Songklanakarin J. Sci. Technol.
40 (5), 1055-1060, Sep. - Oct. 2018

http://www.sjst.psu.ac.th

Original Article

ps-ro fuzzy α-irresolute functions

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Received: 10 March 2017; Revised: 27 May 2017; Accepted: 22 June 2017

Abstract

The aim of this paper is to initiate and study a new class of functions termed as ps-ro fuzzy α-irresolute functions. It is found that this class of functions is independent of the existing class of fuzzy α-irresolute, ps-ro fuzzy irresolute and ps-ro fuzzy continuous functions. Also the ps-ro fuzzy α-irresoluteness is found to be stronger than ps-ro fuzzy semicontinuity, ps-ro fuzzy irresoluteness and ps-rofuzzy α-continuity. Several characterizations of these functions are obtained in terms of newly introduced concept of ps-α interior (closure) operators, ps-ro fuzzy α-nbd, ps-ro fuzzy dense set and their graphs.

Keywords: ps-ro fuzzy topology, ps-ro fuzzy α-irresolute functions, ps-ro fuzzy α-nbd, ps-ro fuzzy dense set

1. Introduction

While studying the interplay among a fuzzy topological space (for short, fts) (X,τ) and its corresponding general topology called strong α-level topology, ps-ro fuzzy topology was introduced, whose members and their complements are called ps-ro open and ps-ro closed fuzzy sets respectively (Deb Ray & Chettri, 2010). In terms of these, a class of functions called ps-ro fuzzy continuous functions was introduced and explored (Deb Ray & Chettri, 2011, 2016). The notions of ps-ro fuzzy semi continuity, ps-ro fuzzy irresoluteness and ps-ro fuzzy α-continuity were introduced and their different properties and interrelations with the existing allied concepts were studied in (Chettri & Gurung, 2016; Chettri, Gurung, & Halder, 2014; Chettri, Gurung, & Katwal, 2016).

In this paper, we introduce the notions of ps-ro fuzzy α-irresoluteness, determine their various characterizations and establish their relation with existing notion of various types of fuzzy functions.

We state a few known definitions and results here that we require subsequently. A fuzzy point x_t is a fuzzy set where $0 < t \leq 1$ and defined as $x_t(z)$ equals to t for $z = x$, otherwise its value is 0. A fuzzy point x_t is called q-coincident with a fuzzy set B, written as $x_t q B$ if $t + B(x) > 1$ (Pao-Ming & Ying-Ming, 1980). For a function g from a set A into a set B, the following holds:

(i) $g^{-1}(1-V) = 1 - g^{-1}(V)$, for any fuzzy set Von B.

(ii) $U_1 \leq U_2 \Rightarrow g(U_1) \leq g(U_2)$, for any fuzzy sets $U_1$and U_2 on A. Also, $V_1 \leq V_2 \Rightarrow g^{-1}(V_1) \leq g^{-1}(V_2)$, for any fuzzy sets $V_1$and V_2on B.

(iii) For any fuzzy set U and Von A and B, $gg^{-1}(V) \leq V$ and $g^{-1}g(U) \geq U$. Equality holds if g is onto and one-to-one respectively (Chang, 1968).

Let g be a function between two non-empty sets A and B and U, V be fuzzy sets on A and B respectively, then $1-U$ (called complement of U), $g(U)$ and $g^{-1}(V)$ are fuzzy sets on A, B and A respectively, defined by $(1-U)(x) = 1 - U(x)\ \forall\ x \in A$, $g(U)(y) = \begin{cases} \sup_{z \in g^{-1}(y)} U(z), when\ g^{-1}(y) \neq \emptyset \\ 0,\ otherwise \end{cases}$ and $g^{-1}(V)(x) = V(g(x))$ (Zadeh, 1965).

A fuzzy topology τ is a family of fuzzy sets on A if (a) $0, 1 \in \tau$ (b) arbitrary union and finite intersection of members of τ belongs to τ.Then (A,τ) is called a fts. Members of the fuzzy topology and their complements are termed as fuzzy open and closed sets respectively on A (Chang, 1968).

For a fuzzy set γ on A, the set $\gamma^\alpha = \{x \in A : \gamma(x) > \alpha\}$ is termed as strong α-level set of A. In a fts (A,τ), the family $i_\alpha(\tau) = \{\gamma^\alpha : \gamma \in \tau\}$ for all $\alpha \in I_1 = [0,1)$ forms a

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strong α-level topology on A (Kohli & Prasan nan, 2001; Lowen, 1976). A fuzzy open set γ on a $fts\ (A,\tau)$ is said to be pseudo regular open fuzzy set if γ^{α} is regular open in $(A, i_{\alpha}(\tau))$, $\forall \in I_1$. The collection of all pseudo regular open fuzzy sets form a fuzzy topology on A called ps-ro fuzzy topology on A, whose members are called ps-ro open and their complements as ps-ro closed fuzzy sets on (A,τ) (Deb Ray & Chettri, 2010). A fuzzy set Uon a $fts\ (A,\tau)$ is called fuzzy α-open if $U \leq int(cl(int(U)))$ (Bin Sahana, 1991).

Fuzzy ps-closure and ps-interior of U are denoted by ps-$cl(U)$ and ps-$int(U)$ respectively and are given by ps-$cl(U) = \wedge\{V: U \leq V,\ V$ is ps-ro closed fuzzy set on $A\}$ and ps-$int(U) = \vee\{V: V \leq U,\ V$ is ps-ro open fuzzy set on $A\}$ (Deb Ray & Chettri, 2011, 2016). A fuzzy set Uon a $fts\ (A,\tau)$ is said to be ps-ro semiopen (ps-roα-open) fuzzy set if $U \leq ps$-$cl(ps$-$int(U))$(resp. $U \leq ps$-$int(ps$-$cl(ps$-$int(U))))$ (Chettri & Gurung, 2016; Chettri, Gurung, & Halder, 2014).

A function g from a $fts\ (A,\tau_1)$ to another $fts\ (B,\tau_2)$ is

(i) fuzzy α-irresolute if the inverse image of every fuzzy α-open set on B is also so on A (Prasad, Thakur, & Saraf, 1994).

(ii) ps-ro fuzzy continuous (ps-ro semicontinuous, ps-ro α-continuous) if the inverse image of every ps-ro open fuzzy set on B is ps-ro open (resp. ps-ro semiopen, ps-ro α-open) fuzzy set on A (Chettri & Gurung, 2016; Chettri, Gurung, & Halder, 2014; Deb Ray & Chettri, 2011, 2016).

(iii) ps-ro fuzzy irresolute if the inverse image of every ps-ro semi open fuzzy set on B is also so on A (Chettri, Gurung, & Katwal, 2016).

2. ps-ro Fuzzy α-Irresolute Function

Definition 2.1: A function g between two $fts\ (A,\tau_1)$ and (B,τ_2) is called ps-ro fuzzy α-irresolute if $g^{-1}(V)$ is ps-ro α-open fuzzy set on A for each ps-ro α-open fuzzy set V on B.

We discuss some examples below to establish interrelations of the newly defined function with few well known existing functions between two fts.

Example 2.1: Let $A = \{a,b,c\}$ and $B = \{x,y,z\}$. Let U, V and W be fuzzy sets on A defined by $U(a) = 0.2, U(b) = 0.2, U(c) = 0.3$; $V(r) = 0.2, \forall\ r \in A$; $W(r) = 0.5, \forall\ r \in A$. Let P, Q, R and S be fuzzy sets on B given by $P(s) = 0.4, \forall\ s \in B$; $Q(x) = 0.5, Q(y) = 0.5$, $Q(z) = 0.6$; $R(s) = 0.3, \forall\ s \in B$ and $S(x) = 0.3, S(y) = 0.3, S(z) = 0.4$. Clearly, $\tau_1 = \{0,1,U,V,W\}$ and $\tau_2 = \{0,1,P,Q,R,S\}$ are fuzzy topologies on A and B respectively. In the corresponding strong α-level topological space $(A, i_{\alpha}(\tau_1)) \forall \alpha \in I_1$, the open sets are $\emptyset, A, U^{\alpha}, V^{\alpha}$ and $W^{\alpha}$where

$$U^{\alpha} = \begin{cases} A\ for\ \alpha < 0.2 \\ \{c\}\ for\ 0.2 \leq \alpha < 0.3 \\ \emptyset\ for\ \alpha \geq 0.3 \end{cases},\quad V^{\alpha} = \begin{cases} A\ for\ \alpha < 0.2 \\ \emptyset\ for\ \alpha \geq 0.2 \end{cases} \text{ and}$$

$$W^{\alpha} = \begin{cases} A\ for\ \alpha < 0.5 \\ \emptyset\ for\ \alpha \geq 0.5 \end{cases}$$

For $0.2 \leq \alpha < 0.3$, the closed sets on $(A, i_{\alpha}(\tau_1))$ are $\emptyset, A$ and $A - \{c\}$. As $int(cl(U^{\alpha})) = A$, A^{α} is not regular open on $(A, i_{\alpha}(\tau_1))$for $0.2 \leq \alpha < 0.3$ and hence U is not pseudo regular open fuzzy set on (A,τ_1). $int(cl(V^{\alpha})) = V^{\alpha}$ and $int(cl(W^{\alpha})) = W^{\alpha}$. So, V^{α} and W^{α} are regular open on $(A, i_{\alpha}(\tau_1))$, $\forall\ \alpha \in I_1$. $0, 1, V$ and W are pseudo regular open fuzzy sets on (A,τ_1) and thus ps-ro fuzzy topology on A is $\{0,1,V,W\}$. Again, Q and S are not pseudo regular open fuzzy sets for $0.5 \leq \alpha < 0.6$ and $0.3 \leq \alpha < 0.4$ respectively on B. So, the ps-ro fuzzy topology on B is $\{0,1,P,R\}$. Let g be a function from $fts\ (A,\tau_1)$ to $fts\ (B,\tau_2)$ defined by $g(a) = x, g(b) = y$ and $g(c) = z$. $0, 1, P$ and R are ps-ro open and hence ps-ro α-open fuzzy sets on B. Any fuzzy set T on B satisfying $R \leq T \leq P$ is also ps-ro α-open fuzzy set on B. For all ps-ro α-open fuzzy set H on B, $g^{-1}(H)$ is also so on A. Hence, g is ps-ro fuzzy α-irresolute. Here, Q is fuzzy α-open set on B but $g^{-1}(Q)$ is not so on A. So, g is not fuzzy α-irresolute.

Example 2.2: Let $A = \{a,b,c\}$ and $B = \{x,y,z\}$. Let U, V and W be fuzzy sets on A defined by $U(r) = 0.5, \forall\ r \in A$; $V(a) = 0.1, V(b) = 0.1, V(c) = 0.2$; $W(r) = 0.3, \forall\ r \in A$. Let P, Q, R, S and T be fuzzy sets on B defined by $P(x) = 0.3$, $P(y) = 0.3$, $P(z) = 0.4$; $Q(s) = 0.2, \forall\ s \in B$; $R(s) = 0.4, \forall\ s \in B$; $S(x)$=0.1, $S(y) = 0.2$, $S(z) = 0.2$ and $T(x) = 0.4$, $T(y)$ =0.4, $T(z) = 0.5$. Clearly, $\tau_1 = \{0,1,U,V,W\}$ and $\tau_2 = \{0,1,P,Q,R,S,T\}$ are fuzzy topologies on A and B respectively. V is not pseudo regular open fuzzy set for $0.1 \leq \alpha < 0.2$ on A. So, the ps-ro fuzzy topology on A is $\{0,1,U,W\}$ and that on B is $\{0,1,Q,R\}$. Let g be a function between two $fts\ (A,\tau_1)$ and (B,τ_2) given by $g(a) = x, g(b) = x$ and $g(c) = y$. For each fuzzy α-open set E on B, $g^{-1}(E)$ is also so on A. Therefore, g is fuzzy α-irresolute. Here, Q is ps-ro α-open fuzzy set on B but $g^{-1}(Q)$ is not so on A. Thus, g is not ps-ro fuzzy α-irresolute.

Remark 2.1: From Example (2.1) and Example (2.2), we can conclude that ps-ro fuzzy α-irresolute and fuzzy α-irresolute functions are independent of each other.

Example 2.3: Let $A = \{a,b,c\}$ and $B = \{x,y,z\}$. Let U, V, W and D be fuzzy sets on A defined by $U(a) = 0.2, U(b) = 0.3, U(c) = 0.3$; $V(r) = 0.4, \forall r \in A$; $W(r) = 0.1, \forall r \in A$ and $D(r) = 0.8, \forall r \in A$. Let P, Q and R be fuzzy sets on B defined by $P(s) = 0.4, \forall s \in B$; $Q(x) = 0.2$, $Q(y) = 0.2$ and $Q(z) = 0.3$ and $R(s) = 0.1, \forall s \in B$. Clearly, $\tau_1 = \{0,1,U,V,W,D\}$ and $\tau_2 = \{0,1,P,Q,R\}$ are fuzzy topologies on A and B respectively. U is not pseudo regular open fuzzy set for $0.2 \leq \alpha < 0.3$ on A. So, the ps-ro fuzzy topology on A is $\{0,1,V,W,D\}$. Again, Q is not pseudo regular open fuzzy set for $0.2 \leq \alpha < 0.3$ on B. So, the ps-ro fuzzy topology on B is $\{0,1,P,R\}$. Let g be a function from the $fts\ (A,\tau_1)$ to the $fts\ (B,\tau_2)$ defined by $g(a) = x, g(b) = y$ and $g(c) = z$. Here, $g^{-1}(T)$ is ps-ro open fuzzy set on A for every ps-ro open fuzzy set Ton B, proving that g is ps-ro fuzzy continuous . Q is ps-ro α-open fuzzy set on B but $g^{-1}(Q)$ is not so on A. Hence, g is not ps-ro fuzzy α-irresolute.

Remark 2.2: In Example (2.1), P is a ps-ro open fuzzy set on B but $g^{-1}(P)$ is not a ps-ro open fuzzy set on A. Hence, g is not ps-ro fuzzy continuous but g is ps-ro fuzzy α-irresolute.

Combining this with Example (2.3), ps-ro fuzzy α-irresolute and ps-ro fuzzy continuous functions do not imply each other.

Remark 2.3: ps-ro fuzzy α-irresoluteness implies ps-ro fuzzy semicontinuity but the converse is not true is illustrated by the example as follows.

Example 2.4: Let $A = \{a, b, c\}$ and $B = \{x, y, z\}$. Let $U, V$and W be fuzzy sets on A defined by $U(a) = 0.1, U(b) = 0.2, U(c) = 0.2$; $V(r) = 0.3, \forall\, r \in A; W(s) = 0.1, \forall\, s \in B$. Let P, Q, R and S be fuzzy sets on B defined by $P(x) = 0.3$, $P(y) = 0.3$, $P(z) = 0.4; Q(s) = 0.4$, $\forall\, s \in B; R(x) = 0.1$, $R(y) = 0.1, R(z)$=0.2 and $S(s) = 0.2, \forall\, s \in B$. Clearly, $\tau_1 = \{0,1, U, \text{V}, W\}$ and $\tau_2 = \{0,1, P, Q, R, S\}$ are fuzzy topologies on A and B respectively. The ps-ro fuzzy topologies on A and B are $\{0,1, V, W\}$ and $\{0,1, Q, S\}$ respectively. Let g be a function from the $fts\ (A, \tau_1)$ to the $fts\ (B, \tau_2)$ defined by $g(a) = x, g(b) = y$ and $g(c) = y$. Here, g is ps-ro fuzzy semi continuous. Q is ps-ro open and hence a ps-ro α-open fuzzy set on B but $f^{-1}(Q)$ is not so on A. Hence, g is not ps-ro fuzzy α-irresolute.

Remark 2.4: In Example (2.4), g is not ps-ro fuzzy α-irresolute. For all ps-ro semiopen fuzzy set T on B, $g^{-1}(T)$ is also so on A. So, g is ps-ro fuzzy irresolute. In Example (2.1), $g^{-1}(Q)$ is not a ps-ro semiopen fuzzy set on A although Q is so on B. So, g is not ps-ro fuzzy irresolute. Hence, ps-ro fuzzy α-irresolute and ps-ro fuzzy irresolute functions are independent of each other.

Remark 2.5: Clearly, ps-ro fuzzy α-irresoluteness implies ps-ro fuzzy α-continuity but this converse is not true as shown below.

In Example (2.3), $g^{-1}(T)$ is ps-ro α-open fuzzy set on A for all ps-roopen fuzzy set T on B. So, g is a ps-ro fuzzy α-continuous function. Here, Q is a ps-ro α-open fuzzy set on B but $f^{-1}(Q)$ is not so on A, proving that g is not ps-ro fuzzy α-irresolute function.

All above discussed interrelations can be put together in an arrow diagram is given as Figure (1).

Theorem 2.1: A function g between two $fts\ (X, \tau_1)$ and (Y, τ_2) is ps-ro fuzzy α-irresolute iff for any fuzzy point x_t of $X$and any ps-ro α-open fuzzy set B on Y satisfying $g(x_t)qB$, $\exists$ a ps-ro α-open fuzzy set A on Xsuch that $x_tqA \le g^{-1}(B)$.

Proof: Let g be ps-ro fuzzy α-irresolute. Let x_t be any fuzzy point on X and B be a ps-ro α-open fuzzy set on Y with $g(x_t)qB$. Thus $g^{-1}(B)$ is ps-ro α-open fuzzy set on X and $Bg(x) + t > 1$. So, $x_tqg^{-1}(B)$. Taking $g^{-1}(B) = A$, the result follows.

Conversely, let B be a ps-ro α-open fuzzy set on Y and x_t be a fuzzy point on $g^{-1}(B)$. Then $x_t \le g^{-1}(B)$ and $g(x_t) \le g(g^{-1}(B)) \le B$. Choose a fuzzy point $x_t{}'$ such that $x_t{}'(x) = 1 - x_t(x)$. Now,$B(y) + g(x_t{}')(y) = B(y) + g(1 - x_t)(y) \ge B(y) + (1 - B)(y) = 1$. So, $g(x_t{}')qB$. Then $\exists$ ps-roα-open fuzzy set A on X such that $x_t{}'qA \le g^{-1}(B)$. $x_t{}'(x) + A(x) = 1 - x_t(x) + A(x) > 1$. So, $x_t \le A$. Hence, $x_t \le A \le g^{-1}(B)$. As x_t is arbitrary, taking union of all such relations, $\vee\{A : x_t \in g^{-1}(B)\} = g^{-1}(B)$ which shows that $g^{-1}(B)$ is ps-ro α-open fuzzy set on X. So, g is ps-ro fuzzy α-irresolute.

Theorem 2.2: A function g between two $fts\ (A, \tau_1)$ and (B, τ_2) is ps-ro fuzzy α-irresolute iff for each ps-ro α-closed fuzzy set U on B, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(U)))) \le g^{-1}(U)$.

Proof: The proof is straightforward and hence omitted.

Theorem 2.3: Let A, B and C be three fts and $f : A \to B$ and $g : B \to C$ be two functions then:

(a) If f and g are ps-ro fuzzy α-irresolute function then $g \circ f$ is also so.

(b) If f is ps-ro fuzzy α-irresolute and g is ps-ro fuzzy α-continuous, then $g \circ f$ is ps-ro fuzzy α-continuous.

Proof: The proof is straightforward and hence omitted.

Definition 2.2: For any fuzzy set U on a $fts\ (A, \tau)$, we define, $ps\text{-}\alpha\ int(U) = \vee\{V : V \le U, V$ is ps-ro α-open fuzzy set on $A\}$ and $ps\text{-}\alpha\ cl(U) = \wedge\{V : U \le V, V$ is ps-ro α-closed fuzzy set on $A\}$.

Some properties of $ps\text{-}\alpha\ int$ and $ps\text{-}\alpha\ cl$ operators are furnished below. The proofs are straightforward and hence omitted.

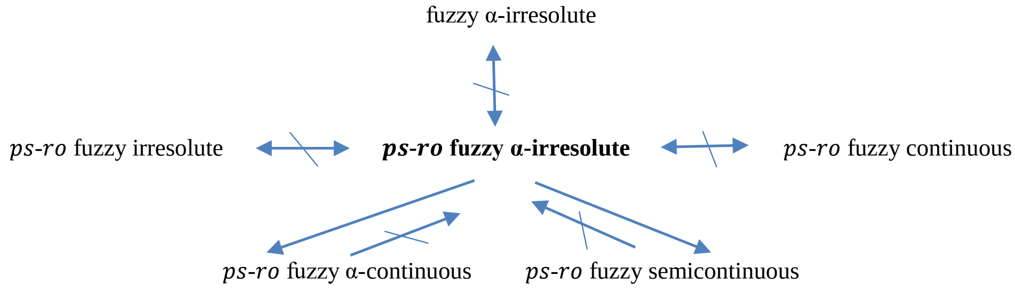

Figure 1. ps-ro fuzzy α-irresolute arrow diagram

Theorem 2.4: For any fuzzy set U of a $fts\ (X,\tau)$, the following results are hold:

(a) $ps\text{-}\alpha\ int(U)$ is the largest $ps\text{-}ro$ α-open fuzzy set which is contained in U and $ps\text{-}\alpha\ cl(U)$ is the smallest $ps\text{-}ro$ α-closed fuzzy set which contains U.

(b) A is $ps\text{-}ro$ α-open fuzzy set iff $U = ps\text{-}\alpha\ int(U)$ and A is $ps\text{-}ro$ α-closed fuzzy set iff $U = ps\text{-}\alpha\ cl(U)$.

(c) $ps\text{-}\alpha\ int(ps\text{-}\alpha\ int(U)) = ps\text{-}\alpha\ int(U)$ and $ps\text{-}\alpha\ cl(ps\text{-}\alpha\ cl(U)) = ps\text{-}\alpha\ cl(U)$.

(d) $ps\text{-}\alpha\ int(U) \le ps\text{-}\alpha\ int(V)$ and $ps\text{-}\alpha\ cl(U) \le ps\text{-}\alpha\ cl(V)\ if\ U \le V$.

(e) $1 - ps\text{-}\alpha\ int(U) = ps\text{-}\alpha\ cl(1-U)$and $1 - ps\text{-}cl(U) = ps\text{-}\alpha\ int(1-U)$.

Proof: The proof is straightforward and hence omitted.

Theorem 2.5: For a function g from a $fts\ (X,\tau_1)$ to another $fts\ (Y,\tau_2)$ the following statements are equivalent.

(a) g is a $ps\text{-}ro$ fuzzy α-irresolute function.

(b) The inverse image of every $ps\text{-}ro$ α-closed fuzzy set on Y is $ps\text{-}ro$ α-closed fuzzy set on X.

(c) For any fuzzy point x_t on X and each $ps\text{-}ro$ α-open fuzzy set B on Y and $g(x_t) \in B$, $\exists$ $ps\text{-}ro$ α-open fuzzy set A on X satisfying $x_t \in A$ and $g(A) \le B$.

(d) $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(B)))) \le g^{-1}(ps\text{-}\alpha\ cl\ (B))$, for all fuzzy set B on Y.

(e) $g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ (A)))) \le ps\text{-}\alpha\ cl\big(g(A)\big)$, for all fuzzy set A on X.

(f) $g(ps\text{-}\alpha\ cl(A)) \le ps\text{-}\alpha\ clg(A)$ for all fuzzy set A on X.

(g) $ps\text{-}\alpha\ cl(g^{-1}(B)) \le g^{-1}(ps\text{-}\alpha\ cl(B))$ for all fuzzy sets B on Y.

(h) $g^{-1}(ps\text{-}\alpha\ intB) \le ps\text{-}\alpha\ int(g^{-1}(B))$ for all fuzzy set B on Y.

Proof:$(a) \Rightarrow (b)$ and $(b) \Rightarrow (a)$ The proof is straightforward and hence omitted.

$(a) \Rightarrow (c)$ Let x_t be any fuzzy point on X and B be any $ps\text{-}ro$ α-open fuzzy set on Y with $g(x_t) \in B$. Since f is $ps\text{-}ro$ fuzzy α-irresolute, it implies that $g^{-1}(B)$ is $ps\text{-}ro$ α-open fuzzy set on X which contains x_t. Taking $g^{-1}(B) = A$, we get the required result.

$(c) \Rightarrow (a)$ Let the given condition hold and B be a $ps\text{-}ro$ α-open fuzzy set on Y. If $g^{-1}(B) = 0$, then the result is true. If $g^{-1}(B) \neq 0$, then $\exists$ a fuzzy point x_t on $g^{-1}(B)$. So, $\exists$ $ps\text{-}ro$ α-open fuzzy set U_{x_t}on X which contains x_t such that $x_t \in U_{x_t} \le g^{-1}(B)$. x_t being arbitrary, taking union of all such relations, we get $g^{-1}(B) = \vee\{x_t : x_t \in g^{-1}(B)\} \le \vee\{U_{x_t} : x_t \in g^{-1}(B)\} \le g^{-1}(B)$. $g^{-1}(B) = \vee\{U_{x_t} : x_t \in g^{-1}(B)\}$ which shows that $g^{-1}(B)$ is $ps\text{-}ro$ α-open fuzzy set on X. Hence, g is $ps\text{-}ro$ fuzzy α-irresolute.

$(b) \Rightarrow (d)$ For any fuzzy set B on Y, $ps\text{-}\alpha\ cl(B)$ is $ps\text{-}ro$ α-closed fuzzy set on Y. So by given hypothesis, $g^{-1}(ps\text{-}\alpha\ cl(B))$ is $ps\text{-}ro$ α-closed fuzzy set on X which imply that $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(ps\text{-}\alpha\ cl\ B)))) \le g^{-1}(ps\text{-}\alpha\ cl(B)$. Thus for a fuzzy set A on X, as $A \le ps\text{-}\alpha\ cl(A)$,we have $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(B)))) \le g^{-1}(ps\text{-}\alpha\ cl(B)$.

$(d) \Rightarrow (e)$ Let Abe fuzzy set on X and $g(A) = B$. Then, $A \le g^{-1}(B)$. So by our hypothesis, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(B)))) \le g^{-1}(ps\text{-}\alpha\ cl(B))$. So, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ (A))) \le ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(B)))) \le g^{-1}(ps\text{-}\alpha\ cl(B)) = g^{-1}(ps\text{-}\alpha\ cl(g(A)))$. This gives $g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ A))) \le gg^{-1}(ps\text{-}\alpha\ cl(g(A)))) \le ps\text{-}\alpha\ cl(g(A))$. Therefore, $g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ A))) \le ps\text{-}\alpha\ cl\big(g(A)\big)$.

$(e) \Rightarrow (b)$ Let B be any $ps\text{-}ro$ α-closed fuzzy set on Y and $A = g^{-1}(B)$. Then $g(A) \le B$ and by given hypothesis,$g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ A))) \le ps\text{-}\alpha\ cl(g(A) \le ps\text{-}\alpha\ cl(B) = B$. So,$g^{-1}(g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ A)))) \le g^{-1}(B)$. This gives $ps\text{-}cl(ps\text{-}int(ps\text{-}cl\ A)) \le g^{-1}(B)$ and hence, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(B)))) \le g^{-1}(B)$, proving that $g^{-1}(B)$ is $ps\text{-}ro$ α-closed fuzzy set on X.

$(b) \Rightarrow (f)$ For any fuzzy set A on X, $A \le g^{-1}(f(A)) \le g^{-1}(ps\text{-}\alpha\ cl(g(A)))$. As $ps\text{-}\alpha\ cl\big(g(A)\big)$is $ps\text{-}ro$ α-closed fuzzy set on Y, $g^{-1}(ps\text{-}\alpha\ cl(g(A)))$ is also so on X. Now, $ps\text{-}\alpha\ cl(A) \le g^{-1}(ps\text{-}\alpha\ cl(g(A)))$ and $g(ps\text{-}\alpha\ cl(A)) \le g(g^{-1}(ps\text{-}\alpha\ cl(g(A))) \le ps\text{-}\alpha\ cl(g(A))$. Thus, $g(ps\text{-}\alpha\ cl\ A) \le ps\text{-}\alpha\ clg(A)$.

$(f) \Rightarrow (g)$ Let B be any fuzzy set on Y and $A = g^{-1}(B)$. By hypothesis, $g(ps\text{-}\alpha\ cl(g^{-1}B))) \le ps\text{-}\alpha\ clg\big(g^{-1}(B)\big) \le ps\text{-}\alpha\ cl(B)$ and $ps\text{-}\alpha\ cl(g^{-1}(B)) \le g^{-1}(g(ps\text{-}\alpha\ cl(g^{-1}(B)))) \le g^{-1}(ps\text{-}\alpha\ cl(B))$. Thus $ps\text{-}\alpha\ cl(g^{-1}(B) \le g^{-1}(ps\text{-}\alpha\ cl(B))$.

$(g) \Rightarrow (h)$ The proof is straightforward and hence omitted.

$(h) \Rightarrow (a)$ Let B be a $ps\text{-}ro$ α-open fuzzy set on Y. $B = ps\text{-}\alpha\ intB$. Then $g^{-1}(ps\text{-}\alpha\ intB) = g^{-1}(B) \le ps\text{-}\alpha\ int(g^{-1}(B))$. Also, $ps\text{-}\alpha\ int(g^{-1}(B)) \le g^{-1}(B)$. Therefore, $ps\text{-}\alpha\ int(g^{-1}(B)) = g^{-1}(B)$which imply that $g^{-1}(B)$is $ps\text{-}ro$α-open fuzzy set on X. Therefore, g is $ps\text{-}ro$fuzzy α-irresolute function.

Theorem 2.6: A bijective function g from a $fts\ (X,\tau_1)$ to another $fts\ (Y,\tau_2)$ is $ps\text{-}ro$ fuzzy α-irresolute iff for any fuzzy set U on X, $ps\text{-}\alpha\ int(g(U)) \le g(ps\text{-}\alpha\ int(U)$.

Proof: Let g be $ps\text{-}ro$ fuzzy α-irresolute and U a fuzzy set on X. $g^{-1}(ps\text{-}\alpha\ int\big(f(U)\big)$ is $ps\text{-}ro$α-open fuzzy set on X. g being one-to-one, $g^{-1}(ps\text{-}\alpha\ int\big(f(U)\big) \le ps\text{-}\alpha\ int\ (g^{-1}(g(U))) = ps\text{-}\alpha\ int(U)$. Again, g being onto, $g(g^{-1}(ps\text{-}\alpha\ int(g(U))) = ps\text{-}\alpha\ int(g(U)) \le g(ps\text{-}\alpha\ int(U))$.

Conversely, let V be any $ps\text{-}ro$α-open fuzzy set on Y. g being onto, $V = ps\text{-}\alpha\ int(V) = ps\text{-}\alpha\ int(g(g^{-1}(V))$. By hypothesis, $ps\text{-}\alpha\ int(g(g^{-1}(V)) \le g(ps\text{-}\alpha\ int(g^{-1}(V))$. As g is one to one, $g^{-1}(V) \le g^{-1}(g(ps\text{-}\alpha\ int(g^{-1}(V)))) = ps\text{-}\alpha\ int(g^{-1}(V))$. As, $ps\text{-}\alpha\ int\big(g^{-1}(V)\big) \le g^{-1}(V)$, $ps\text{-}\alpha\ int\big(g^{-1}(V)\big) = g^{-1}(V)$ showing that $g^{-1}(V)$ is $ps\text{-}ro$ α-open fuzzy set on X. Hence, g is $ps\text{-}ro$fuzzy α-irresolute function.

Theorem 2.7: For a $ps\text{-}ro$ fuzzy α-irresolute function $g: (X,\tau_1) \to (Y,\tau_2)$, $ps\text{-}cl\ (ps\text{-}int\ (ps\text{-}cl\ (g^{-1}(V)))) \le g^{-1}(ps\text{-}cl(V))$, $ps\text{-}\alpha\ cl(g^{-1}(V)) \le g^{-1}(ps\text{-}cl(V))$ forall fuzzy set V on Y and $g(ps\text{-}cl(ps\text{-}int(ps\text{-}cl(U)))) \le ps\text{-}cl(g(U))$, $g(ps\text{-}\alpha\ cl(U)) \le ps\text{-}clg(U)$ for any fuzzy set Uon X.

Proof: Let g be $ps\text{-}ro$ fuzzy α-irresolute and V be a fuzzy set on Y. $ps\text{-}cl(V)$ is $ps\text{-}ro$ closed and hence $ps\text{-}ro$ α-closed fuzzy set on Y. $g^{-1}(ps\text{-}cl(V))$ is $ps\text{-}ro$ α-closed fuzzy set on X which imply that $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(ps\text{-}cl\,V)))) \leq g^{-1}(ps\text{-}cl(V))$. As, $U \leq ps\text{-}cl(V)$, for a fuzzy set U on X, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(g^{-1}(V)))) \leq g^{-1}(ps\text{-}cl(V)$. Again, $V \leq ps\text{-}cl(V)$, $g^{-1}(V) \leq g^{-1}(ps\text{-}cl(V)$ which gives $ps\text{-}\alpha\, cl(g^{-1}(V)) \leq ps\text{-}\alpha\, cl(g^{-1}(ps\text{-}cl(V))) = g^{-1}(ps\text{-}cl(V))$. Similarly another part can be proved.

Theorem 2.8: $g^{-1}(ps\text{-}int(V)) \leq ps\text{-}\alpha\, int(g^{-1}(V))$ for a $ps\text{-}ro$ fuzzy α-irresolute function $g:(X,\tau_1) \rightarrow (Y,\tau_2)$, for any fuzzy set V on Y.

Proof: Let g be $ps\text{-}ro$ fuzzy α-irresolute and V be a fuzzy set on Y. $ps\text{-}int(V)$ is $ps\text{-}ro$ open and hence $ps\text{-}ro$ α-open fuzzy set on Y. $g^{-1}(ps\text{-}int(V))$ being $ps\text{-}ro$ α-open fuzzy set on X, $g^{-1}(ps\text{-}int(V)) = ps\text{-}\alpha\, int(g^{-1}(ps\text{-}int(V))) \leq ps\text{-}\alpha\, int\, (g^{-1}(V)$, as $ps\text{-}int(V) \leq V$. Therefore, $g^{-1}(ps\text{-}int(V)) \leq ps\text{-}\alpha\, int(g^{-1}(V))$.

Lemma 2.1 (Azad (1981), Lemma 2.4, p.17**]:** Let $h: X \rightarrow X \times Y$ be the graph of a function $g:(X,\tau_1) \rightarrow (Y,\tau_2)$ given by $h(x) = (x, g(x))$. If U and V be fuzzy sets on X and Y respectively, then $h^{-1}(U \times V) = U \wedge g^{-1}(V)$.

Theorem 2.9: For a function g from a fts (X,τ_1) to another fts (Y,τ_2), if the graph $h: X \rightarrow X \times Y$ of g is $ps\text{-}ro$ fuzzy α-irresolute then g is also $ps\text{-}ro$ fuzzy α-irresolute, where $(X,\tau_1)$and(Y,τ_2) are two fts.

Proof: Let B be any $ps\text{-}ro$ α-open fuzzy set on Y. Using Lemma (2.1) we get,

$g^{-1}(B)= 1 \wedge g^{-1}(B) = h^{-1}\,(1 \times B)$. $(1 \times B)$ is $ps\text{-}ro$ α-open fuzzy set on $(X \times Y)$ and as h is $ps\text{-}ro$ fuzzy α-irresolute function, $h^{-1}\,(1 \times B)$ is $ps\text{-}ro$ α-open fuzzy set on X. Thus, $g^{-1}(B)$ is $ps\text{-}ro$ α-open fuzzy set on X. Hence g is $ps\text{-}ro$ fuzzy α-irresolute.

Definition 2.3: A fuzzy set U on a fts (X,τ) is called $ps\text{-}ro$ fuzzy dense set if $ps\text{-}cl(U) = 1$ and U is called a nowhere $ps\text{-}ro$ fuzzy dense set if $ps\text{-}int(ps\text{-}cl(U)) = 0$.

Theorem 2.10: If a function $g:(X,\tau_1) \rightarrow (Y,\tau_2)$ is $ps\text{-}ro$ fuzzy α-irresolute, where (X,τ_1) and (Y,τ_2) are two fts then $g^{-1}(A)$ is a $ps\text{-}ro$ α-closed fuzzy set on X for any nowhere $ps\text{-}ro$ fuzzy dense set U on Y.

Proof: Let U be a nowhere $ps\text{-}ro$ fuzzy dense set on Y. $ps\text{-}cl(ps\text{-}int(1-U)) = 1$. $ps\text{-}int(ps\text{-}cl(ps\text{-}int(1-U))) = 1$. So, $1-U \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(1-U))) = 1$ and hence $1-U$ is $ps\text{-}ro$ α-open fuzzy set on Y. $g^{-1}(1-U) = 1 - g^{-1}(U)$, $f^{-1}(U)$ is $ps\text{-}ro$ α-closed fuzzy set on X.

Definition 2.4: A fuzzy set U is said to be a $ps\text{-}ro$ fuzzy α-nbd of a fuzzy point $x_t$if $\exists$ a $ps\text{-}ro$ α-open fuzzy set V such that $x_t \in V \leq U$.

Theorem 2.11: For a function f between two fts (X,τ_1) and (Y,τ_2), the following are equivalent:

(a) f is $ps\text{-}ro$ fuzzy α-irresolute function.

(b) For any fuzzy point x_t on X, the inverse of each $ps\text{-}ro$ fuzzy α-nbd V of $f(x_t)$ on Y is a $ps\text{-}ro$ fuzzy α-nbd of x_t on X.

(c) For any fuzzy point $x_t$on X and any $ps\text{-}ro$ fuzzy α-nbd V of $f(x_t)$ on Y, $\exists$ a $ps\text{-}ro$ fuzzy α-nbd U of $x_t$on X satisfying $f(U) \leq V$.

Proof:

$(a) \Rightarrow (b)$ Let f be $ps\text{-}ro$ fuzzy α-irresolute. Let x_t be a fuzzy point on X and V be a $ps\text{-}ro$ fuzzy α-nbd of $f(x_t)$ on Y. Then $\exists$ a $ps\text{-}ro$ α-open fuzzy set W on Y such that $f(x_t) \in W \leq V$. Now, f being a $ps\text{-}ro$ fuzzy α-irresolute, $f^{-1}(W)$ is $ps\text{-}ro$ α-open fuzzy set on X. Then $x_t \in f^{-1}(W) \leq f^{-1}(V)$ which proves that $f^{-1}(V)$ is a $ps\text{-}ro$ fuzzy α-nbd of x_t on X.

$(b) \Rightarrow (c)$ Let $x_t$be any fuzzy point on X and V be a $ps\text{-}ro$ fuzzy α-nbd of $f(x_t)$ on Y. $f^{-1}(V)$ is a $ps\text{-}ro$ fuzzy α-nbd of $x_t$on X. Let $f^{-1}(V) = U$. Then $f(U) \leq V$.

$(c) \Rightarrow (a)$ Let x_t be any fuzzy point on X and V be a $ps\text{-}ro$ α-open fuzzy set on Y with $f(x_t) \in V$ then $\exists$ a $ps\text{-}ro$ fuzzy α-nbd W of x_t on X with $f(W) \leq V$. As W is a $ps\text{-}ro$ fuzzy α-nbd of $x_t$on X, $\exists$ a $ps\text{-}ro$ α-open fuzzy set U on X satisfying $x_t \in U \leq W$ i.e., $f(x_t) \in f(U) \leq f(W) \leq V$. Hence, $f(U) \leq V$. Now, using $(c) \Rightarrow (a)$ of Theorem (2.5), the result follows.

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