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Original Article

Spatial object modeling in soft topology

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Abstract

This paper gives fundamental concepts and properties of a soft spatial region. We provide a theoretical framework for both dominant ontologies used in GIS.

Keywords: soft sets, soft topology, soft spatial region, GIS

1. Introduction

Molodtsov (1999) introduced the concept of soft sets as a new mathematical tool for dealing with uncertainty. Molodtsov (1999, 2001, and 2004) applied this theory to several directions, and then formulated the notions of soft number, soft derivative, soft integral, etc in Molodtsov *et al.* (2006). The soft sets theory has been applied to many fields of sciences. Maji *et al.* (2003) worked on theoretical study of soft sets in detail, and also presented an application of soft set in the decision making problem using the reduction of rough sets (see Maji *et al.*, 2002).

Recently, Shabir and Naz (2011) initiated the study of the notion of soft topological spaces. They defined soft topology as a collection τ of soft sets over X and gave basic notions of soft topological spaces such as soft open and closed sets, soft subspace, soft closure operator, soft neighborhood and soft T_i-spaces, for i = 1,2,3,4. After that many authors (Aygünoğlu and Aygün (2012), Min (2011), Zorlutuna *et al.* (2012), Hussain and Ahmad (2011), Varol and Aygün (2013)) studied some of the basic concepts and properties of soft topological spaces.

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A topological relation is a relation that is invariant under homeomorphisms. Topological relations have an important significance in GIS modeling since they are the basis for spatial modeling, spatial query, analysis and reasoning.

Recently, topological relations have been much investigated in the crisp topological space. White (1980) introduced the algebraic topological models for spatial objects. Allen (1983) identified thirteen topological relations between two temporal intervals. The famous 4-intersection approach (see Egenhofer (1989), Egenhofer and Franzosa (1991)) and the 9-intersection approach Egenhofer and Herring (1990) were proposed for formalism of topological relations between two simple regions, which are defined in the crisp topological space. These approaches were then generalized in different ways and different applications (Egenhofer and Franzosa (1995), Egenhofer *et al.* (1994), Chen *et al.* (2000, 2001)). Most of these theoretical results has been applied in GIS software such as maintaining of topological consistency (Egenhofer and Sharma (1993), Kainz (1995), Oosterom (1997)) and spatial reasoning (see Dutta (1991), Al-Taha (1992)).

Many ways can be used to identify the topological relations. In Kainz et al. (1993), some topological relations are investigated based on poset and lattice theory. They pointed out that the poset and lattice can be used in GIS modeling and can reduce the complexity of some queries. In 1992, Randell *et al.* (1992) deduced the topological relations based on logic. Eight relations were identified based on their region connection calculus (RCC) theory. It is also appropriate for their theory to be adopted for spatial reasoning (see Bennett et al. (1997), Bennett and Cohn (1999)).

In this paper, we will model and analyze the spatial relationships from the view point of soft sets theory. We demonstrate how it can provide model for soft region and a 9-intersection model.

2. Soft Sets and Soft Topological Spaces

Throughout this section, we recall the concepts of soft set and soft topology which has been discussed in Molodtsov (1999, 2001, and 2004), Molodtsov *et al.* (2006), Maji *et al.* (2003), Shabir and Naz (2011), Aygünoğ lu and Aygün (2012), Min (2011), Zorlutuna *et al.* (2012), Hussain and Ahmad (2011), Varol and Aygün (2013).

2.1 Soft sets

Definition 2.1 Let X be an initial universe, E be a set of parameters, $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$.

This means that a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2 Let (F, A) and (G, B) be two soft sets over a common universe X. Then:

(1) (F, A) is a soft subset of (G, B) (denoted by $(F, A) \tilde{\subset} (G,B)$ iff $A \subset B$ and $F(e) \subset G(e)$ for each $e \in A$.

(2) (F, A) is a soft equal to (G, B) (denoted by $(F,A) \tilde{=} (G,B)$) iff $(F,A) \tilde{\subset} (G,B)$ and $(G,B) \tilde{\subset} (F,A)$.

(3) The union of (F, A) and (G, B) (denoted by $(H,C) \tilde{=} (F,A) \tilde{\cup} (G,B)$) is defined by $C = A \cup B$ and

$$H(e) = \begin{cases} F(e), & if\ e \in A - B; \\ G(e), & if\ e \in B - A; \\ F(e) \cup G(e), & if\ e \in A \cap B. \end{cases}$$

for each $e \in C$.

(4) The intersection of (F, A) and (G, B) (denoted by $(H,C) \tilde{=} (F,A) \tilde{\cap} (G,B)$) is defined by $C = A \cap B$ and $H(e) = F(e) \cap G(e)$ for each $e \in C$.

Definition 2.3 A soft set (F, A) over the initial universe X and parameters set E, where $A \subset E$ is called:

(1) A null soft set (denoted by $\tilde{\varnothing}$) if $F(e) = \varnothing$ for each $e \in A$.

(2) An absolute soft set (denoted by $\tilde{X}$) if $F(e) = X$ for each $e \in A$.

Definition 2.4 Let (F, A) be a soft set over the initial universe X and parameters set E where $A \subset E$. Then:

(1) The relative complement of (F, A) (denoted by $(F, A)^C$) is defined by $(F,A)^c \tilde{=} (F^c, A)$, where $F^c : A \to P(X)$ is given by $F^c\ (e) = X - F(e)$ for each $e \in A$.

(2) $x \in (F, A)$ (read as x belongs to a soft set (F, A)) where $x \in X$ iff $x \in F(e)$ for each $e \in A$ and $x \notin (F, A)$ whenever $x \notin F(e)$ for some $e \in A$.

Definition 2.5 Let X be an initial universe and E set of parameters such that $A \subset E$ and $x \in X$. The soft set (x, A) is defined by $x(e) = \{x\}$ for each $e \in A$.

2.2 Soft topology

Definition 2.6 Let X be an initial universe and E set of parameters. A collection τ of soft sets defined over X with respect to E is called a soft topology iff it contains $\tilde{\varnothing}, \tilde{X}$ and closed under finite soft intersections and arbitrary soft unions. The triplet (X, τ, E) is called a soft topological space over X.

Let (X, τ, E) be a soft topological space over X, then the members of τ are said to be open soft sets and its relative complement are closed soft sets.

Proposition 2.7 Let (X, τ, E) be a soft topological space over X. Then the collection $\tau_e = \{F(e) : (F, E) \in \tau\}$ for each $e \in E$, defines a topology on X.

It is clear from the obvious proposition that the soft topology on X is a parameterized family of topologies on X where E is the set of parameters.

Definition 2.8 Let (X, τ, E) be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E) (denoted by $\overline{(F,E)}$) is the intersection of all closed soft super sets of (F, E). Clearly $\overline{(F,E)}$ is the smallest closed soft set over X which contains (F, E).

Theorem 2.9 If (X, τ, E) is a soft topological space and (F, E), (G, E) are two soft sets over X, then we have:

(1) $\overline{\tilde{\phi}} \tilde{=} \tilde{\phi}$ and $\overline{\tilde{X}} \tilde{=} \tilde{X}$.

(2) $(F,E) \tilde{\subset} \overline{(F,E)}$.

(3) (F, E) is closed soft subset iff $\overline{(F,E)} \cong (F,E)$.

(4) $\overline{\overline{(F,E)}} \cong \overline{(F,E)}$.

(5) If $(F,E) \tilde{\subset} (G,E)$, then $\overline{(F,E)} \tilde{\subset} \overline{(G,E)}$.

(6) $\overline{(F,E) \tilde{\cup} (G,E)} = \overline{(F,E)} \tilde{\cup} \overline{(G,E)}$.

(7) $\overline{(F,E) \tilde{\cap} (G,E)} = \overline{(F,E)} \tilde{\cap} \overline{(G,E)}$..

Definition 2.10 Let (X,τ,E) be a soft topological space over the universe X, (G, E) be a soft set over X, and $x \in X$. Then x is a soft interior point of (G, E) if there exists a soft open set (F, E) such that $x \in (F,E) \tilde{\subseteq} (G,E)$. In this case, the soft set (G, E) is called a soft neighborhood of x.

Definition 2.11 Let (X,τ,E) be a soft topological space over X, then the soft interior of the soft set (F, E) (denoted by $(F, E)^{\circ}$) is the union of all soft open sets contained in (F, E). Clearly, $(F, E)^{\circ}$ is the largest soft open set contained in (F, E).

Theorem 2.12 If (X,τ,E) is a soft topological space and $(F, E),(G, E)$ are two soft sets over X, then we have:

(1) $\tilde{\phi}^{\circ} \cong \tilde{\phi}$ and $\tilde{X}^{\circ} \cong \tilde{X}$.

(2) $(F,E)^{\circ} \tilde{\subseteq} (F,E)$.

(3) (F, E) is open soft subset iff $(F,E)^{\circ} \cong (F,E)$.

(4) $((F,E)^{\circ})^{\circ} \cong (F,E)^{\circ}$.

(5) If $(F,E) \tilde{\subseteq} (G,E)$, then $(F,E)^{\circ} \tilde{\subseteq} (G,E)^{\circ}$.

(6) $(F,E)^{\circ} \tilde{\cap} (G,E)^{\circ} \cong ((F,E) \tilde{\cap} (G,E))^{\circ}$.

(7) $((F,E) \tilde{\cup} (G,E))^{\circ} \tilde{\subseteq} (F,E)^{\circ} \tilde{\cup} (G,E)^{\circ}$.

Theorem 2.13 Let (X,τ,E) be a soft topological space and (F, E) be a soft subset over X. Then

(1) $\left((F,E)'\right)^{\circ} \cong \left(\overline{(F,E)}\right)'$ and $\overline{(F,E)'} \cong \left((F,E)^{\circ}\right)'$.

(2) $(F,E)^{\circ} \cong \left(\overline{(F,E)'}\right)'$ and $\overline{(F,E)'} \cong \left((F,E)^{\circ}\right)'$.

Definition 2.14 Let (X,τ,E) be a soft topological space defined over the universe X. Then

(1) The soft exterior of a soft subset (F, E) (denoted by $(F,E)_{\circ}$) is defined by $(F,E)_{\circ} \cong \left((F,E)'\right)^{\circ}$. Clearly, $(F,E)_{\circ}$ is the largest open soft subset contained in $\left(F,E\right)'$.

(2) The soft boundary of a soft subset (F, E) (denoted by $\partial(F,E)$) is defined by $\partial(F,E) \cong \overline{(F,E)} \tilde{\cap} \overline{(F,E)'}$.

Theorem 2.15 Let (X,τ,E) be a soft topological space and (F, E) be a soft subset defined over X. Then:

(1) $\left(\partial(F,E)\right)' \cong (F,E)^{\circ} \tilde{\cup} \left((F,E)'\right)^{\circ} \cong (F,E)^{\circ} \tilde{\cup} (F,E)_{\circ}$.

(2) $\overline{(F,E)} \cong (F,E)^{\circ} \tilde{\cup} \partial(F,E)$.

(3) $\partial(F,E) \cong \overline{(F,E)} \tilde{\cap} \overline{(F,E)'} \cong \overline{(F,E)} - (F,E)^{\circ}$.

(4) $(F,E)^{\circ} \cong (F,E) \setminus \partial(F,E)$.

Definition 2.16 Let (X,τ,E) be a soft topological space, (F, E) and (G, E) be two soft subsets defined over X, then (F, E) and (G, E) are a soft separation of $\tilde{X}$ if

(1) $(F,E) \not\cong \tilde{\varnothing}$ and $(G,E) \not\cong \tilde{\varnothing}$,

(2) $(F,E) \tilde{\cup} (G,E) \cong \tilde{X}$,

(3) $\overline{(F,E)} \tilde{\cap} (G,E) \cong \tilde{\varnothing}$ and $(F,E) \tilde{\cap} \overline{(G,E)} \cong \tilde{\varnothing}$.

Definition 2.17 A soft topological space (X,τ,E) is said to be soft disconnectedness if there exists a soft separation of $\tilde{X}$, otherwise (X,τ,E) is said to be soft connected. A soft subset (F, E) of a soft topological space (X,τ,E) is soft connected, if it is soft connected as a soft subspace.

Theorem 2.18 Let (G, E) be a soft connected subset of a soft topological space (X,τ,E) and (F, E) be a soft subset of X such that $(G,E) \tilde{\subseteq} (F,E) \tilde{\subseteq} \overline{(G,E)}$. Then (F, E) is a soft connected subset.

Theorem 2.19 For any pair (F, E), (G, E) of soft subsets of the soft topological space (X,τ,E) and soft connected subspace (H, E) of (X,τ,E), we have $(H,E) \tilde{\subseteq} (F,E)$ or $(H,E) \tilde{\subseteq} (G,E)$.

Definition 2.20 A soft subset (F, E) of a soft topological space (X,τ,E) is said to soft separate X if $\left(F,E\right)'$ is soft disconnected subset.

Theorem 2.21 Let (X,τ,E) be a soft topological space and (F, E) be a soft subset defined over X. If $(F,E)^{\circ} \not\cong \tilde{\varnothing}$ and $\overline{(F,E)} \not\cong \tilde{X}$, then $(F,E)^{\circ}$ and $\overline{(F,E)}'$ form a soft separation of $\partial(F,E)'$, *i.e.*, $\partial(F,E)'$ soft separates X.

Proof. Since $(F,E)^{\circ} \not\cong \tilde{\varnothing}$ and $\overline{(F,E)}' \not\cong \tilde{X}$, then $(F,E)^{\circ}$ and $\overline{(F,E)}'$ are soft open subsets such that $(F,E)^{\circ} \tilde{\cap} \overline{(F,E)}' \cong \tilde{\varnothing}$. By using Theorem 2., we have $\partial(F,E)' \cong (F,E)^{\circ} \tilde{\cup} \overline{(F,E)}'$. Then $(F,E)^{\circ}$ and $\overline{(F,E)}'$ form a soft separation of $\partial(F,E)'$.

3. A Framework for the Description of Soft Topological Spatial Relation

Our model describes the soft topological spatial relations between two soft subsets (F, E) and (G, E) of the soft topological spaces (X,τ,E) based on the four soft intersections of the soft boundaries and soft interiors of (F, E) and (G, E), i.e.,

$$\partial(F,E) \tilde{\cap} \partial(G,E), \quad (F,E)^{\circ} \tilde{\cap} (G,E)^{\circ},$$

$$\partial(F,E) \tilde{\cap} (G,E)^{\circ}, \text{ and } (F,E)^{\circ} \tilde{\cap} \partial(G,E).$$

Definition 3.1 For any two soft subsets (F, E) and (G, E) of a soft topological space (X,τ,E), the soft topological spatial relation between (F, E) and (G, E) is described by a four-tuple of values of soft topological invariants associated to each of the four subsets $\partial(F,E) \tilde{\cap} \partial(G,E), (F,E)^{\circ} \tilde{\cap} (G,E)^{\circ}$, $\partial(F,E) \tilde{\cap} (G,E)^{\circ}$ and $(F,E)^{\circ} \tilde{\cap} \partial(G,E)$, respectively.

The soft topological spatial relation is denoted by a four-tuple $\left(-,-,-,-\right)$ where the entries will be the values of the soft topological invariants associated to the four soft subset-intersections.

In this paper, we restricted our attention to the binary soft topological spatial relations defined by assigning the appropriate values of null ($\tilde{\varnothing}$)/ non-null ($\neg\tilde{\varnothing}$) soft subsets to the entries in the four-tuple. The following table summarizes the 16 possibilities of these combinations.

Table 1. The 16 possibilities of the binary soft topological spatial relations (briefly, *sr*) based on the criteria of null/non-null soft intersection of the soft boundaries and soft interiors.

	$\partial \tilde{\cap} \partial$	$^\circ \tilde{\cap} ^\circ$	$\partial \tilde{\cap} ^\circ$	$^\circ \tilde{\cap} \partial$
sr_0	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_1	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_2	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_3	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_4	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_5	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_6	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_7	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$
sr_8	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_9	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{10}	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{11}	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{12}	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{13}	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{14}	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$
sr_{15}	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$

For any pair of soft subsets (*F, E*) and (*G, E*) defined over a common universe *X*, there is exactly one of the 16 topological spatial relations, i.e., the 16 topological spatial relations are mutually exclusive.

4. Soft Topological Relations between Spatial Soft Regions

Definition 4.1 Let (X, τ, E) be a connected soft topological space. A spatial soft region in *X* is a non-null soft subset (*F, E*) such that:

(1) $(F,E)^\circ$ is soft connected.

(2) $\overline{(F,E)^\circ} \cong (F,E)$.

The soft set defined over is a parameterized family of subsets of *X*, i.e., the soft set defined over is a collection of subsets of *X* such that each subset of this collection its elements involved in a common characteristic. In other words, soft region is a collection of traditional regions.

Figure 1 shows a soft region (*F, E*) defined over *X* where $E = \{e_1, e_2, e_3\}$ is the set of parameters. It is clear that *F*(*e*) is a region for each $e \in E$.

Proposition 4.2 The soft boundary of each spatial soft region is non-null.

Proof. Let (*F, E*) be a soft region in *X*. Since $(F,E)^\circ \not\cong \tilde{\varnothing}$, $\overline{(F,E)} \cong (F,E)$ and $(F,E) \tilde{\not\cong} \tilde{X}$, then $(F,E)^\circ$ and $(F,E)'$ form a soft separation of $\partial(F,E)'$. Suppose that $\partial(F,E) \cong \tilde{\varnothing}$, then $(F,E)^\circ$ and $(F,E)'$ form a soft separation of which is a contradiction since is connected soft topology.

Proposition 4.3 For any pair of spatial soft region, the spatial soft relations $sr_2, sr_4, sr_5, sr_8, sr_9, sr_{12}$ and sr_{13} can not occur.

Proof. Let (*F, E*) and (*G, E*) be two spatial soft regions such that $\partial(F,E) \tilde{\cap} (G,E)^\circ \not\cong \tilde{\varnothing}$. By using Theorem 2.15, $(F,E)^\circ \tilde{\cup} \partial(F,E) \cong \overline{(F,E)}$ and $(F,E)^\circ \tilde{\cup} \partial((F,E)^\circ) \cong \overline{(F,E)^\circ}$. Since $\overline{(F,E)} \cong (F,E) \cong \overline{(F,E)^\circ}$, then $(F,E)^\circ \tilde{\cup} \partial(F,E) \cong (F,E)^\circ \tilde{\cup} \partial((F,E)^\circ)$. Also, we have $(F,E)^\circ \tilde{\cap} \partial((F,E)^\circ) \cong \tilde{\varnothing}$ and $(F,E)^\circ \tilde{\cap} \partial(F,E) \cong \tilde{\varnothing}$. It follows that $\partial((F,E)^\circ) \cong \partial(F,E)$. Let $x \in \partial(F,E) \tilde{\cap} (G,E)^\circ$, then $x \in \partial((F,E)^\circ)$. Since $(G,E)^\circ$ is open soft subset such that $x \in (G,E)^\circ$, it follows that $(F,E)^\circ \tilde{\cap} (G,E)^\circ \not\cong \tilde{\varnothing}$. Hence the soft boundary-soft interior or soft interior-soft boundary intersection is non-null, then the soft interior-soft interior intersection between the same two soft regions is also non-null. i.e., the six soft topological soft relations $sr_4, sr_5, sr_8, sr_9, sr_{12}$ and sr_{13} can not occur.

Now, let (*F, E*) and (*G, E*) be two spatial soft regions such that $\partial(F,E) \tilde{\cap} \partial(G,E) \cong \tilde{\varnothing}$ and $(F,E)^\circ \tilde{\cap} (G,E)^\circ \tilde{\not\cong} \tilde{\varnothing}$, then $(F,E)^\circ \tilde{\cap} \partial(G,E) \tilde{\not\cong} \tilde{\varnothing}$. Suppose that $\partial(F,E) \tilde{\cap} (G,E)^\circ \cong \tilde{\varnothing}$. Since $(G,E) \cong (G,E)^\circ \tilde{\cup} \partial(G,E)$, then $\partial(F,E) \tilde{\cap} (G,E) \cong \tilde{\varnothing}$. Therefore $(G,E) \tilde{\subset} \partial(F,E)'$. Since $(F,E)^\circ$ and $(F,E)'$ form a separation of $\partial(F,E)'$. Since (*G, E*) is soft connected, then $(G,E) \tilde{\subset} (F,E)^\circ$ or $(G,E) \tilde{\subset} (F,E)'$. Since $(F,E)^\circ \tilde{\cap} (G,E)^\circ \not\cong \tilde{\varnothing}$, it follows that $(G,E) \tilde{\subset} (F,E)^\circ$, therefore $\partial(G,E) \subset (F,E)^\circ$. i.e., $\partial(G,E) \tilde{\cap} (F,E)^\circ \not\cong \tilde{\varnothing}$. Hence the soft boundary-soft boundary intersection is null and soft interior-soft interior intersection is non-null, then either the soft boundary-soft interior or the soft interior-soft boundary intersection is non-null.

The descriptive terms for the nine soft topological spatial relations between two soft regions are given in the following table.

Proposition 4.4 Let (*F, E*) and (*G, E*) be two spatial soft regions in X. If $(F,E)^\circ \tilde{\cap} (G,E)^\circ \not\cong \tilde{\varnothing}$ and $(F,E)^\circ \tilde{\cap} \partial(G,E) \cong \tilde{\varnothing}$, then $(F,E)^\circ \tilde{\subset} (G,E)^\circ$ and $(F,E) \tilde{\subset} (G,E)$.

Proof. Since $(F,E)^\circ$ is connected soft set, then by using Theorem 2.21 $(G,E)^\circ$ and $(G,E)'$ form a soft separation of

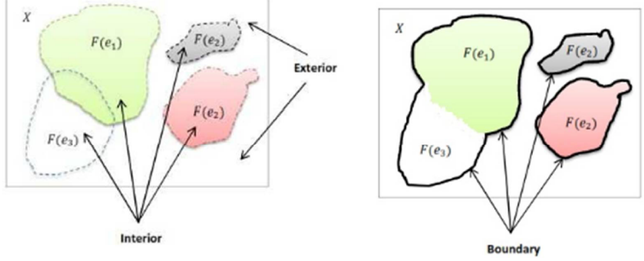

Figure 1. shows a soft region defined over where is the set of parameters. It is clear that is a region for each .

Table 2. The nine relations between the two spatial soft regions (*F, E*) and (*G, E*).

	$\partial\tilde{\cap}\partial$	$^\circ\tilde{\cap}^\circ$	$\partial\tilde{\cap}^\circ$	$^\circ\tilde{\cap}\partial$	
sr_0	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	(*F, E*) and (*G, E*) are disjoint
sr_1	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	(*F, E*) and (*G, E*) touch
sr_3	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\tilde{\varnothing}$	(*F, E*) equal (*G, E*)
sr_6	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	(*F, E*) is inside (*G, E*) or (*G, E*) contains (*F, E*)
sr_7	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	(*F, E*) is covered by (*G, E*) or (*G, E*) covers (*F, E*)
sr_{10}	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	(*F, E*) contains (*G, E*) or (*G, E*) is inside (*F, E*)
sr_{11}	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	(*F, E*) covers (*G, E*) or (*G, E*) is covered by (*F, E*)
sr_{14}	$\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	(*F, E*) and (*G, E*) or (*G, E*) overlap with disjoint soft boundaries
sr_{15}	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	$\neg\tilde{\varnothing}$	(*F, E*) and (*G, E*) or (*G, E*) overlap with intersecting soft boundaries

X. Moreover, $(F,E)^\circ\tilde{\cap}\partial(G,E)\tilde{=}\tilde{\varnothing}$ implies that $(F,E)^\circ\tilde{\subseteq}(G,E)^\circ\tilde{\cup}(G,E)'$. Also, Theorem 2.19. implies that either $(F,E)^\circ\tilde{\subseteq}(G,E)^\circ$ or $(F,E)^\circ\tilde{\subseteq}(G,E)'$. Since $(F,E)^\circ\tilde{\cap}(G,E)^\circ\tilde{\neq}\tilde{\varnothing}$, $(F,E)^\circ\tilde{\subseteq}(G,E)^\circ$, and it follows that $\overline{(F,E)^\circ}\tilde{\subseteq}\overline{(G,E)^\circ}$. Then $(F,E)\tilde{\subseteq}(G,E)$.

Corollary 4.5 Let (*F, E*) and (*G, E*) be two spatial soft regions. If the soft spatial relation between (*F, E*) and (*G, E*) is sr_3, then (*F, E*) = (*G, E*).

Proof. Since $(F,E)^\circ\tilde{\cap}(G,E)^\circ\tilde{\neq}\tilde{\varnothing}$ and $(F,E)^\circ\tilde{\cap}\partial(G,E)\tilde{=}\tilde{\varnothing}$, by using Proposition 4.4 we have $(F,E)\tilde{\subseteq}(G,E)$. Also, $\partial(F,E)\tilde{\cap}(G,E)^\circ\tilde{=}\tilde{\varnothing}$ and Proposition 4.4 implies $(G,E)\tilde{\subseteq}(F,E)$. Thus $(F,E)\tilde{=}(G,E)$.

Corollary 4.6 Let (*F, E*) and (*G, E*) be two spatial soft regions. If the spatial relation between (*F, E*) and (*G, E*) is sr_6, then $(F,E)\tilde{\subseteq}(G,E)^\circ$.

Proof. From Proposition 4.4, we have $(F,E)^\circ\tilde{\subseteq}(G,E)^\circ$ and $(F,E)\tilde{\subseteq}(G,E)$. Also, from Theorem 2.15 we have $(F,E)\tilde{=}(F,E)^\circ\tilde{\cup}\partial(F,E)$ and $(G,E)\tilde{=}(G,E)^\circ\tilde{\cup}\partial(G,E)$. Thus $\partial(F,E)\tilde{\subseteq}(G,E)$. Since $\partial(F,E)\tilde{\cap}\partial(G,E)\tilde{=}\tilde{\varnothing}$, it follows that $\partial(F,E)\tilde{\subseteq}(G,E)^\circ$. Thus $(F,E)\tilde{\subseteq}(G,E)^\circ$.

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