



*Original Article*

## Performance analysis and comparison of $m \times n$ zero forcing and MMSE equalizer based receiver for mimo wireless channel

N. Sathish Kumar\* and K. R. Shankar Kumar

*Department of Electronics and Communication Engineering,  
SriRamakrishna Engineering College, Coimbatore, TN, India -641022*

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### Abstract

Wireless transmission is affected by fading and interference effects which can be combated with equalizer. The use of MIMO system promises good improvement in terms of spectral efficiency, link reliability and Signal to Noise Ratio (SNR). The effect of fading and interference always causes an issue for signal recovery in wireless communication. Equalization compensates for Intersymbol Interference (ISI) created by multipath within time dispersive channels. This paper analyses the performance of Zero forcing and MMSE equalizer for MIMO wireless chanels. The simulation results are obtained using MatLab tool box version 7.0 at RF signal processing lab. The Bit Error Rate (BER) characteristics for the various transmitting and receiveing antennna is simulated in matlab tool box and many advantages and disdvantages of the system is descrbed. The simulation results show that the equalizer based zero forcing receiver is good for noise free channel and is successful in remving ISI, but MMSE is a better choice than ZF in terms of BER charateristics and under Noise performance.

**Keywords:** MIMO, Zero forcing Equalizer, ISI, BER, linear equalization, MMSE

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### 1. Introduction

Wireless communication systems has shown that using multiple antennas at both transmitter and receiver and thus provides the possibility of higher data rates compared to single antenna systems (David Gesbert *et al.*, 2003; Zhang *et al.*, 2006; Tse *et al.*, 2005). The system with multiple antennas at the transmitter and receiver is commonly referred is to as multiple input multiple output (MIMO) systems. The multiple antennas are used to increase data rates through multiplexing or to improve performance through diversity. This technique offers higher capacity to wireless systems and the capacity increases linearly with the number of antennas and link range with out additional bandwidth and power requirements. MIMO can reduce fading and improve higher spectral efficiency and link reliability or diversity (Jindal,

2005). In MIMO wireless communication, an equalizer is employed which is a network that makes an attempt to recover a signal that has suffers with an Inter symbol Interference (ISI).

### 2. Zero Forcing Equalizer Mathematics

Zero Forcing Equalizer is a linear equalization algorithm used in communication systems, it inverts the frequency response of the channel, which was proposed by Robert Lucky. The Zero-Forcing Equalizer applies the inverse of the channel to the received signal, to restore the signal before the channel. The name Zero forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero in a noise free case. This will be useful when ISI is more predominant when comparing to the noise (Zang *et al.*, 2006; yi Jiang *et al.*, 2011). For a channel with frequency response  $F(f)$  the zero forcing equalizer  $C(f)$  is constructed by  $C(f) = 1 / F(f)$ . Thus the combination of channel and equalizer gives a flat frequency response and linear phase  $F(f)C(f) = 1$ .

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\* Corresponding author.

Email address: [nsk20022002@gmail.com](mailto:nsk20022002@gmail.com)

Consider a 2x 2 MIMO channel, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad 1.1$$

The received signal on the Second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad 1.2$$

Where

- $y_1, y_2$  are the received symbol on the first and second antenna respectively,
- $h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,
- $h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,
- $h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,
- $h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna,
- $x_1, x_2$  are the transmitted symbols and
- $n_1, n_2$  are the noise on 1<sup>st</sup> and 2<sup>nd</sup> receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad 1.3$$

Equivalently,

$$Y = HX + N \quad 1.4$$

Where,

- $Y$  = Received Symbol Matrix.
- $H$  = Channel matrix .
- $X$  = Transmitted symbol Matrix .
- $N$  = Noise Matrix.

To solve for  $x$ , we need to find a matrix  $W$  which satisfies  $WH = I$ . The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad 1.5$$

Where

- $W$  - Equalization Matrix and
- $H$  - Channel Matrix

This matrix is known as the Pseudo inverse for a general  $m \times n$  matrix where

$$H^H H = \begin{pmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad 1.6$$

Note that the off diagonal elements in the matrix  $H^H H$  are not zero, because the off diagonal elements are non zero in values. Zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e. when solving for  $x_1$  the interference from  $x_2$  is tried to be nulled and vice versa. While doing so, there can be amplification of noise. Hence Zero forcing equalizer is not the best possible equalizer. (Jinag *et al.*, 2011) However, it is simple and reasonably easy to implement. For BPSK Modulation in Rayleigh fading channel, the BER is defined as

$$P_b = \frac{1}{2} \left( 1 - \sqrt{\frac{(E_b / N_o)}{(E_b / N_o) + 1}} \right) \quad 1.7$$

Where

- $P_b$  - Bit Error Rate
- $E_b / N_o$  - Signal to noise Ratio

### 3. MMSE Equalizer Mathematics

A Minimum Mean Square Error (MMSE) estimator describes the approach which minimizes the mean square error (MSE), which is a common measure of estimator quality. The main feature of MMSE equalizer, is that it does not usually eliminate ISI completely but, minimizes the total power of the noise and ISI components in the output (Sathish Kumar, *et al.*, 2011; Jinag *et al.*, 2011). Let  $x$  be an unknown random variable, and let  $y$  be a known random variable. An estimator  $\hat{x}(y)$  is any function of the measurement  $y$ , and its mean square error is given by

$$MSE = E \{(\hat{x}(y) - x)^2\} \quad 1.8$$

Where the expectation is taken over both  $x$  and  $y$ .

The MMSE estimator is then defined as the estimator achieving minimal MSE. In many cases, it is not possible to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimizing the MSE within a particular class, such as the class of linear estimators (Cho *et al.*, 2002). The linear MMSE estimator is the estimator achieving minimum Mean square error among all estimators of the form  $AY + b$ . If the measurement  $Y$  is a random vector,  $A$  is a matrix and  $b$  is a vector.

Let us now try to understand the mathematics for extracting the two symbols which interfered with each other.

In the first time slot, the received signal on the first receive antenna

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad 1.9$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad 2.0$$

Where

- $y_1, y_2$  are the received symbol on the first and second antenna respectively,
- $y_1, y_2$  are the received symbol on the first and second antenna respectively,
- $h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,
- $h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,
- $h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,
- $h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna,
- $x_1, x_2$  are the transmitted symbols and
- $n_1, n_2$  are the noise on 1<sup>st</sup> and 2<sup>nd</sup> receive antennas.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \quad 2.1$$

Equivalently,

$$Y = HX + N \quad 2.2$$

Where,

- $Y$  = Received Symbol Matrix.
- $H$  = Channel matrix .
- $X$  = Transmitted symbol Matrix .
- $N$  = Noise Matrix.

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient  $W$  which minimizes the criterion,

$$E \{ [W_{y-x}] [W_{y-x}]^H \}$$

Where

- $W$  - Equalization Matrix
- $H$  - Channel Matrix and
- $n$  - Channel noise
- $y$  - Received signal.

To solve for  $x$ , we need to find a matrix  $W$  which satisfies  $WH = I$ . The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by,

$$W = [H^H H + N_o I]^{-1} H^H \quad 2.3$$

This matrix is known as the pseudo inverse for a general  $m \times n$  matrix

Where

$$H^H H = \begin{pmatrix} h_{1,1}^* & h_{1,2}^* \\ h_{2,1}^* & h_{2,2}^* \end{pmatrix} \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad 2.4$$

When comparing the eq.(2.3) to the eq.(1.5) in Zero Forcing equalizer, apart from  $N_o I$  the term both the equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer.

#### 4. Simulation Results and Discussions

##### Simulation Analysis 1:

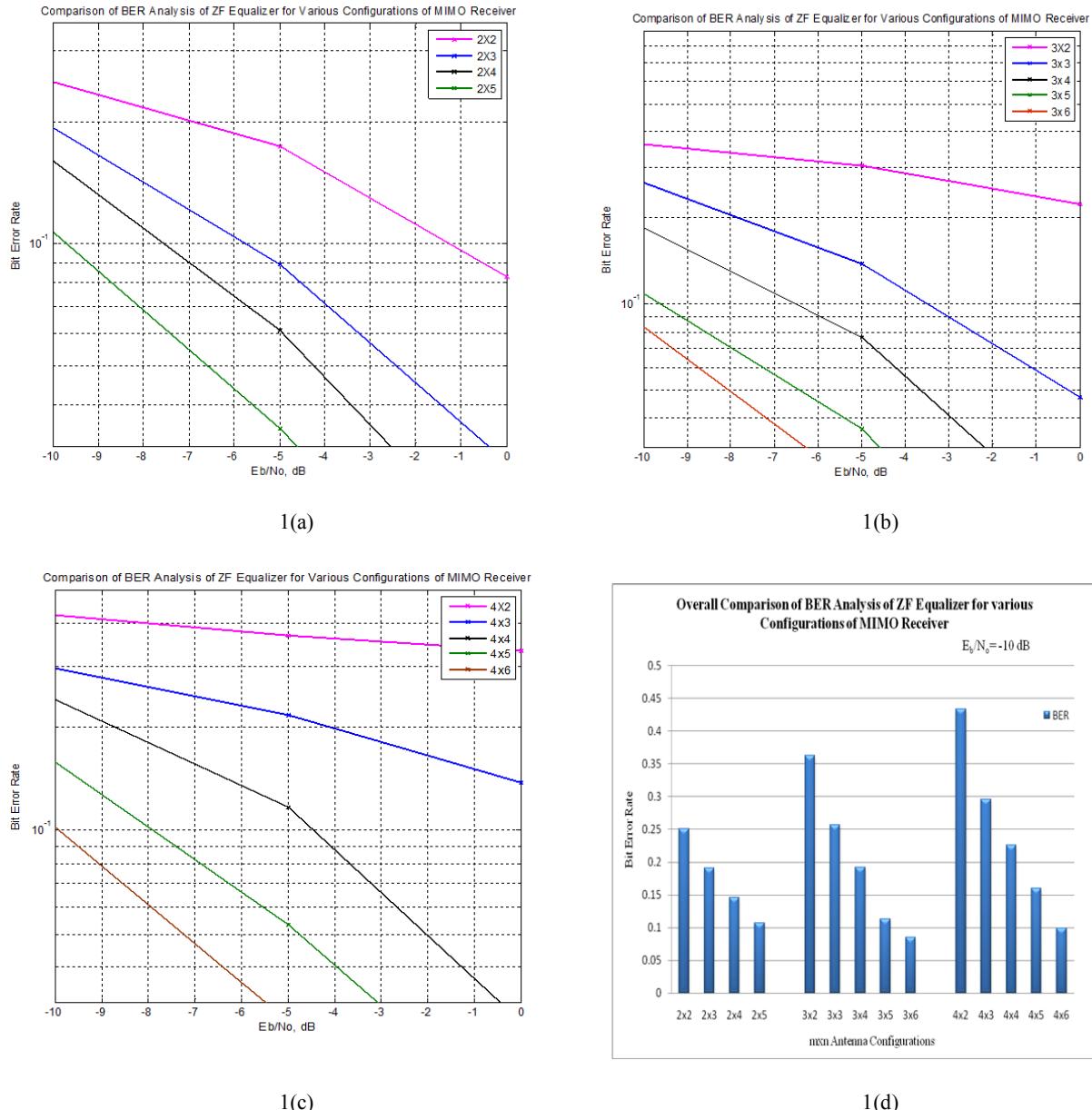
The simulations were carried out at RF signal processing lab Now let us study the simulation results of ZF equalizer receiver BER performance characteristics. Let us vary the receiver antenna keeping transmitter antenna constant for equalizer based receiver at a particular  $e_b/N_0$  value using BPSK modulation method. Figure 1(a-c) and table 1 show, as the no of receivers ( $n$ ) is increased keeping the no of transmitters ( $m$ ) as constant for a zero forcing receiver. Figure 1(a) shows  $m$  is fixed with 2 and  $n$  is varied. Similarly figure 1(b) and 1 (c) also shows the  $m$  is fixed with 2, and 3 and  $n$  is varied respectively. Figure 1(d) shows the consolidated result in the form of bar chart comparison. It is evident that the Bit Error Rate (BER) decreases as the number of transmitter increases, in Zero Forcing Equalizer.

The following observations are made. The Zero Forcing Equalizer removes all ISI and is ideal only when the channel is noiseless. When the channel is noisy, the Zero Forcing Equalizer will amplify the noise greatly at frequencies  $f$  where the channel response  $H(j2\delta f)$  has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. The ZF equalizer thus neglects the effect of noise altogether, and is not often for wireless links. However, it performs well for static channels with high SNR.

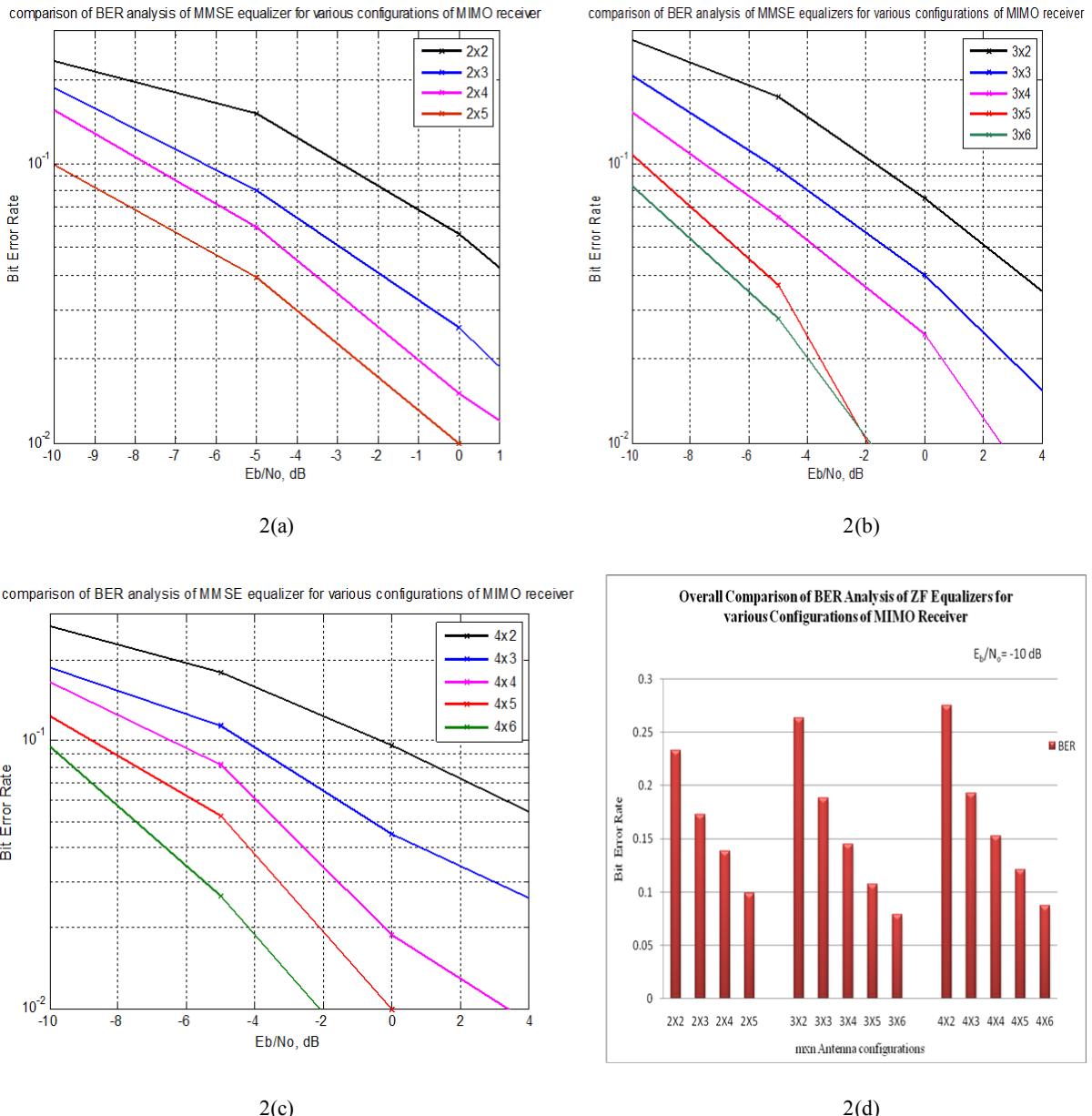
##### Simulation Analysis 2:

Figure 2 (a-c) and table 2 shows that as the no of receivers ( $n$ ) is increased keeping the no of transmitters ( $m= 2, 3$  and  $4$ ) as constant. It is observed that the Bit Error Rate (BER) decreases in MMSE equalizer . Figure 2(d) shows the consolidated result in the form of bar chart comparison.

From the above following observations are made .A more balanced linear equalizer is the Minimum Mean Square Error Equalizer, which is not eliminate ISI completely but instead minimizes the total power of the noise and ISI com-

Figure 1. Comparison of BER analysis for  $m \times n$  Antenna configurations of ZF Equalizer.Table 1. Bit Error Rate values for  $mn$  antenna configurations of ZF Equalizer

Sl.No	Bit Error Rate value for $= E_b / N_o - 10$ dB					
	mn	Value	mn	Value	mn	Value
1	2x2	0.251	3x2	0.3627	4x2	0.434
2	2x3	0.1907	3x3	0.257	4x3	0.2960
3	2x4	0.146	3x4	0.1919	4x4	0.2265
4	2x5	0.107	3x5	0.113	4x5	0.1605
5	-	-	3x6	0.08556	4x6	0.0992

Figure 2. Comparison of BER analysis for  $m \times n$  Antenna configurations MMSE EqualizerTable 2. Bit Error Rate values for  $mxn$  antenna configurations of MMSE Equalizer

Sl.No	Bit Error Rate value for $= E_b / N_o - 10 \text{ dB}$					
	$mxn$	Value	$mxn$	Value	$mxn$	Value
1	2x2	0.233	3x2	0.2633	4x2	0.275
2	2x3	0.1727	3x3	0.1883	4x3	0.193
3	2x4	0.1385	3x4	0.1448	4x4	0.152
4	2x5	0.099	3x5	0.1077	4x5	0.121
5	-	-	3x6	0.0791	4x6	0.087

ponents in the output. Hence from the above graphs it is evident that the BER decreases as the number of receiving antenna increases with respect to number of transmitting antenna in MMSE equalizer based MIMO receiver.

## Conclusion

This paper presents a simulation study on the performance comparison of ZR and MMSE Equalizer based MIMO receiver. This paper compares the mathematics and simulation results of two equalizers based MIMO receivers. The simulation results show that the BER characteristics for the two different types of equalizers namely ZF, and MMSE. Two types of simulation analysis are carried at RF signal processing lab. The Simulation analysis 1 discuss about  $m \times n$  ZF Equalizer analysis. This is by varying the receiver antenna configuration and keeping transmitter antenna constant for a particular type of  $E_b/N_0$  value using BPSK modulation method. The Zero Forcing Equalizer removes all ISI and is ideal only when the channel is noiseless. When the channel is noisy, the Zero Forcing Equalizer will amplify the noise greatly at frequencies  $f$  where the channel response  $H(j2\pi f)$  has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely.

From the simulation results its is summarized that Zero forcing equalization fails in the most of application due to the following.

- Already the channel impulse response has finite length but the impulse response of the equalizer need to be infinitely long.
- The channel may consist of zeros in its frequency response but that cannot be inverted.
- Some frequencies may be small, upon compensation it grows large.
- Addition of noise also gets boosted up and thus spoils the over all signal to noise ratio. Hence it is considered to a good receiver under noise free conditions.

Simulation analysis 2 discusses about MMSE Equalizer based MIMO receiver. Based on the mathematical model and simulation results the it is inferred that MMSE equalizer based receiver is removes a marginal noise but does not eliminate completely also doesn't amplify as the case of zero forcing. Hence MMSE equalizer based is a best choice than Zero forcing equalizer based receiver.

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