Songklanakarin J. Sci. Technol.
41 (1), 216-221, Jan. - Feb. 2019

http://www.sjst.psu.ac.th

Original Article

Generalized fuzzy closed sets in generalized fuzzy topological spaces

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Received: 8 April 2016; Revised: 8 August 2017; Accepted: 20 October 2017

Abstract

This paper aims to put forward one more difference between generalized topological space and generalized fuzzy topological space via generalized closed set in the respective fields. Also we claim and prove that to a certain extent, the role of generalized closed set in a fuzzy topological space is not similar in nature with that of generalized closed set in a generalized fuzzy topological space. Generalized closed sets coincide with closed sets under some appropriate restrictions though the implication does not hold in general in a generalized fuzzy topological space.

Keywords: generalized fuzzy g_X-closed set, fuzzy g_X-no where dense set, generalized fuzzy g_X-open set, generalized fuzzy (g_X, g_Y)-continuity

1. Introduction and Preliminaries

The generalized closed set is the most common but important and interesting concepts in topological spaces as well as fuzzy topological spaces. The concept of generalized closed set in general topological space was first instigated by Levine (1970), which has been extensively used as an excel-lent tool for studying different concepts in the said space. In fuzzy setting, the concept of generalized fuzzy closed set was initiated by Balasubramanian *et al*. (1997). Subsequently, ma-ny authors have devoted their work to the study of various forms of generalized fuzzy closed set, for instance Saraf *et al*. (2005) and Park *et al*. (2003). On the other hand, Császár (20 02) introduced the notion of generalized neighborhood sys-tems and generalized topological spaces (in short, GTS's). Very recently a number of researchers are attempting to ex-tend the idea of generalized closed set in generalized topology which is a broader framework of general topology. Moreover, Maragathavalli *et al*. (2010) have studied generalized closed set and its fundamental properties in generalized topological spaces. Before that, Chetty (2008) has extended the concept of generalized topological space in fuzzy environment and named it generalized fuzzy topological space. Furthermore, Császár (2011) defined the concept of weak structure which is a weaker form of generalized topology. Afterwards various re-searchers have worked on that field namely Ghareeb *et al*. (2015), Zaharan *et al*. (2012), and Zakari *et al*. (2017) and studied various properties of weak structures in various di-rections.

The main objective of this paper is to define ge-neralized closed set in the field of generalized fuzzy topolo-gical space and study various properties of it. Difference is ex-posed in between fuzzy topological spaces and generalized fuzzy topological spaces (resp. generalized fuzzy topological spaces and generalized topological spaces) with the help of generalized closed set.

Further, the relevant application of generalized fuz-zy (g_X, g_Y)-continuity is also shown by using fuzzy contra (g_X, g_Y)-continuity which is defined by Chakraborty *et al*. (20 17).

To make the exposition self enclosed as far as feasible, we illuminate a few prerequisites as follows:

Let X be a nonempty set and g_X be a collection of fuzzy subsets of X. Then g_X is called a generalized fuzzy topology on X iff $0_X \in g_X$ and $G_i \in g_X$ for $i \in I \neq \phi$ implies $G = \vee_{i \in I} G_i \in g_X$. The pair (X, g_X) is called a generalized

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fuzzy topological space (in short, GFTS). The elements of g_X are called the fuzzy g_X-open sets and the complements are called the fuzzy g_X-closed sets. The collection of all the fuzzy g_X-open sets and fuzzy g_X-closed sets are denoted by $\mathrm{GFO}(X, g_X)$ and $\mathrm{GFC}(X, g_X)$ respectively. The g_X-closure of a fuzzy subset λ of X is denoted by $c_{g_X}(\lambda)$, defined to be the intersection of all the fuzzy g_X-closed sets including λ and the g_X-interior of λ, denoted by $i_{g_X}(\lambda)$, defined as the union of all the fuzzy g_X-open sets contained in λ. The complement of a fuzzy set λ is denoted by λ^c or $1_X - \lambda$.

1.1 Definition (Chakraborty et al., 2017) A fuzzy set λ of a GFTS (X, g_X) is called fuzzy g_X-dense if $c_{g_X}(\lambda) = 1_X$.

1.2 Definition (Chakraborty *et al.*, 2017) A GFTS (X, g_X) is said to be generalized fuzzy submaximal if each fuzzy g_X-dense subset is a fuzzy g_X-open set.

1.3 Definition (Chakraborty *et al.*, 2017) A fuzzy set δ of a GFTS (X, g_X) is called a fuzzy g_X-locally closed set if there exists a fuzzy g_X-open set λ and fuzzy g_X-closed set μ such that $\delta = \lambda \wedge \mu$.

1.4 Definition (Chakraborty *et al.*, 2017) Let (X, g_X) be a GFTS. A fuzzy set λ is called

(i) fuzzy g_X-semiopen set if $\lambda \leq c_{g_X}\, i_{g_X}(\lambda)$.

(ii) fuzzy g_X-regular open set if $\lambda = i_{g_X}\, c_{g_X}(\lambda)$.

The complement of fuzzy g_X-semiopen and fuz-zy g_X- regular open sets are called fuzzy g_X-semiclosed set and fuz-zy g_X- regular closed set respectively.

1.5 Definition (Maragathavalli *et al.*, 2010) A subset A in a GTS (X, k) is called g_K-closed if $c_k(\mathrm{A}) \leq M$ whenever $\mathrm{A} \leq M$ and $M \in k$.

1.6 Definition (Chakraborty *et al.*, 2017) Let (X, g_X) and (Y, g_Y) be any two GFTSs. Then a function $f : X \rightarrow Y$ is said to be fuzzy (g_X, g_Y)-continuous if for each fuzzy g_Y-open set λ in Y, $f^{-1}(\lambda)$ is a fuzzy g_X-open set in X.

1.7 Definition (Chakraborty *et al.*, 2017) A function $f: (X, g_X) \rightarrow (Y, g_Y)$ from a GFTS to another GFTS is called a fuzzy contra (g_X, g_Y)-continuous if for each fuzzy g_Y-open set λ in Y, $f^{-1}(\lambda)$ is a fuzzy g_X-closed set in X.

2. Characterizations of Generalized Closed Sets in GFTS

In this section, we study the characteristics of gene-ralized closed set in generalized fuzzy topological spaces. We emphasize that the collection of all the generalized closed sets is not closed under the operations of the union as well as inter-section in the context of generalized fuzzy topological spaces. Also the behavior of generalized closed sets in the generalized topological space and generalized fuzzy topological space is dissimilar under some particular features.

2.1 Definition (Chakraborty *et al.*, 2017) A fuzzy set λ in a GFTS (X, g_X) is called a generalized fuzzy g_X-closed (briefly, gf g_X-closed) if $c_{g_X}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and $\mu \in \mathrm{GFO}\,(X, g_X)$.

2.2 Remark Every fuzzy g_X-closed set is gf g_X-closed set but the converse is not true.

2.3 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $\mu = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$. In the GFTS (X, g_X), μ is a gf g_X-closed set but not a fuzzy g_X-closed set.

Balasubramanian and Sundaram (1997) have shown that the union of two generalized fuzzy closed sets is a ge-neralized fuzzy closed set in the field of fuzzy topological space. However, it is not true in the perspective of GFTSs and justified in the following examples.

2.4 Remark The union of two gf g_X-closed sets need not be a gf g_X-closed set in a GFTS (X, g_X).

2.5 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.4), (b, 0.7), (c, 0.5)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.7), (c, 0.5)\}\}$. We suppose that

$\lambda_1 = \{(a, 0.4), (b, 0.7), (c, 0.5)\}$,

$\lambda_2 = \{(a, 0.6), (b, 0.3), (c, 0.5)\}$ and

$\lambda_3 = \{(a, 0.6), (b, 0.7), (c, 0.5)\}$. We have $c_{g_X}(\lambda_1) = \lambda_1 \leq \lambda_1$ and $c_{g_X}(\lambda_2) = \lambda_2 \leq \lambda_2$. Also $\lambda_1 \vee \lambda_2 = \lambda_3$ but $c_{g_X}(\lambda_1 \vee \lambda_2) = 1_X \nleq \lambda_3$. Therefore, λ_1 and λ_2 are two gf g_X-closed sets while $\lambda_1 \vee \lambda_2$ is not a gf g_X-closed set.

2.6 Remark The intersection of two gf g_X-closed sets may not be a gf g_X-closed set in a GFTS (X, g_X).

2.7 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, \{(a, 1), (b, 0), (c, 0)\}\}$, $\lambda_1 = \{(a, 1), (b, 0.5), (c, 0)\}$ and $\lambda_2 = \{(a, 1), (b, 0), (c, 0.6)\}$. It is easy to verify that both λ_1 and λ_2 are gf g_X-closed sets in GFTS (X, g_X) but the intersection $\lambda_1 \wedge \lambda_2 = \{(a, 1), (b, 0), (c, 0)\}$ is not a gf g_X-closed set.

Mukherjee and Das (2010) defined the difference of two fuzzy sets in the following way:

2.8 Definition (Mukherjee & Das, 2010) For any two fuzzy sets A and B in an fuzzy topological space X, their difference is denoted by $(A - B)$ and defined as

$$(A - B)(x) = \begin{cases} A(x) - B(x), & if A(x) > B(x) \\ 0 & if A(x) < B(x), \end{cases}$$

where $x \in X$.

2.9 Theorem (Maragathavalli *et al.*, 2010) Let (X, k) be a GTS. Then a subset A is g_k-closed iff $c_k(A) - A$ does not contain any nonempty k-closed set.

It is very interesting to disprove the above theorem in the context of GFTS and for the purpose we define the difference of two fuzzy sets in GFTS following the direction of Mukherjee and Das mentioned above.

2.10 Definition For any two fuzzy sets λ and μ in a GFTS (X, g_X), their difference is denoted by $(\lambda - \mu)$ and defined as

$$(\lambda - \mu)(x) = \begin{cases} \lambda(x) - \mu(x) & if \ \lambda(x) \geq \mu(x), \\ 0_X & if \ \lambda(x) < \mu(x), \end{cases}$$

where $x \in X$.

Application of this definition in the following two consecutive examples establishes our claim.

2.11 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, \mu_1, \mu_2, \mu_3\}$, where $\mu_1 = \{(a, 0.8), (b, 1)\}$, $\mu_2 = \{(a, 0.2), (b, 0.3)\}$ and $\mu_3 = \{(a, 0.6), (b, 0.5)\}$.

Also let $\lambda_1 = \{(a, 0.2), (b, 0.4)\}$. Simple calculation gives that the fuzzy subset λ_1 is a gf g_X-closed set whereas $c_{g_X}(\lambda_1) - \lambda_1 = \{(a, 0.2), (b, 0.1)\}$ and it contains the nonempty fuzzy g_X-closed set $\{(a, 0.2), (b, 0)\}$.

2.12 Example Let $X = \{a, b\}$, $g_X = \{0_X, \mu_1, \mu_2, \mu_1 \vee \mu_2\}$, where $\mu_1 = \{(a, 0.4), (b, 0.6)\}$, $\mu_2 = \{(a, 0.5), (b, 0.3)\}$ and $\mu_1 \vee \mu_2 = \{(a, 0.5), (b, 0.6)\}$. Let us suppose that $\lambda_1 = \{(a, 0.5), (b, 0.2)\}$. We have, $c_{g_X}(\lambda_1) - \lambda_1 = \{(a, 0), (b, 0.2)\}$, and it does not contain any nonempty fuzzy g_X-closed set in GFTS (X, g_X), while λ_1 is not a gf g_X-closed set.

2.13 Theorem If λ be any gf g_X-closed set and $\lambda \le \mu \le c_{g_X}(\lambda)$, then μ is also a gf g_X-closed set.

Proof. Since λ is a gf g_X-closed set then there exist a fuzzy g_X-open set β such that $\lambda \le \beta$ with $c_{g_X}(\lambda) \le \beta$. We suppose that $\mu \le \beta$. From the given statement, we have $c_{g_X}(\lambda) = c_{g_X}(\mu)$. This implies that μ is a gf g_X-closed set.

2.14 Definition A fuzzy set λ in a GFTS (X, g_X) is called a generalized fuzzy g_X-open (briefly, gf g_X-open) iff $1_X - \lambda$ is a gf g_X-closed.

2.15 Theorem A fuzzy set λ in a GFTS (X, g_X) is gf g_X-open iff $\mu \le i_{g_X}(\lambda)$, whenever $\mu \le \lambda$ and μ is a fuzzy g_X-closed.

Proof. Let λ be any gf g_X-open set and μ be a fuzzy g_X-closed set such that $\mu \le \lambda$. So $\mu \le \lambda$ implies that $1_X - \mu \ge 1_X - \lambda$ and also $1_X - \lambda$ is a gf g_X-closed set. Therefore $c_{g_X}(1_X - \lambda) \le 1_X - \mu$. It implies that
$1_X - c_{g_X}(1_X - \lambda) \ge 1_X - (1_X - \mu) = \mu$. But we know that
$1_X - c_{g_X}(1_X - \lambda) = i_{g_X}(\lambda)$. Therefore $\mu \le i_{g_X}(\lambda)$.

Conversely, we suppose that λ is a fuzzy set such that $\mu \le i_{g_X}(\lambda)$, whenever $\mu \le \lambda$ and μ is a fuzzy g_X-closed set. Therefore, we have $1_X - \lambda$ is a gf g_X-closed set. Let $1_X - \lambda \le \mu$, where μ is a fuzzy g_X-open set. Now $1_X - \lambda \le \mu$ implies that $1_X - \mu \le \lambda$. Hence, by the assumption, we have $1_X - \mu \le i_{g_X}(\lambda)$ that is $1_X - i_{g_X}(\lambda) \le \mu$. But $1_X - i_{g_X}(\lambda) = c_{g_X}(1_X - \lambda)$. Hence $c_{g_X}(1_X - \lambda) \le \mu$. This shows that $1_X - \lambda$ is a gf g_X-closed set.

2.16 Theorem If λ_1 and λ_2 are two gf g_X-open sets with $\lambda_1 \wedge c_{g_X}(\lambda_2) = \lambda_2 \wedge c_{g_X}(\lambda_1) = 0_X$ in a GFTS (X, g_X), then $\lambda_1 \vee \lambda_2$ is a gf g_X-open set.

Proof. Let μ be any fuzzy g_X-closed set in a GFTS (X, g_X) such that $\mu \le \lambda_1 \vee \lambda_2$. Then we have $\mu \wedge c_{g_X}(\lambda_1) \le \lambda_1$ because $c_{g_X}(\lambda_1) \wedge \lambda_2 = 0_X$ and $\mu \wedge c_{g_X}(\lambda_1) \le i_{g_X}(\lambda_1)$ using theorem 2.15. Similarly, one can show that $\mu \wedge c_{g_X}(\lambda_2) \le i_{g_X}(\lambda_2)$. Now we have $\mu = \mu \wedge (\lambda_1 \vee \lambda_2) \le \left(\mu \wedge c_{g_X}(\lambda_1)\right) \wedge \left(\mu \wedge c_{g_X}(\lambda_2)\right) \le i_{g_X}(\lambda_1) \vee i_{g_X}(\lambda_2) \le i_{g_X}(\lambda_1 \vee \lambda_2)$. Using theorem 2.15, we conclude that $\lambda_1 \vee \lambda_2$ is a gf g_X-open set.

2.17 Remark Generalized fuzzy g_X-closed sets and fuzzy g_X-locally closed sets are two generalizations of fuzzy g_X-closed sets but both are completely independent to each other.

2.18 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}$ and $\mu = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$. In the GFTS (X, g_X), μ is a gf g_X-closed but not a fuzzy g_X-locally closed set.

2.19 Example Let $= \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $\mu = \{(a, 0.5), (b, 0.4), (c, 0.3)\}$. In the GFTS (X, g_X), μ is a fuzzyg_X-locally closed but not a gf g_X-closed set.

2.20 Theorem A gf g_X-closed set in a GFTS (X, g_X) is a fuzzy g_X-closed iff it is a fuzzy g_X-locally closed set.

Proof. Let λ be any gf g_X-closed set and fuzzy g_X-locally closed set in a GFTS (X, g_X). Therefore $\lambda = \beta \wedge \mu$, where β is fuzzy g_X-open set and μ is a fuzzy g_X-closed set. So we have $\lambda \le \beta$ and $\lambda \le \mu$. Again, λ is a gf g_X-closed set which shows $c_{g_X}(\lambda) \le \beta$. Also we have $c_{g_X}(\lambda) \le c_{g_X}(\mu) = \mu$. Thus, we get $c_{g_X}(\lambda) \le \beta \wedge \mu = \lambda$. It implies that $c_{g_X}(\lambda) = \lambda$. Therefore λ is a fuzzy g_X-closed set.

Conversely, let λ be any fuzzy g_X-closed set then it is obviously a fuzzy g_X-locally closed set as well as gf g_X-closed set.

2.21 Definition A fuzzy set λ of a GFTS (X, g_X) is called fuzzy g_X-nowhere dense if $i_{g_X} c_{g_X}(\lambda) = 0_X$.

2.22 Proposition Every fuzzy g_X-nowhere dense set is a gf g_X-closed set.

Proof. Let λ be any fuzzy g_X-nowhere dense set in a GFTS (X, g_X). Therefore, we have $i_{g_X} c_{g_X}(\lambda) = 0_X$ and it means that there does not exist any fuzzy g_X-open set in between λ and $c_{g_X}(\lambda)$. Also, let us suppose that $\lambda \le \mu$, where μ is fuzzy g_X-open set and then obviously $c_{g_X}(\lambda) \le \mu$. Therefore λ is a gf g_X-closed set.

2.23 Remark Every gf g_X-closed set may not be a fuzzy g_X-nowhere dense set.

2.24 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $\mu = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$. In the GFTS (X, g_X), μ is a gf g_X-closed set but not a fuzzy g_X-nowhere dense set.

2.25 Proposition Every fuzzy g_X-semiclosed set is a gf g_X-closed set.

Proof. The argument is straightforward from the definition of fuzzy g_X-semiclosed set.

2.26 Remark However the converse of the above proposition is not true in general.

2.27 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $\mu = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$. In the GFTS (X, g_X), μ is a gf g_X-closed but not a fuzzy g_X-semiclosed set.

2.28 Proposition If λ be any fuzzy g_X-semiclosed set in a GFTS (X, g_X), then $i_{g_X}(\lambda)$ is a fuzzy g_X-regular open set.

Proof. Let us suppose that λ be any fuzzy g_X-semiclosed set in a GFTS(X, g_X). Therefore, by the hypothesis we have $i_{g_X}(\lambda) = i_{g_X} c_{g_X}(\lambda)$. It implies that $i_{g_X}(\lambda)$ is a fuzzy g_X-regu-lar open set.

2.29 Corollary For any gf g_X-closed set, if there is no fuzzy g_X-open set in between λ and $i_{g_X}(\lambda)$ then $i_{g_X}(\lambda)$ is a fuzzy g_X-regular open set.

2.30 Definition (Chakraborty *et al*., 2017) A GFTS (X, g_X) is said to be generalized fuzzy $T_{\frac{1}{2}}$ if every gf g_X-closed set in X is fuzzy g_X-closed in X.

2.31 Proposition In a GFTS (X, g_X), the following conditions are equivalent:

(i) GFTS (X, g_X) is generalized fuzzy $T_{\frac{1}{2}}$.

(ii) Every gf g_X-closed set is fuzzy g_X-locally closed set.

Proof. (i)⇒(ii) Given that the GFTS (X, g_X) is a generalized fuzzy $T_{\frac{1}{2}}$ space. Hence by the definition of generalized fuzzy $T_{\frac{1}{2}}$ space, every gf g_X-closed set is fuzzy g_X-closed set.

(ii)⇒(i) Using theorem 2.20, it can be proved easily.

2.32 Definition A fuzzy set λ in a GFTS (X, g_X) is said to be regular generalized fuzzy g_X-closed (briefly, rgf g_X-closed) if $c_{g_X}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is fuzzy g_X-regular open set.

2.33 Remark Every gf g_X-closed set is rgf g_X-closed set but the converse is not true.

2.34 Example Let $X = \{a, b, c\}$, $g_X = \{0_X, \{(a, 0.4), (b, 0.4), (c, 0.7)\}, \{(a, 0.6), (b, 0.6), (c, 0.8)\}\}$ and $\mu = \{(a, 0.4), (b, 0.4) (c, 0.3)\}$. In the GFTS (X, g_X), μ is a rgf g_X-closed but not a gf g_X-closed set.

2.35 Definition (Chakraborty *et al*., 2016) A GFTS (X, g_X) is called a generalized fuzzy extremally disconnected if closure of every fuzzy g_X-open set is fuzzy g_X-open set.

2.36 Theorem Let (X, g_X) be a GFTS. Then (X, g_X) is a generalized fuzzy extremally disconnected if every fuzzy g_X-regular closed set is fuzzy g_X-open set.

Proof. Let (X, g_X) be a generalized fuzzy extremally disconnected space. Suppose that μ is a fuzzy g_X-regular closed set, then $\mu = c_{g_X}\, i_{g_X}(\mu)$. Since $i_{g_X}(\mu)$ is fuzzy g_X-open set, then by the hypothesis, $c_{g_X}\, i_{g_X}(\mu)$ is also a fuzzy g_X-open set. Therefore $i_{g_X}\left(c_{g_X}\, i_{g_X}(\mu)\right) = c_{g_X}\, i_{g_X}(\mu) = \mu$. Hence μ is a fuzzy g_X-open set.

Conversely, we suppose that every fuzzy g_X-regular closed set is fuzzy g_X-open set in a GFTS (X, g_X). Let μ be any fuzzy g_X-open set in a GFTS (X, g_X). Since $c_{g_X}(\mu) = c_{g_X}\, i_{g_X}(\mu)$ is fuzzy g_X-regular closed set, then by the hypothesis $i_{g_X}\, c_{g_X}(\mu) = c_{g_X}(\mu)$. Therefore, $c_{g_X}(\mu)$ is fuzzy g_X-open set. Hence (X, g_X) is generalized fuzzy g_X-extremally disconnected space.

2.37 Proposition In a generalized fuzzy extremally disconnected space (X, g_X), any fuzzy subset λ is rgf g_X-closed set.

Proof. Let $\lambda \leq \mu$, where μ is fuzzy g_X-regular open set in (X, g_X). Then $c_{g_X}(\lambda) \leq c_{g_X}(\mu) = \mu$, since (X, g_X) is generalized fuzzy extremally disconnected space and in generalized fuzzy extremally disconnected space every fuzzy g_X-regular open set is fuzzy g_X-closed set. Therefore, every fuzzy subset is rgf g_X-closed in generalized fuzzy extremally disconnected space (X, g_X).

3. Generalized Fuzzy Continuous Functions in GFTSs

This section is devoted to study continuity between two GFTSs via gf g_X-closed set as a weaker form of continuity. It is also established that generalized fuzzy (g_X, g_Y)-continuity and fuzzy almost (g_X, g_Y)-continuity are independent of each other. Nevertheless, their sameness is proved up to a possible extent.

3.1 Definition (Chakraborty *et al*., 2017) A function $f : (X, g_X) \to (Y, g_Y)$ is called generalized fuzzy (g_X, g_Y)-continuous if the inverse image of every fuzzy g_Y-closed set λ in Y is gf g_X- closed set in X.

3.2 Example Let $X = Y = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $g_Y = \{0_Y, \{(a, 0.4), (b, 0.3), (c, 0.5)\}\}$.

We consider the func-tion $f: (X, g_X) \to (Y, g_Y)$ such that $f(a) = a$, $f(b) = b$, $f(c) = c$. Then the function f is generalized fuzzy (g_X, g_Y)-continuous function.

3.3 Remark Every fuzzy (g_X, g_Y)-continuous function is generalized fuzzy (g_X, g_Y)-continuous function.

The following example demonstrates that the contrary of the above remark is false.

3.4 Example Let $X = Y = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $g_Y = \{0_Y, \{(a, 0.4), (b, 0.3), (c, 0.5)\}\}$.

We consider the function $f: (X, g_X) \to (Y, g_Y)$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. It is a generalized fuzzy (g_X, g_Y)-continuous function but not a fuzzy (g_X, g_Y)-continuous function as $f^{-1}\{(a, 0.6), (b, 0.7), (c, 0.4)\} = \{(a, 0.6), (b, 0.7), (c, 0.4)\}$, which is a gf g_X-closed set but not a fuzzy g_X-closed set in X.

3.5 Definition Let (X, g_X) and (Y, g_Y) be any two GFTSs. Then a function $f : (X, g_X) \to (Y, g_Y)$ is called fuzzy almost (g_X, g_Y)-continuous if $f^{-1}(\mu)$ (λ) is a fuzzy g_X-open set in X for each fuzzy g_Y-regular open set μ of Y.

3.6 Remark Generalized fuzzy (g_X, g_Y)-continuity and fuzzy almost (g_X, g_Y)- continuity between two GFTSs are of independent in nature.

3.7 Example Let $X = Y = \{a, b, c\}$, $g_X = \{0_X, 1_X, \{(a, 0.5), (b, 0.4), (c, 0.3)\}, \{(a, 0.6), (b, 0.3), (c, 0.5)\}, \{(a, 0.6), (b, 0.4), (c, 0.5)\}\}$ and $g_Y = \{0_Y, \{(a, 0.4), (b, 0.3), (c, 0.5)\}\}$. The function $f: (X, g_X) \to (Y, g_Y)$, defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, is gene-ralized fuzzy (g_X, g_Y)-continuous function but not a fuzzy almost (g_X, g_Y) -continuous function as $f^{-1}\{(a, 0.6), (b, 0.7), (c, 0.5)\} = \{(a, 0.6), (b, 0.7), (c, 0.5)\}$ is gf g_X-closed set but not a fuzzy g_X-regular closed set in X.

3.8 Example Let $X = Y = \{a, b, c\}$, $g_X = \{0_X, \{(a, 0.4), (b, 0.7) (c, 0.5)\}, \{(a, 0.6), (b, 0.7), (c, 0.5)\}\}$ and $g_Y = \{0_Y, \{(a, 0.4), (b, 0.6), (c, 0.5)\}, \{(a, 0.4), (b, 0.7), (c, 0.5)\}\}$. Here the function $f: (X, g_X) \to (Y, g_Y)$, defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is a fuzzy almost (g_X, g_Y)-continuous function but not a generalized fuzzy (g_X, g_Y)-continuous function as $f^{-1}\{(a, 0.6), (b, 0.4), (c, 0.5)\} = \{(a, 0.6), (b, 0.4), (c, 0.5)\}$, which is not a gf g_X-closed set in X.

3.9 Proposition Let (X, g_X) be a generalized fuzzy $T_{\frac{1}{2}}$ space. We assume $f: (X, g_X) \to (Y, g_Y)$ be a generalized fuzzy (g_X, g_Y)-continuous function, then f is a fuzzy almost (g_X, g_Y)-continuous function.

Proof. It can be established easily using definition 2.30.

3.10 Proposition Let $f: (X, g_X) \to (Y, g_Y)$ be a generalized fuzzy (g_X, g_Y)-continuous and fuzzy contra (g_X, g_Y)-continuous function from X to Y, then f is a fuzzy (g_X, g_Y)-continuous function.

Proof. It can be easily verified from the definition of generalized fuzzy (g_X, g_Y)-continuity and fuzzy contra (g_X, g_Y)-continuity.

3.11 Proposition Let $f: (X, g_X) \to (Y, g_Y)$ be a fuzzy (σ, g_Y)-continuous function from X to Y. Then $f^{-1}(\lambda) = i_{g_X} c_{g_X} (f^{-1}(\lambda))$ for any fuzzy g_Y-closed subset λ of Y.

Proof. Let $f: (X, g_X) \to (Y, g_Y)$ be a fuzzy (σ, g_Y)-continuous function. Suppose that λ is a fuzzy g_Y-closed subset of (Y, g_Y). This implies that $f^{-1}(\lambda)$ is fuzzy g_X-semiclosed set in X. From proposition 2.28, we get $i_{g_X}(f^{-1}(\lambda))$ is a fuzzy g_X-regular open set in (X, g_X). Therefore $f^{-1}(\lambda) = i_{g_X} c_{g_X}(f^{-1}(\lambda))$ for any fuzzy g_Y-closed subset λ of Y.

4. Conclusions

A novel theory of gf g_X-closed set has been presented in this paper in the context of GFTS. Also, it is shown that the interior of any fuzzy g_X-semiclosed set is fuzzy g_X-regular open set. Usually, generalized fuzzy (g_X, g_Y)-continuous function is not a fuzzy (g_X, g_Y)-continuous function in respect of all contexts. Nevertheless, we have adopted one particular approach along which the implication is true. Comparison between different spaces is established and can be extended in the direction of L-fuzzy topological space. Zaharn *et al*. (2014) introduced generalized closed set in weak structure. So there is a further scope to study different kinds of generalized closed set in weak structures.

Acknowledgements

The author would like to thank the referees for their valuable comments and suggestions.

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