Original Article

Ergonomic workforce scheduling under complex worker limitation and task requirements: Mathematical model and approximation procedure

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Abstract

The ergonomic workforce scheduling problem (WPS) is addressed in this paper. Unlike its previous related works, the problem considers realistic worker limitation and task requirements that include heterogeneous workforce with limited task flexibility, varying worker team sizes, and pre-defined task operation schedules. Its main objective is to find a daily rotating work schedule solution using a minimum number of workers such that all workers' ergonomics hazard exposures do not exceed a permissible limit. Initially, the ergonomic WPS is explained. Its mathematical model and approximation procedure to obtain the workforce schedule solution are described. From the results of the computation experiment, it can be concluded that the approximation procedure is both efficient and effective in solving large-sized ergonomic workforce scheduling problems.

Keywords: workforce scheduling, ergonomics, hazard exposure reduction, optimization, approximation procedure

1. Introduction

The workforce scheduling problem (WSP) involves assigning a group of workers to perform a set of tasks over a given time period. Although the WSP is aimed to develop a feasible worker-task timetable under a number of restrictions, there are wide ranges of objectives depending on a key problem of each application. For example, the problem can be constructed so as to minimize either the number of workers (Narasimhan, 1997; Lagodimos and Leopoulos, 2000) or the total cost (Fowler *et al.*, 2008) to accomplish certain tasks, or to maximize the productivity performance (Chu, 2007). The WSP has been studied extensively in various service systems, for example, the scheduling of airline crew and bus/train drivers (Pinedo and Chao, 1999; Kwan, 2004; Qi *et al.*, 2004).

According to Baker (1976) and Ernst *et al.* (2004), shift scheduling or day-off scheduling is a common problem in the healthcare service system. Many researchers included the concept of rotating work schedules in their studies. Alfares (1998) developed an efficient two-phase algorithm for cyclic days-off scheduling. Later, he developed a new integer programming model and a two-stage solution method for the flexible 4-day workweek scheduling problem with weekend work frequency constraints (Alfares, 2003). Musliu *et al.* (2002) constructed a new framework that includes four main steps with backtracking algorithm for rotating workforce schedules. For large-sized problems, Musliu (2003) and Mora and Musliu (2004) applied methods based on the heuristic, GA, and tabu search to obtain rotating work schedules. Very few researchers, however, considered an ergonomics issue when they developed work schedules.

Industrial noise, thermal, and physical workloads are examples of common ergonomics hazards in the workplace. To avoid excessive exposure to any concerned ergonomics hazard, workers are either rotated among different work-stations or assigned to perform industrial tasks at different

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work areas within the same workday. For effective hazard exposure reduction, it is necessary to determine the work schedules such that no workers are exposed to the concerned hazard beyond a permissible daily limit. Nanthavanij and Yenradee (1999) proposed a quantitative approach to job rotation by developing a mathematical model for the problem with equal numbers of workers and tasks. Their solution described the rotating work schedules such that the maximum noise hazard exposure is minimized. Nanthavanij and Yenradee (2000a) investigated the effect of work period length on the noise hazard reduction. Later, they developed a mathematical model to determine the minimum number of workers for job rotation (Nanthavanij and Yenradee, 2000b). For the complex safety-based job rotation problem, a genetic algorithm (GA) approach was applied to obtain the minimax work assignment solution (Nanthavanij and Kullpattara-nirun, 2001; Kullpattaranirun and Nanthavanij, 2005).

Yaoyuenyong (2006) showed that when the minimum number of workers for job rotation is to be determined, the WSP is a variant of the classic bin packing problem, which is a well known *NP* hard problem. Thus, the optimal rotating work schedule solution is obtainable in reasonable amount of time only when the problem size is relatively small. For large problems, a heuristic approach has been a popular choice among researchers. Yaoyuenyong and Nanthavanij (2006) developed a hybrid procedure to determine an optimal workforce without being exposed to excessive noise hazard in the manufacturing environment. Additionally, they developed heuristic job rotation procedures for workers who are exposed to single-limit and multiple-limit occupational hazards (Yaoyuenyong and Nanthavanij, 2008). Nanthavanij *et al.* (2010) included the productivity issue in their study and developed a heuristic procedure to find appropriate work schedules such that workers are assigned to the tasks that they can perform competently.

This paper addresses the ergonomic WSP under complex worker limitation and task requirements. Specifically, heterogeneous workforce with limited task flexibility, varying worker team sizes, and pre-defined task operation schedules are considered. The paper is organized as follows. First, the ergonomic WSP is explained. Then, an integer linear programming model representing this problem is developed. An approximation procedure for solving large-sized problems is proposed. Using a hypothetical example, solutions from the optimization approach and approximation procedure are compared. Finally, the efficiency and effectiveness of the approximation procedure are evaluated.

2. Ergonomic Workforce Scheduling Problem

As required by virtually all safety laws, workers must not be exposed to a given occupational hazard beyond a permissible limit within each workday. When job rotation is applied, workers are rotated to perform several tasks (preferably at different work locations or areas) during the workday to reduce their hazard exposures. The effectiveness of job rotation depends on the number of utilized workers and their daily work schedules. When the hazard levels of the concerned tasks are high, it might be necessary to increase the size of workforce to help alleviate the daily hazard exposure of each worker.

Since increasing the number of workers also increases the cost of manpower, it is important to find the right workforce size for the job rotation. Most recent works in ergonomic workforce scheduling assume that the workforce is homogeneous. In other words, any worker can be assigned to perform a given task. While this assumption simplifies the workforce scheduling problem, it is undoubtedly unrealistic. Generally, workers are different in terms of the number of tasks that can be assigned to them. If a worker is assigned to the task that he/she cannot perform, the work system performance could be seriously affected and such assignment will not be acceptable. Thus far, little attention has been given to this worker limitation. Moreover, most studies assume that each concerned task needs only one worker to perform. In practice, there are numerous tasks or workstations that require two or more workers to work together. It is reasonable to assume that these workers receive the same amount of ergonomics hazard. Failure to consider the above mentioned worker limitation and task requirements could, to some extent, hinder the applications of job rotation.

Specifically, the worker limitation and task requirements considered in this study are as follows: 1) Workers are not equally flexible. Some workers are well trained and can perform many tasks, while some might be able to perform only one or two tasks. 2) The numbers of workers assigned to perform individual tasks do not have to be equal. 3) Not all tasks need to be performed on a full-day basis. Some tasks might be performed only part of the day.

In brief, a feasible daily rotating work schedule solution must satisfy the following conditions: 1) All workers' daily hazard exposures must not exceed the permissible limit. 2) Workers must not be assigned to the tasks that they cannot perform. 3) The number of workers assigned to any task must exactly match the number of workers required by that task.

3. Mathematical Model and Approximation Procedure

The ergonomic WSP can be mathematically formulated as an integer linear programming problem. Its objective is to find a minimum set of workers for job rotation to satisfy all ergonomics, worker limitation, and task requirements constraints.

The model formulation requires the following assumptions: 1) A workday is divided into equal work periods. Job rotation is allowed only at the end of the work period. 2) The number of workers is equal to or greater than the number of tasks. 3) For any given work period, a worker can be assigned to perform at most one task. 4) In any given work period, a task may or may not be performed depending on its operation schedule. 5) The number of workers required to perform

different tasks do not have to be equal. 6) The numbers of tasks that the workers can perform do not have to be equal. 7) The permissible daily limit of hazard exposure is known and is the same for all workers.

3.1 Mathematical model

The model parameters, variables, and decision variables are listed below.
The parameters are:

I number of available workers for job rotation
J number of tasks to be performed
K number of equal work periods per workday
L permissible daily limit of hazard exposure

The variables are:

a_{ij} 1 if worker i can perform task j
     0 otherwise
h_j amount of hazard exposure per work period of task j
t_{jk} 1 if task j has to be performed in work period k
     0 otherwise
w_j number of workers required to perform task j
Y total number of utilized workers for job rotation

The decision variables are:

x_{ijk} 1 if worker i is assigned to perform task j in work period k
     0 otherwise
y_i 1 if worker i is chosen from the group of available workers
     0 otherwise

$$\text{Minimize} \quad Y = \sum_{i=1}^{I} y_i \qquad (1)$$

subject to

$$\sum_{j=1}^{J} \sum_{k=1}^{K} h_j x_{ijk} \leq L \qquad \text{for } \forall\, i \qquad (2)$$

$$\sum_{j=1}^{J} x_{ijk} \leq 1 \qquad \text{for } \forall\, i,k \qquad (3)$$

$$\sum_{i=1}^{I} x_{ijk} = w_j \cdot t_{jk} \qquad \text{for } \forall\, j,k \qquad (4)$$

$$x_{ijk} \leq y_i \qquad \text{for } \forall\, i,j,k \qquad (5)$$

$$x_{ijk} \leq a_{ij} \qquad \text{for } \forall\, i,j,k \qquad (6)$$

$$x_{ijk} \leq t_{jk} \qquad \text{for } \forall\, i,j,k \qquad (7)$$

$$x_{ijk}, y_i \in \{0,1\} \qquad \text{for } \forall\, i,j,k \qquad (8)$$

The first equation is an objective function, which sums up the number of utilized workers. Constraint 2 requires that the worker's daily hazard exposure does not exceed the permissible limit. Constraint 3 specifies that a worker can be assigned to perform at most one task per work period. Constraint 4 ensures that when any task is to be performed, the number of assigned workers must be equal to that required by the task. Constraint 5 shows that the utilized workers are drawn from the group of available workers. Constraint 6 does not permit any worker to be assigned to the task that one cannot perform. Constraint 7 specifies that a worker can only be assigned to the task that is being performed. Finally, constraint 8 defines the decision variables.

3.2 Approximation procedure

Since WSP is a well known *NP* hard problem, it is not possible to solve large problems to optimality in reasonable time. An approximation procedure to obtain the ergonomic work schedule solution is developed. First, a lower bound of the number of workers for job rotation R can be computed using the following steps.

1. For each work period k where $k = 1$ to K, compute

$$r_k = \sum_{j=1}^{J} w_j \cdot t_{jk}.$$

2. Set $R' = \max\{r_k \,|\, k = 1,...,K\}$.

3. Compute $E = \dfrac{1}{L} \sum_{j=1}^{J} \sum_{k=1}^{K} h_j \cdot t_{jk}$.

4. If $E \leq J$, set $R = R'$. Otherwise, compute $R = \left(R' \cdot \dfrac{E}{J}\right)$. In case R is fraction, round the result to the nearest integer.

The approximation procedure consists of two phases. In Phase 1, an initial daily rotating work schedule solution is constructed. In Phase 2, task exchanges between work periods among different workers are evaluated in order to reduce the number of utilized workers.

Initialization:

0.1 Let H_i be the sum of hazard exposure h_j's from all tasks currently assigned to worker i in all work periods. Initially, set $H_i = 0$ for all i's.

0.2 List the concerned tasks in decreasing order of h_j where $j = 1$ to J. If there is a tie, break the tie arbitrarily.

0.3 Let A_i be the number of tasks that worker i can be assigned to perform, where for all i's.

0.4 List the workers in decreasing order of A_i for all i's.

0.5 Create a set of workers W consisting of the first R workers on the list in Step 0.4.

0.6 Set $j = 1$ and $k = 1$.

Phase 1: Assigning workers to tasks

1.1   Check if task j satisfies the following two conditions:

a)   task j needs to be performed in the current work period k ($t_{jk} = 1$), and

b)   task j still needs a worker (i.e., its required number of workers w_j is not met).

If both conditions are satisfied, then task j needs to be considered in the current work period k. Proceed to the next step. Otherwise, go to Step 1.4.

1.2   Find any available worker i in W whose $H_i \leq L$ and is currently the smallest. If there is a tie, choose the worker whose A_i is larger (i.e., who can be assigned to perform many tasks). If all workers in W have been considered but there still are some tasks left to be assigned, a new worker (from those not previously included in W), whose A_i is the largest and who can perform the concerned task, will be added.

1.3   Check if worker i satisfies the following three conditions: a) worker i can perform task j ($a_{jk} = 1$), b) worker i is still available in the current worker period k ($x_{ijk} = 0$), and c) the new sum of hazard exposure does not exceed the permissible limit ($H_i + h_j \leq L$). If all three conditions are satisfied, then task j can be assigned to worker i in the current work period k. Assign task j to worker i, update the worker's sum of hazard exposure H_i, and proceed to the next step. Otherwise, return to Step 1.2 to find another worker to perform task j. If only Condition b is not satisfied, consider re-assigning the task already assigned to worker i in work period k to another work period, say work period c, that worker i is available. First, find another worker, say worker a, who is previously assigned to perform that task in work period c and is still available in work period k. If such worker a exists, then move the task already assigned to worker i in work period k to work period c and move the same task already assigned to worker a in work period c to work period k. Now that worker i is available in work period k, assign task j to worker i, update the worker's sum of hazard exposure H_i, and proceed to the next step. Otherwise, return to Step 1.2 to find another worker to perform task j.

1.4   Set $k = k + 1$. If $k \leq K$, return to Step 1.1 to consider the next work period. If $k > K$, proceed to the next step.

1.5   Set $j = j + 1$. If $j > J$, go to Phase 2 (Step 2.1). Otherwise, reset $k = 1$ and return to Step 1.2.

Phase 2: Exchanging tasks between work periods

2.1   Consider the first worker having the smallest H_i.

2.2   Set $i = 1$ and $k = 1$.

2.3   If worker i is assigned to any task in work period k, proceed to the next step. Otherwise, set $k = k + 1$ and repeat this step. If $k > K$, set $i = i + 1$ and repeat this step. If all workers have been considered, go to Step 2.12.

2.4   Find another worker, say worker a ($a \neq i$), who can perform the task currently assigned to worker i, say task j, in work period k. In case there are such several workers, list

them in ascending order of the total hazard exposure H. If there is a tie, break the tie arbitrarily. Select the worker having the smallest H as worker a. If worker a is available in work period k and $L - H_a \geq h_j$, go to Step 2.7. If worker a is available in work period k but $L - H_a < h_j$, then consider re-assigning the tasks currently assigned to worker a to other workers so that $L - H_a \geq h_j$. If the task exchanges to reduce H_a are successful, go to Step 2.7. However, if no other workers are found, discard the current worker a from consideration. Select another worker who can perform the considered task in work period k. Then, repeat Step 2.4. If worker a is unavailable in work period k, proceed to the next step.

2.5   Find another worker, say worker b ($b \neq i, a$), who can perform the task currently assigned to worker a in work period k. Call the work period in which worker b performs that task as work period c. Worker b must still be available in work period k. Then, proceed to the next step. However, if no such worker b exists, discard the current worker a from consideration. Select another worker who can perform the considered task in work period k. Then, repeat Step 2.4.

2.6   Perform the following exchanges: a) Reassign the task currently assigned to worker a in work period k to worker b (in the same work period). b) Switch the tasks currently to workers a and b in work period c.

2.7   Re-assign task j to worker a and add the hazard exposure amount h_j to H_a.

2.8   Compute the new total hazard exposure H_i for all worker i's.

2.9   If there is at least one worker whose $H_i = 0$, proceed to the next step. Otherwise, go to Step 2.11.

2.10   Delete such worker(s) from the utilized worker group, keep the task exchanges, and set $k = k + 1$. If $k \leq K$, return to Step 2.3. Otherwise, set $i = i + 1$ and return to Step 2.3.

2.11   Keep the task exchanges temporarily. Set $i = i + 1$ and return to Step 2.3. If all worker i's have been considered but the number of utilized workers is not decreased, reset the current work schedule solution to the initial solution.

2.12   The current number of workers in the utilized worker group is the minimum number of workers required for job rotation. Phase 2 is terminated.

4. Numerical Example

Let us consider a hypothetical workplace with a certain ergonomics hazard and assume that there are five tasks (T1, …, T5) to be performed. A workday is divided into four equal work periods. At each task location, the hazard exposure amount per work period is known and not time-dependent. For simplicity, it is assumed that the permissible daily hazard exposure limit L is 1.0000. Table 1 shows the hazard exposure amounts per work period, numbers of required workers, and required operation periods of these five tasks. Currently, there are 20 workers (W1, …, W20)

Table 1. Task data

Task	Hazard Exposure Amount per Work Period	Number of Required Workers	Operation Period			
			1	2	3	5
T1	0.3090	2	Y	Y	-	Y
T2	0.1952	3	-	Y	Y	Y
T3	0.4291	2	-	-	Y	Y
T4	0.5937	1	Y	Y	Y	Y
T5	0.2812	1	Y	Y	Y	-

Y = Task will be performed.

Table 2. Worker data

Worker	Task					Worker	Task				
	T1	T2	T3	T4	T5		T1	T2	T3	T4	T5
W1	Y	Y	-	Y	-	W11	Y	-	Y	Y	Y
W2	-	Y	Y	Y	Y	W12	-	Y	Y	Y	Y
W3	Y	Y	-	Y	Y	W13	Y	Y	Y	Y	Y
W4	Y	-	Y	Y	-	W14	-	Y	Y	Y	Y
W5	Y	Y	Y	-	Y	W15	Y	Y	Y	Y	Y
W6	-	Y	Y	Y	Y	W16	Y	Y	Y	Y	Y
W7	Y	Y	Y	-	Y	W17	Y	Y	Y	Y	Y
W8	Y	-	Y	Y	Y	W18	Y	Y	Y	Y	Y
W9	Y	Y	-	Y	-	W19	Y	Y	Y	Y	Y
W10	Y	Y	Y	-	Y	W20	Y	Y	Y	Y	Y

Y = Worker can perform the task.

available for job rotation. Table 2 shows all possible worker-task pairings.

From Table 1, it can be seen that if any worker is assigned to perform task T4 throughout the workday, the sum of hazard exposure amounts will exceed 1.0000 (which is the permissible daily limit). It is thus necessary to find a feasible set of rotating work schedules using the minimum number of required workers whereas all daily hazard exposures do not exceed 1.0000.

The problem is formulated as an integer linear programming model and solved using the IBM ILOG CPLEX v.12.1.0 software program. Since the problem size is small, an optimal solution can be obtained. The resulting job rotation requires a minimum number of nine workers to prevent any workers' daily hazard exposures from exceeding 1.0000. The optimal rotating work schedule solution is shown in Table 3.

Next, the approximation procedure described in Section 3.2 is applied to obtain a feasible daily rotating work schedule solution. A lower bound of the number of utilized workers for job rotation is computed using the steps presented in Section 3.2. The resulting lower bound is found to be nine workers, which is equal to the minimum number of workers obtained from ILOG CPLEX.

From the steps in Phase 1, an *initial* feasible work schedule solution requires 10 workers. Table 4 shows the work schedules of the 10 utilized workers. The maximum total hazard exposure is 0.9841 (in workers W2, W3, W13, and W16) and the minimum value is 0.1952 (in worker W5).

Worker W5 who is the last chosen worker is assigned to perform only one task, i.e., T2, in work period 4. The worker's total hazard exposure is only 0.1952. Therefore, it is expected that the number of utilized workers could be reduced by 1 worker if the task T2 currently assigned to worker W5 can be assigned to another worker who is available in the same work period and can perform this task.

From Phase 2, it is found that worker W5 can be dismissed from the workforce. The resulting *improved* work schedule solution is shown in Table 5. The final workforce consists of nine workers. The task exchanges in Phase 2 are able to allocate the five tasks to these nine workers more evenly in terms of the total hazard exposure. The maximum value is 0.9915 (in worker W16) and the minimum value is 0.8581 (in worker W17).

When comparing the two solutions shown in Table 3 (from ILOG CPLEX) and Table 5 (from the approximation procedure), it can be seen that the approximation procedure is nearly as effective as ILOG CPLEX in generating the safe

Table 3.   Optimal work schedule solution (from ILOG CPLEX).

Utilized Worker	Work Period				Daily Hazard Exposure
	1	2	3	4	
W1	-	T2	T4	T2	0.9841
W3	T4	-	-	T1	0.9027
W5	T1	-	T3	T2	0.9332
W6	-	T2	T2	T4	0.9841
W7	-	T1	T2	T3	0.9332
W8	-	T5	T5	T3	0.9915
W10	T5	T1	T2	T2	0.9806
W16	T1	T2	T3	-	0.9332
W20	-	T4	-	T1	0.9027

Table 4.   Initial feasible work schedule solution (from Phase 1).

Utilized Worker	Work Period				Daily Hazard Exposure
	1	2	3	4	
W2	T4	T2	T2	-	0.9841
W3	-	T2	T2	T4	0.9841
W13	-	T2	T4	T2	0.9841
W15	-	T4	T2	T2	0.9841
W16	-	-	T3	T3	0.8581
W17	-	-	T3	T3	0.8581
W18	T1	T1	-	T1	0.9269
W19	T1	T1	-	T1	0.9269
W20	T5	T5	T5	T2	0.8437
W5	-	-	-	T2	0.1952

Table 5.   Improved work schedule solution (from Phase 2).

Utilized Worker	Work Period				Daily Hazard Exposure
	1	2	3	4	
W2	T4	T2	T2	-	0.9841
W3	-	T2	T2	T4	0.9841
W13	-	T2	T4	T2	0.9841
W15	-	T4	T2	T2	0.9841
W16	T5	-	T5	T3	0.9915
W17	-	-	T3	T3	0.8581
W18	T1	T1	-	T1	0.9269
W19	T1	T1	-	T1	0.9269
W20	-	T5	T3	T2	0.9055

rotating work schedules. Both solutions require the same number of workers and yield the same maximum total hazard exposure values. The fact that the gap between the maximum and minimum total hazard exposures in Table 3 is smaller than that in Table 5 indicates that ILOG CPLEX is able to obtain a superior work schedule solution.

5. Computation Experiment

Fifty two test problems (P1 – P52) were generated. The number of workers ranged between 20 and 45 persons and the number of tasks ranged between 5 and 16 tasks. While there were some problems with the same numbers of workers

and tasks, their hazard exposure amounts per work period, numbers of workers per task, and task operation schedules were however different. All test problems were solved using both the optimization approach (by ILOG CPLEX) and the approximation procedure. For convenience, a computer program was written using an Optimization Programming Language (OPL) in ILOG CPLEX based on the computation steps of the approximation procedure in order to solve the 52 test problems.

Only 37 test problems could be solved to optimality by ILOG CPLEX. Among the 15 unsolved test problems, ILOG CPLEX could not solve 13 test problems due to an "out of memory" error. For the other 2 unsolved test problems, the program was terminated after reaching a preset computation time limit of 86,400 seconds. Thus, the optimality of the solutions could not be proved. Table 6 shows the resulting numbers of utilized workers of the 52 test problems from both solution approaches.

From the 37 solved test problems, the approximation procedure could yield the same numbers of utilized workers as those obtained from ILOG CPLEX in 30 test problems (81.08%). For the remaining seven test problems, the approximation procedure obtained the numbers of utilized workers with only 1 worker more than those obtained by ILOG CPLEX. The average computation time was 13.26 seconds, with a standard deviation of 6.14 seconds. It was also observed that the computation time of the approximation procedure increased linearly with the problem size.

6. Conclusion

The ergonomic WSP is addressed in this paper. The problem is intended to develop daily rotating work schedules for workers to alleviate their total hazard exposures and prevent from exceeding the permissible limit. From a given set of tasks, the number of utilized workers is to be minimized.

Table 6.   Summary of numbers of utilized workers.

Problem			TW		CT		Problem			TW		CT	
No.	W	T	O	A	O	A	No.	W	T	O	A	O	A
P1	20	5	14	14	0.44	7.86	P27	30	10	24	25	3.46	11.35
P2	20	5	13	13	0.59	8.35	P28	30	10	n/a	21	8744.14	10.12
P3	20	5	12	12	0.26	7.69	P29	30	10	23	23	14.74	12.89
P4	20	5	11	11	0.50	7.42	P30	30	10	29	29	0.19	13.04
P5	20	5	10	10	0.33	7.51	P31	32	11	28	28	534.93	13.95
P6	20	5	14	14	0.09	8.12	P32	32	11	24	24	2.26	12.30
P7	20	5	10	10	1.83	7.55	P33	32	11	26	26	1.48	14.76
P8	20	5	12	12	0.09	7.29	P34	32	11	28	28	0.36	16.30
P9	20	5	9	9	0.95	7.19	P35	32	11	26	27	1.15	11.92
P10	20	5	10	11	0.44	7.30	P36	32	11	26	26	0.83	14.15
P11	25	8	23	23	2.78	10.97	P37	32	11	26[*]	26	86400.00	12.32
P12	25	8	17	17	0.59	9.06	P38	32	11	29	29	1.08	16.41
P13	25	8	17	17	48.95	8.73	P39	32	11	26	26	1.39	13.29
P14	25	8	21	21	1.34	9.69	P40	32	11	n/a	26	15581.05	11.65
P15	25	8	20	20	6.99	9.24	P41	35	12	n/a	30	14893.85	14.64
P16	25	8	21	21	0.94	9.33	P42	35	12	n/a	28	14615.95	14.81
P17	25	8	21	21	64.22	8.94	P43	35	12	n/a	29	15968.01	12.56
P18	25	8	18	19	2.98	10.14	P44	40	14	n/a	32	8640.77	18.16
P19	25	8	15	15	39.28	9.05	P45	40	14	n/a	32	30143.70	15.69
P20	25	8	22	22	0.61	10.98	P46	40	14	35[*]	35	86400.00	26.10
P21	30	10	20	20	0.70	9.78	P47	43	15	41	41	2.78	28.68
P22	30	10	28	29	5.07	14.22	P48	43	15	n/a	31	31123.04	14.89
P23	30	10	n/a	24	11611.50	11.38	P49	43	15	n/a	36	19867.41	26.62
P24	30	10	23	23	3.09	12.50	P50	45	16	n/a	43	9312.93	33.53
P25	30	10	26	27	6.21	13.01	P51	45	16	n/a	33	4456.62	23.06
P26	30	10	27	28	2.14	13.08	P52	45	16	n/a	40	11671.96	29.95

Note: A = approximation procedure; CT = computation time; n/a = not solvable due to "out of memory" error; O = optimization approach (ILOG CPLEX); T = number of tasks; TW = total number of required workers; W = number of available workers; * = current best solution (terminated due to exceeding the time limit of 24 hours); Computer specifications: Intel Core i5-460M, 2.53 GHz, 4GB RAM.

The problem also considers realistic worker limitation and task requirements. As for the worker limitation, their differences in terms of task flexibility are accounted for. Workers usually have limited task training and can perform only a few tasks competently. Some tasks do not have to be performed on a full-day basis. Also, some tasks need only one worker to perform, while some might need two or more workers working as a worker team.

The ergonomic WSP can be expressed as an integer linear programming model. For small-sized problems, an optimization software program such as the IBM ILOG CPLEX v.12.1.0 can be employed to solve the problems to optimality. The proposed approximation procedure can be applied to solve large problems. From the 52 test problems with 20-45 workers and 5-16 tasks used in the computation experiment, ILOG CPLEX can obtain the optimal solutions for 37 test problems. The remaining 15 problems are unsolvable due to either an "out of memory" error or exceeding the computation time limit (of 86,400 seconds).

The approximation procedure can solve all 52 test problems. The average computation time is 13.26 seconds. It can yield the same numbers of utilized workers as those obtained from the ILOG CPLEX in 30 out of 37 solved test problems (about 81%). For the rest of the solved test problems, the difference is only one worker. From the results, it is reasonable to conclude that the proposed approximation procedure is both efficient (in terms of the computation time) and effective (in terms of the solution quality) in solving the ergonomic WSP with worker limitation and task requirements.

Acknowledgments

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