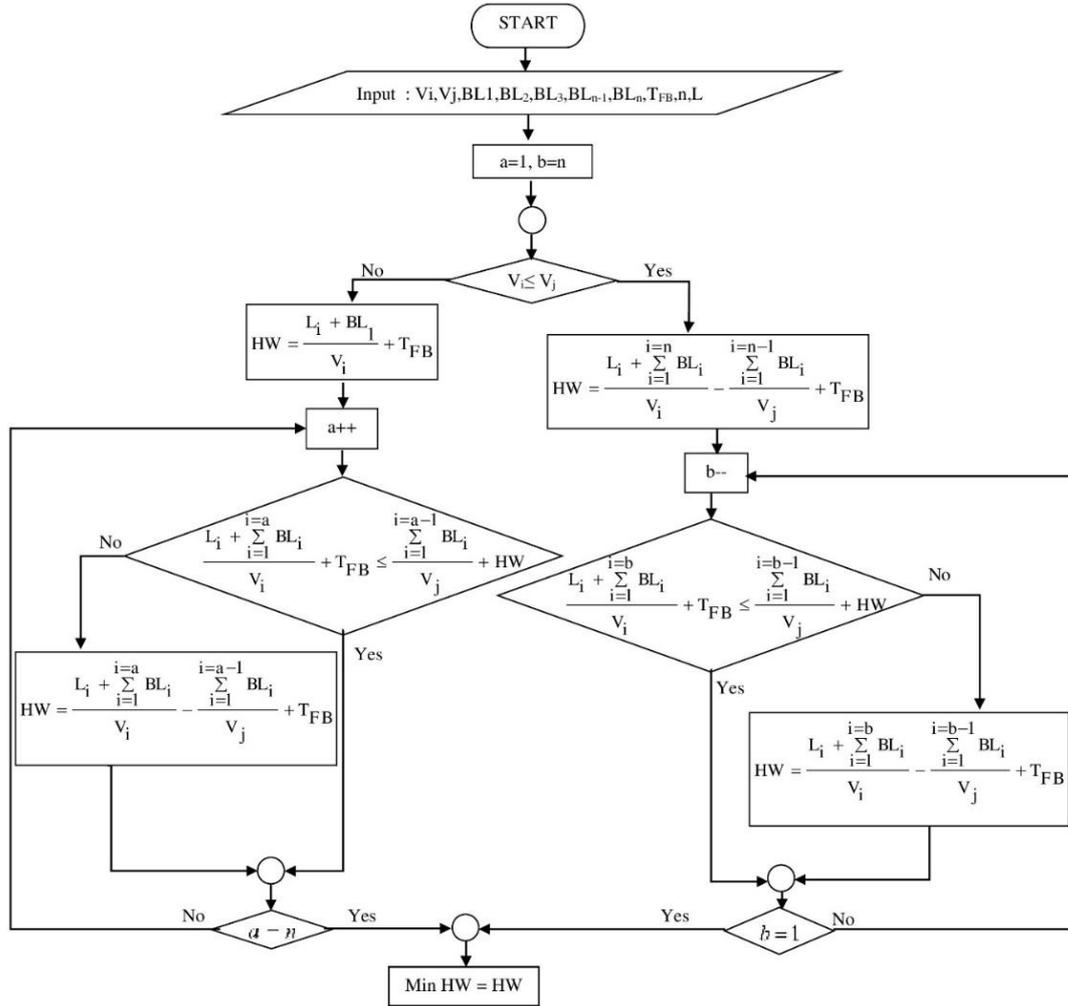


Figure 1. Headway determination diagram.

This study presents the design of a prototype nomograph to facilitate calculation under all conditions. This nomograph uses the PyNomo program, powered by Python script. To create nomographs, tenforms of equations are

normally applied depending on the relationship of sub-equations $F_i(u_i)$. Four forms have been selected to calculate the minimum headways in this study, as shown in Table 2. Nomographs are constructed following the procedures given in Figure 1.

Table 2. Types of nomographs supported by PyNomo.

Type	Form of Equation	Form of Nomogram
Type 1	$F_1(u_1) + F_2(u_2) + F_3(u_3) = 0$	Three parallel lines
Type 2	$F_1(u_1) = F_2(u_2)F_3(u_3)$	Left-tilting “N” or right-tilting “Z”
Type 3	$F_1(u_1) + F_2(u_2) + \dots + F_N(u_N) = 0$	N parallel lines with reference axes
Type 6	$F_1(u_1) = F_2(u_2)$	Scale transforming “Ladder”

3. Research Method

Nomographs use lines to represent variables and distances between lines and scale to represent the relation between variables affecting headway. The ranges on scales are designed to cover the train and track characteristics. These

nomographs are flexible. Therefore, they can be applied to various cases of operation, including changes in route or speed characteristics. The research method involves developing practical nomographs for train minimum headway determination. The results from the nomograph are validated with analytical solution to confirm its accuracy. The research framework is illustrated in Figure 2.

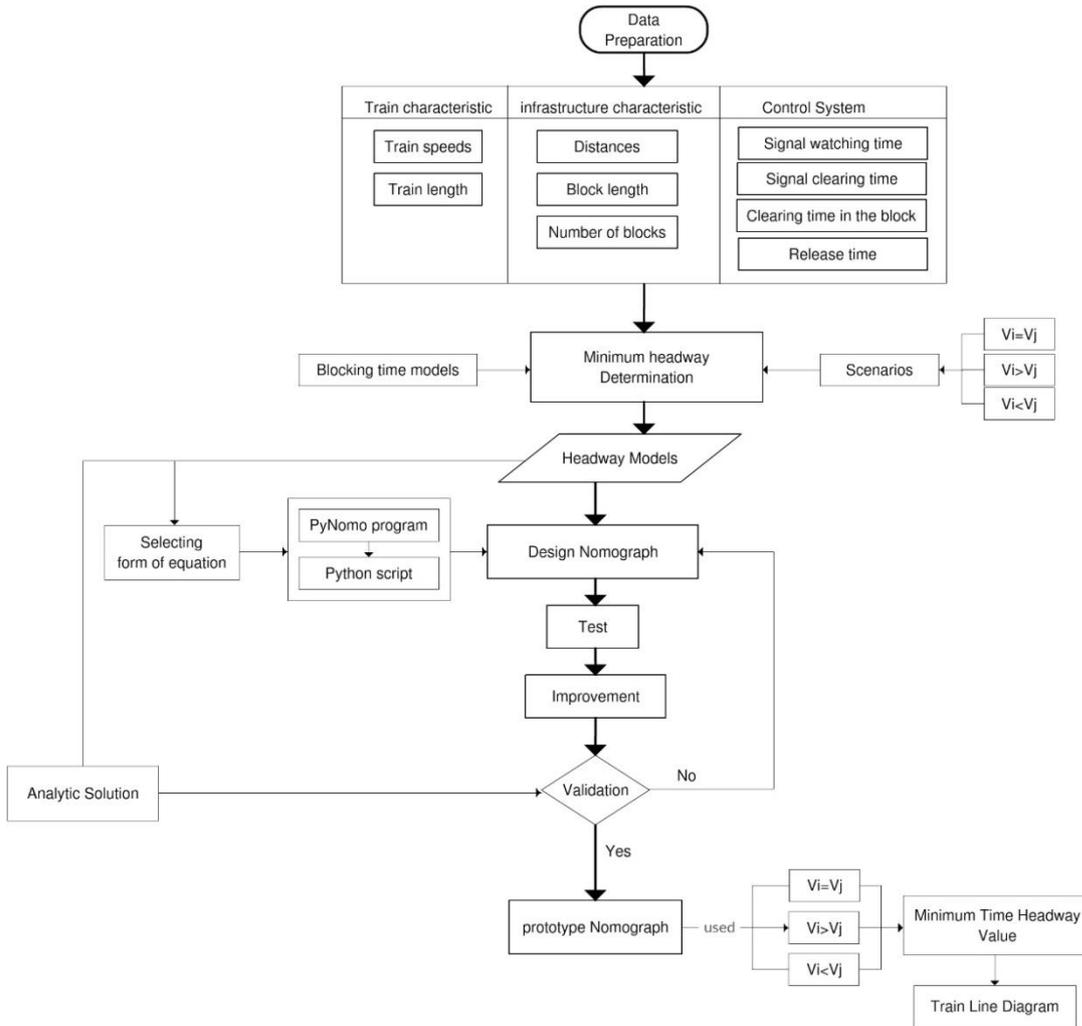


Figure 2. Conceptual framework diagram of analysis and design of nomographs for minimum headway calculation.

3.1 Model 1 for Vi = Vj

The nomograph in Figure 4 is designed based on the critical block determined by maximum block length. This nomograph can be applied to a maximum block length of 70 km, with speeds of 50–150 km/hr. It is suitable for Vi = Vj on routes with equal and unequal block lengths. The following data are required:

- Distance from the origin station to the end of the longest block (D_{bmax}) km, where $D_{bmax} = \sum_{i=1}^k BL_i$ and k is the

position of maximum block length (Figure 3 shows an example of calculating D_{bmax})

- Maximum block length, BL_{max} (km)
- Train length, L (m)
- Leading and following train speeds, V_i and V_j (km/hr), respectively
- Signal-watching time and clearing time in signal plus the release time and clearing time in block, T_{FB} (min) = $wt + t_{fc} + rt + ct$

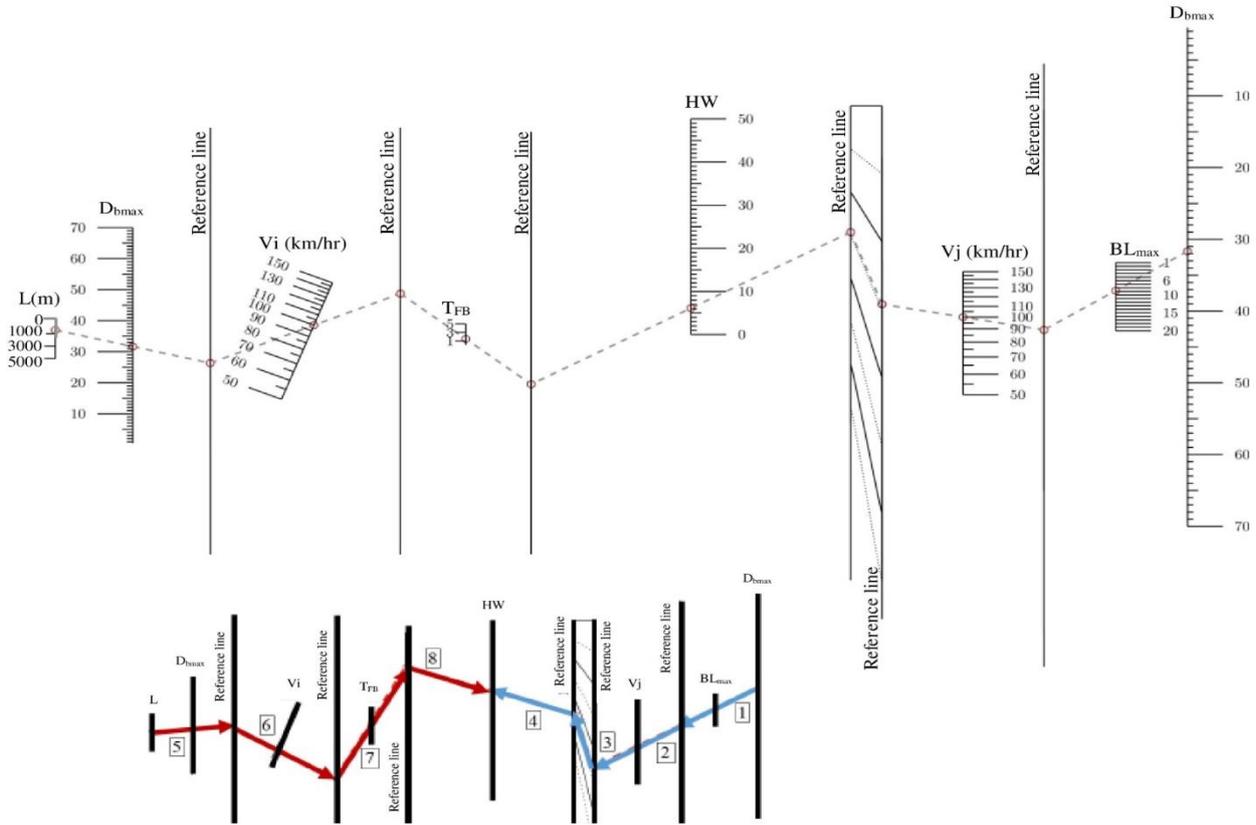


Figure 4. Nomograph Model 1 for $V_i = V_j$.

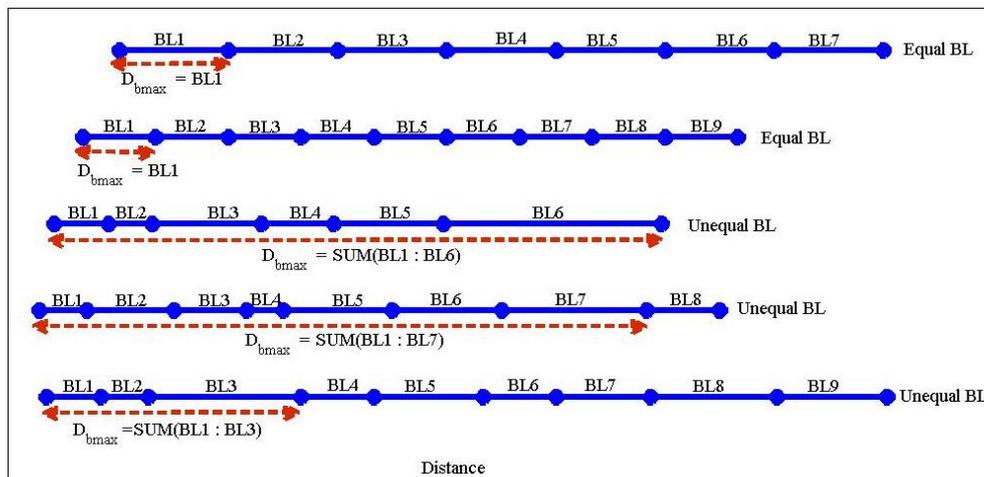


Figure 3. D_{bmax} calculation example.

3.2 Model 2 for $V_i > V_j$

The nomograph in Figure 5 is designed under the assumption that the leading train is faster. The trailing train can be released after the leading train has left the block. Thus, the first block becomes the critical block. This nomograph can be applied to block lengths of up to 10 km with speeds of 40–

140 km/hr. It requires three line connections among the graphs and is suitable for operations under $V_i > V_j$ with equal and unequal block lengths. The following data are required:

- The first block length, BL_1 (km)
- Train length, L (m)
- Leading train speed, V_i (km/hr)
- T_{FB} (min)

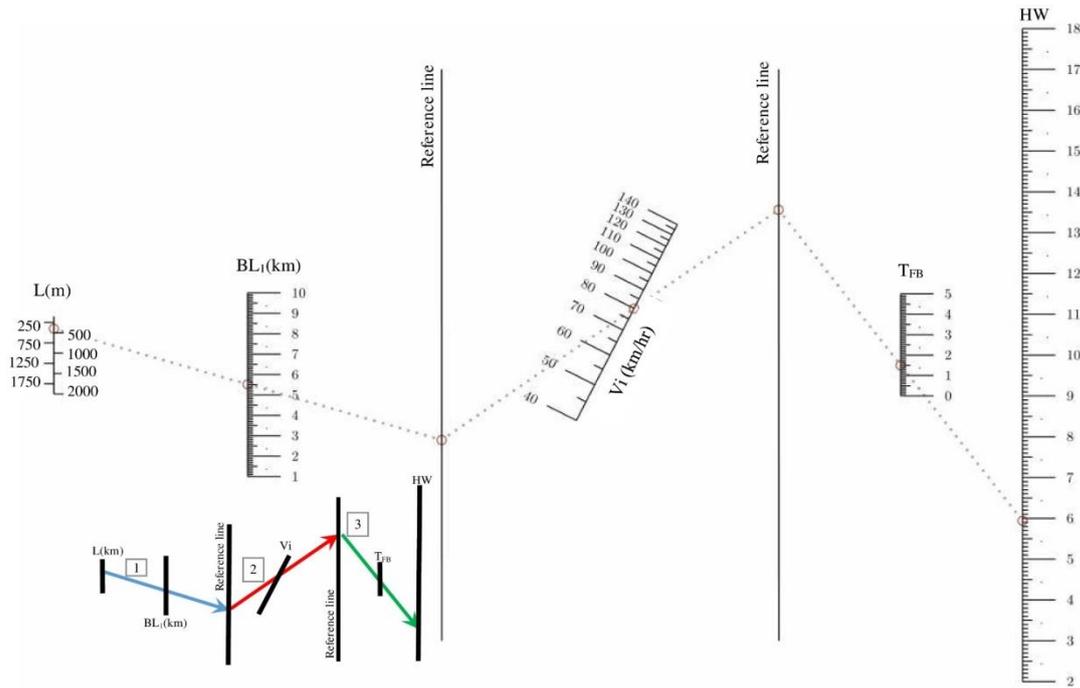


Figure 5. Nomograph Model 2 for $V_i > V_j$.

3.3 Model 3 for $V_i < V_j$

The nomograph in Figure 6 is designed for headway determination when the leading train is slower. Thus, the last block normally defines the critical block unless the blocks have significantly different lengths. The following train has to wait until the leading train arrives at the last block before being safely released from the origin station. This nomograph

can be used for route lengths of up to 100km with speeds of 50–150 km/hr and it is suitable for operations under $V_i < V_j$. The following data are required:

- Distance from origin to destination (D_n) (km) = $\sum_{i=1}^{i=n} b_i$
- Train length, L (m)
- Train speeds, V_i, V_j (km/hr)
- T_{FB} (min)

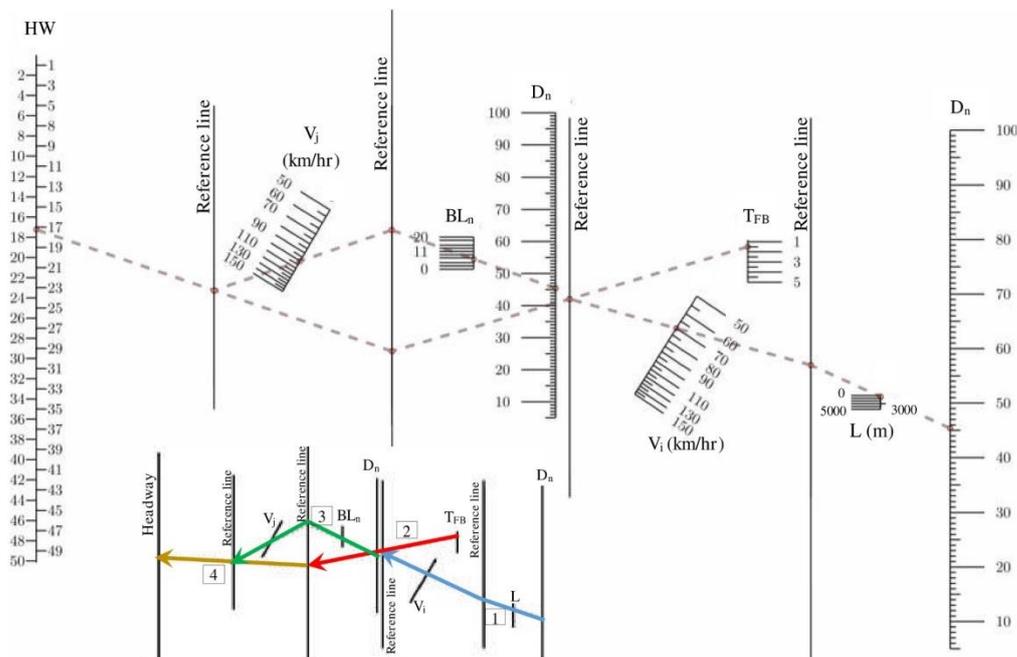


Figure 6. Nomograph Model 3 for $V_i < V_j$.

4. Results and Discussion

The nomographs are validated for a route with unequal block lengths using real distance data from the Nong Nam Khun (NNK)–Nakhon Ratchasima (NR) section of the

State Railway of Thailand (SRT) Northeastern Line. The section comprises eight stations spanned across a total of 45.38 km. A block covers the distance between two adjacent stations. Thus, the block lengths in this section are different, as shown in Figure 7 (a).

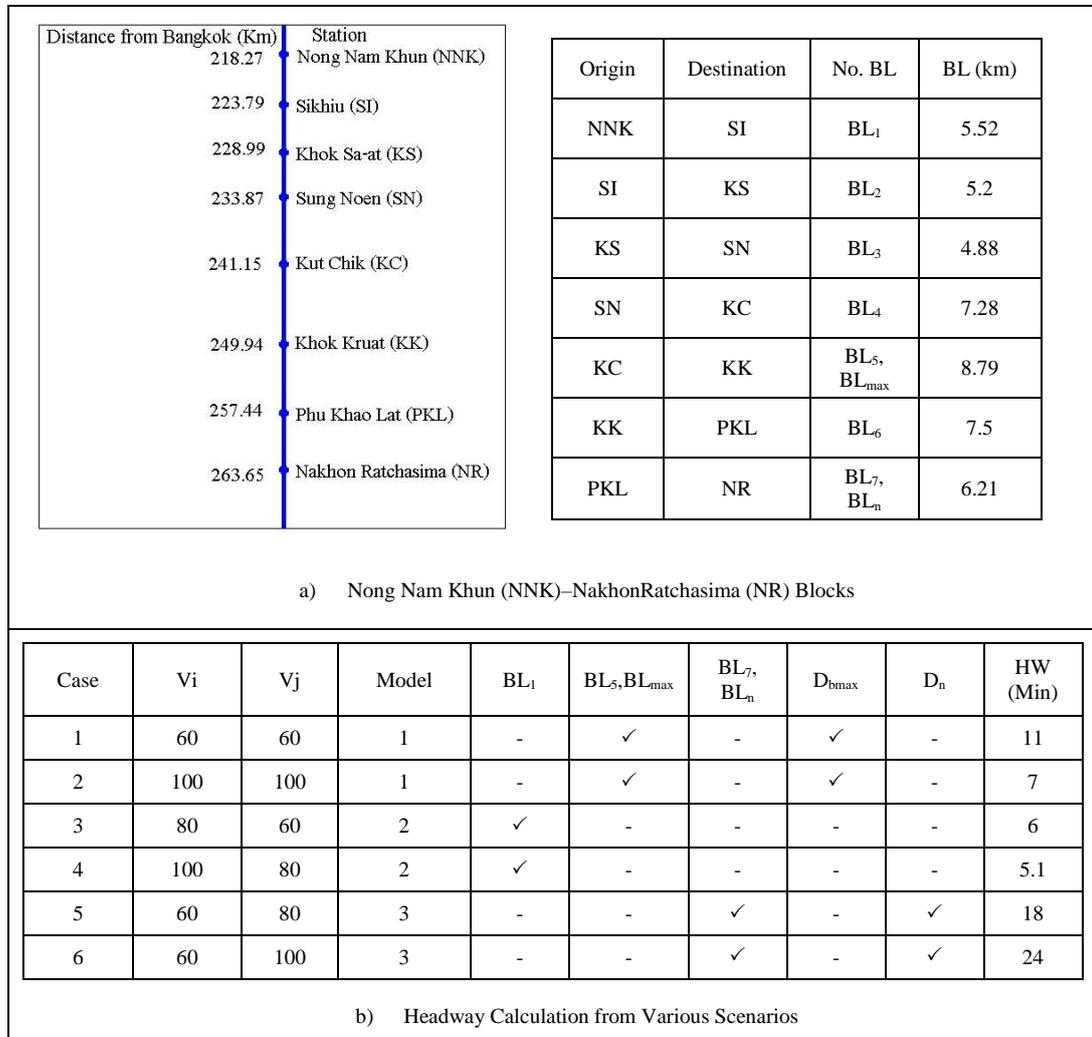


Figure 7. Headway calculation by nomography.

4.1 Nomography application

The validation is conducted with 60, 80, and 100 km/hr speeds for the $V_i = V_j$, $V_i < V_j$, and $V_i > V_j$ cases, assuming a train length (L) of 400 meters and $T_{FB} = 1.5$ min. A total of six cases are tested with combinations of speeds and other variables, as shown in Figure 7 (b). illustrate the determination of the headway using nomographs.

4.2 Operations on a time–space diagram

The nomographs are validated with time–space diagrams using the blocking time on the studied route. Minimum headways obtained from the nomograph are used as initial headways between the two trains for all six cases as shown in Figure 8. Blocking stairways show that both trains can run together without any conflict. Therefore, the headway from the three nomographs can be assumed to be the minimum headway from the critical block consideration. No space is available for further headway reduction.

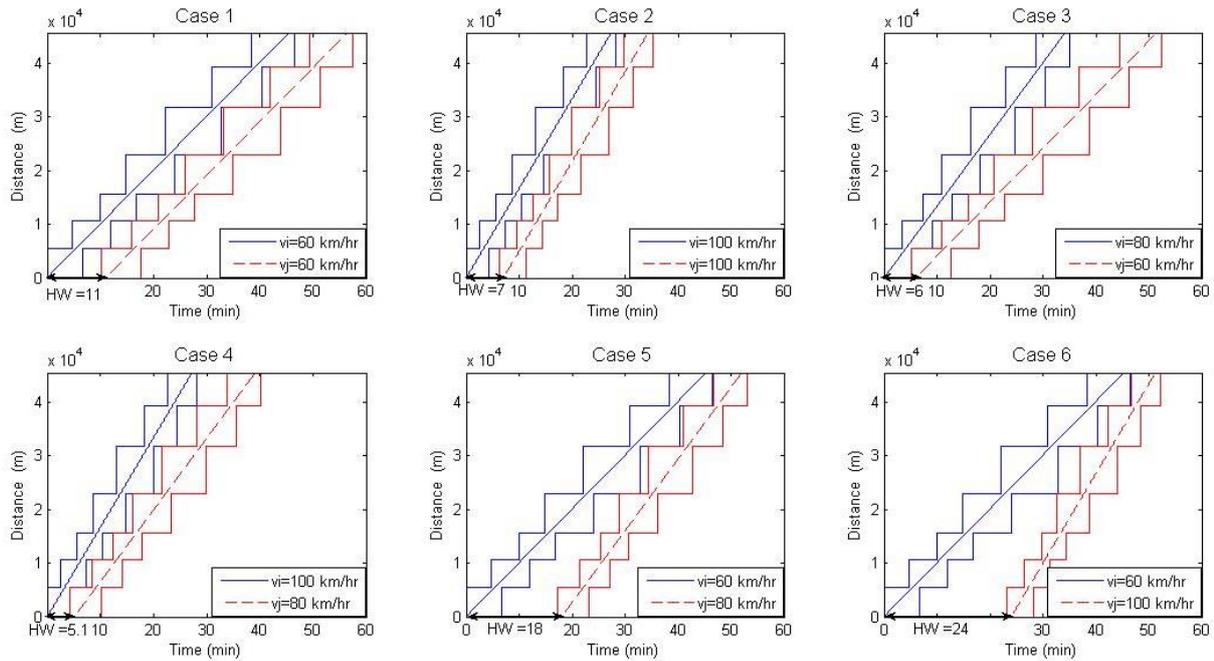


Figure 8. Blocking Time Stairway of Train for All Six Cases As Computed by Nomography

When two trains of different types alternately run on the route, the nomograph application should be divided into two parts to determine the two headways. For example, consider two trains running on the NKK–NR section with speeds of $V_i = 60 \text{ km/hr}$ and $V_j = 100 \text{ km/hr}$. The first part of

the headway between Train 1 and Train 2 can be determined using Model 3 as it is under the $V_i < V_j$ condition, whereas the headway between Train 2 and Train 3 uses Model 2 as it falls in the range of $V_j > V_i$. The speed, V_j , is fixed at 100 km/hr . The train operation diagram is shown in Figure 9.

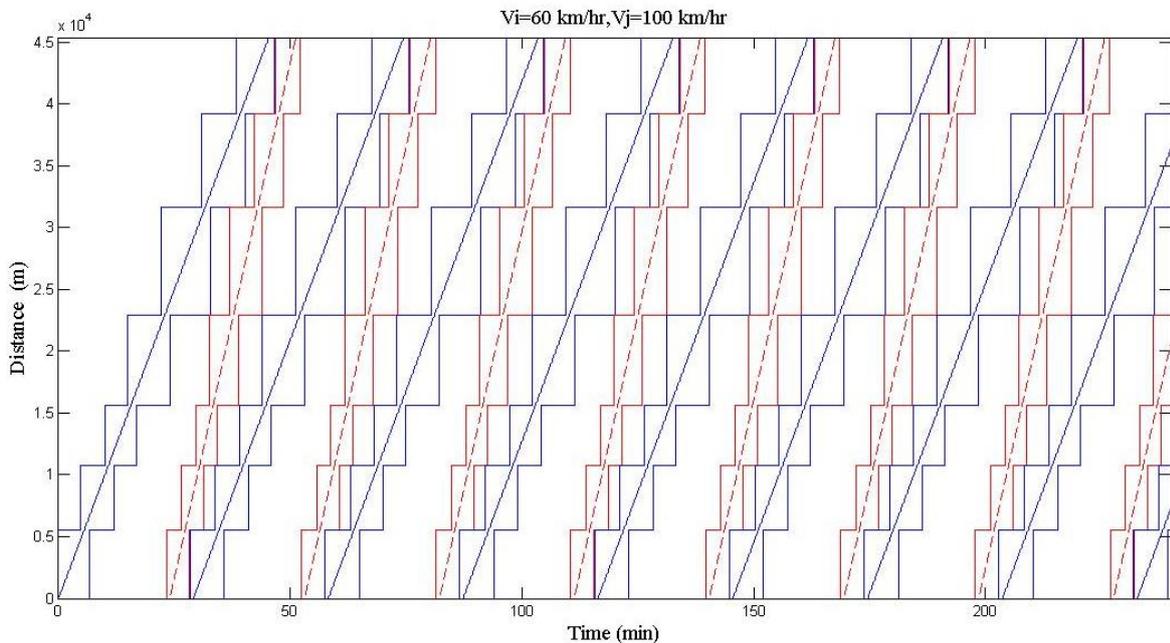


Figure 9. Time-Space diagram for two types of trains with headways calculated from Model 2 and 3.

4.3 Nomography vs. mathematical analysis

Headways calculated from the nomographs are close to those computed by mathematical equations. The differences are only in decimals. Nomographs can be used to effectively estimate headways in both equal and unequal block length cases. More variables, such as buffer time (Büker, 2013) and dwell time, can be added to T_{FB} or as additional lines to increase the efficiency of estimation. In the route where a given block length is more than 2.5 times the other and $V_i \neq V_j$, it is recommended that the result be compared with that obtained using Model 1. One must compare headways from two nomographs and choose the larger value to prevent conflict. For example, when $V_i > V_j$, the maximum value of the headways obtained from Models 1 and 2 must be chosen, and when $V_i < V_j$, the maximum value of the headways obtained from Models 1 and 3 must be chosen.

5. Conclusions

Under operation with equal block lengths, the minimum headway can simply be determined by mathematical equations. When trains are running on different block lengths, the significant variables and conditions become more complicated. To determine the minimum headway, one must consider the hierarchy of conditions and may have to rely on a software package to determine the solution. The nomographs are validated, and it is proved that they yield results close to those obtained by mathematical analysis. In addition, the graphs are sufficiently flexible to be used for any type of operations, including trains with equal and unequal speeds on sections with equal and unequal block lengths. However, this research designs the nomographs to be used as tools for quickly estimating the minimum headway and reducing the complexity of the analysis. In reality, train operation involves the variation of speeds at the shut, stop and between stations constrained by geometry. Further research could add acceleration, deceleration and other types of speed variation to better reflect real operating conditions. Interested individuals can adopt and enhance the use of nomographs in academic and practical analysis of railway projects. Nomograph is a flexible tool that can be customized to solve various systems in the future. Including improved single and double-track railway operation. In can also serve as an effective analytical tools under scenarios with major and minor adjustments in the future systems.

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References

Abril, M., Barber, F., Ingolotti, L., Salido, M. A., Tormos, P., & Lova, A. (2008). An assessment of railway capacity. *Transportation Research Part E: Logistics and Transportation Review*, 44(5), 774-806.

- Auerswald, K., Fiener, P., Martin, W., & Elhaus, D. (2014). Use and misuse of the K factor equation in soil erosion modeling: An alternative equation for determining USLE nomograph soil erodibility values. *CATENA*, 118, 220-225.
- Bandyopadhyay, S. S. (1983). Sample Size of Pavement Deflections by Nomograph. *Journal of Transportation Engineering*, 109(4), 599-604.
- Banks, J. H. (2002). *Introduction to Transportation Engineering, 2nd Edition*. 251-282. New York, NY: Mc Graw-Hill Publishing.
- Büker, T. (2013). Methods of assessing railway infrastructure capacity. *Engineering Science and Technology, an International Journal*, 16(2), 39-51.
- Cantinotti, M., Giordano, R., Scalese, M., Murzi, B., Assanta, N., Spadoni, I., . . . Iervasi, G. (2016). Nomograms for two-dimensional echocardiography derived valvular and arterial dimensions in Caucasian children. *Journal of Cardiology*, 69(1), 208-215.
- deFabris, S., Longo, G., Medeossi, G., & Pesenti, R. (2014). Automatic generation of railway timetables based on a mesoscopic infrastructure model. *Journal of Rail Transport Planning and Management*, 4(1-2), 2-13.
- Dicembre, A., & Ricci, S. (2011). Railway traffic on high density urban corridors: Capacity, signalling and timetable. *Journal of Rail Transport Planning and Management*, 1(2), 59-68.
- Fransoo, J. C., & Bertrand, J. W. M. (2000). An aggregate capacity estimation model for the evaluation of railroad passing constructions. *Transportation Research Part A: Policy and Practice*, 34(1), 35-49.
- Fumasoli, T., Bruckmann, D., & Weidmann, U. (2015). Operation of freight railways in densely used mixed traffic networks – An impact model to quantify changes in freight train characteristics. *Research in Transportation Economics*, 54, 15-19.
- Gluchoff, A. (2012). The History and Development of Nomography by H. A. Evesham (Docent Press). *Historia Mathematica*, 39(4), 469-475.
- Goverde, R. M. P., Corman, F., & D'Ariano, A. (2013). Railway line capacity consumption of different railway signalling systems under scheduled and disturbed conditions. *Journal of Rail Transport Planning and Management*, 3(3), 78-94.
- Hansen, I. A., & Pacht, J. (2014). *Railway Timetabling & Operations: Analysis, Modelling, Optimisation, Simulation, Performance Evaluation* (2nd ed). Hamburg, Germany: Deutscher Verkehrsvlg.
- Harrod, S. (2009). Capacity factors of a mixed speed railway network. *Transportation Research Part E: Logistics and Transportation Review*, 45(5), 830-841.
- Hernando, A., Roanes-Lozano, E., & García-Álvarez, A. (2010). An accelerated-time microscopic simulation of a dedicated freight double-track railway line. *Mathematical and Computer Modelling*, 51(9-10), 1160-1169.
- Huisman, T., & Boucherie, R. J. (2001). Running times on railway sections with heterogeneous train traffic. *Transportation Research Part B: Methodological*, 35(3), 271-292.

- Kanai, S., Shiina, K., Harada, S., & Tomii, N. (2011). An optimal delay management algorithm from passengers' viewpoints considering the whole railway network. *Journal of Rail Transport Planning and Management*, 1(1), 25-37.
- Kawai, K., Ishihara, S., Yamaguchi, H., Sunami, E., Kitayama, J., Miyata, H., . . . Watanabe, T. (2015). Nomograms for predicting the prognosis of stage IV colorectal cancer after curative resection: A multicenter retrospective study. *European Journal of Surgical Oncology*, 41(4), 457-465.
- Landex, A., & Kaas, A. H. (2005). Planning the most suitable travel speed for high frequency railway lines. *1st International Seminar on Railway Operations Modelling and Analysis*. Delft, The Netherlands: Technische Universiteit Delft.
- LEE, C. K. (1997). The minimum headway of a rail transit line. *Journal of the Eastern Asia Society for Transportation Studies*, 2(1).
- Lee, C. K., Goldstein, D., Gibbs, E., Joensuu, H., Zalberg, J., Verweij, J., . . . Rutkowski, P. (2015). Development and validation of prognostic nomograms for metastatic gastrointestinal stromal tumour treated with imatinib. *European Journal of Cancer*, 51(7), 852-860.
- Li, F., Sheu, J. B., & Gao, Z. Y. (2014). Deadlock analysis, prevention and train optimal travel mechanism in single-track railway system. *Transportation Research Part B: Methodological*, 68, 385-414.
- Lindner, T. (2011). Applicability of the analytical UIC Code 406 compression method for evaluating line and station capacity. *Journal of Rail Transport Planning & Management*, 1(1), 49-57.
- Liu, H., Mao, B., Wang, B., Du, P., & Ding, Y. (2011). Optimization of Railway Section Signalling Layout Based on Quasi-Moving Block. *Journal of Transportation Systems Engineering and Information Technology*, 11(4), 103-109.
- Lu, L. F., Huang, M. L., & Zhang, J. (2016). Two axes re-ordering methods in parallel coordinates plots. *Journal of Visual Languages and Computing*, 33, 3-12.
- Mao, B., Liu, J., Ding, Y., Liu, H., & Ho, T. K. (2006). Signalling layout for fixed-block railway lines with real-coded genetic algorithms. *Hong Kong Institution of Engineers, Transactions*, 13(1), 35-40.
- Medeossi, G., Longo, G., & de Fabris, S. (2011). A method for using stochastic blocking times to improve timetable planning. *Journal of Rail Transport Planning and Management*, 1(1), 1-13.
- Mitra, S., Tolliver, D., & Mitra, S. (2010). Estimation of Railroad Capacity Using Parametric Methods. *Journal of the Transportation Research Forum*, 49(2), 111-126.
- Morris, C. K., Myers, J., Froelicher, V. F., Kawaguchi, T., Ueshima, K., & Hideg, A. (1993). Nomogram based on metabolic equivalents and age for assessing aerobic exercise capacity in men. *Journal of the American College of Cardiology*, 22(1), 175-182.
- Mussone, L., & Wolfler Calvo, R. (2013). An analytical approach to calculate the capacity of a railway system. *European Journal of Operational Research*, 228(1), 11-23.
- Pachl, J. (2002). *Railway Operation and Control*. Mountlake Terrace, WA: VTD Rail Publishing.
- Parkinson, T. (1996). Rail Transit Capacity. *Transit capacity and quality of service manual* (2nd Ed.). Retrieved from <http://onlinepubs.trb.org/onlinepubs/tcrp/docs/tcrp100/Part0.pdf>
- Samplaski, M. K., Yu, C., Kattan, M. W., Lo, K. C., Grober, E. D., Zini, A., . . . Jarvi, K. A. (2014). Nomograms for predicting changes in semen parameters in infertile men after varicocele repair. *Fertility and Sterility*, 102(1), 68-74.
- Sangphong, O., Sirdhara, S., & Ratanavaraha, V. (2017). Determining critical rail line blocks and minimum train headways for equal and unequal block lengths and various train speed scenarios. *Engineering Journal*, 21(3), 281-293.
- Thananitayadom, T. (1977). Communication systems Analysis and design using nomographs. *Computer Networks*, 1(3), 147-154.
- Union Internationale des Chemins de fer. (2004). *UIC CODE 406R* (1st Ed.). Paris, France: International Union of Railways.
- Vromans, M. J. C. M., Dekker, R., & Kroon, L. G. (2006). Reliability and heterogeneity of railway services. *European Journal of Operational Research*, 172(2), 647-665.