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Original Article

Coordinating a two-level supply chain with defective items, inspection errors and price-sensitive demand

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Abstract

This paper presents an integrated inventory model consisting of a vendor and a buyer with imperfect product, imperfect inspection and backorder. We intend to study both inventory and pricing decisions in supply chain system by assuming that the demand is sensitive to the buyer's selling price. In addition, the production cost is formulated by considering raw material cost, labor cost and advertisement cost. The objective of the model is to determine the delivery quantity, number of deliveries, buyer's selling price and number of backorders. An iterative procedure is developed to find the optimal solution. A numerical example is presented to illustrate the application of the model and a sensitivity analysis is performed to investigate the effect of the changes in key parameters to the model's solution.The result show that the buyer's selling price is sensitive to the changes of the defect rate and the probability of type I inspection error.

Keywords: supply chain, imperfect product, inspection error, price sensitive demand, backorders

1. Introduction

In recent years, the coordination of production, delivery and ordering along parties of supply chain system has received a great deal of attention from scholars. The parties in supply system may share their informations such as demand and inventory to improve the coordination and collaboration of supply chain system. Now, companies are realising that managing inventories efficiently accross the supply chain can significantly reduce the total cost of the system. The parties may jointly determine the production and inventory decisions by incorporating all parties interest for minimising total cost. The determination of lot sizing decisions in the supply chain system is usually known as joint economic lot size (JELS). For comprehensive review of JELS, the reader can refer to Glock (2012a).

A major stream of research in this area has focused on developing JELS considering defective items. Some researchers have investigated defective items in an integrated vendor-buyer model under various assumptions (Bazan *et al.,* 2014; Bera *et al.,* 2009; Jindal & Solanki, 2016). The previous JELS models have usually assumed that the inspection process conducted by the buyer is free of error. In real system, however, we often obtain a condition where an inspector may classify the defective items as non-defective or vice versa. Thus, some researchers relaxed the assumption of perfect inspection process and extended the previous models by introducing human errors (Jauhari *et al.,* 2016; Khan *et al.,* 2014).

Although imperfect production process and inspection errors have been studied in JELS models, several important aspects in production and delivery processes have not been considered. The drawbacks of previous models on production and inventory decisions are the assumption that, (1) the demand is constant, (2) vendor's on-hand inventory is always sufficient to fulfill the buyer's demand, and (3) the production cost is constant.

In reality, companies often use different types of pricing mechanism to influence the buying behavior of end customer. A pricing mechanism, such as a variable customer price can be adopted by the parties in the supply chain to induce customers to consume more product in the new price

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level. Generally, if the vendor can offer a lower price level, the customers will respond by increasing the consumption of product. However, the company must have a low operating cost to ensure that he can compete on price.Next, in practice, the companies may reduce inventory levels by using a planned stockouts. They assume that the loss in customer goodwill resulting from stockouts can be compensated by a reduction in inventory levels. Customers encountering shortages may want to wait to get the needed product, if their needs are not crucial. Further, the production cost should be treated as a variable and is determined by incorporating some considerations such as the volume of production, labor cost, raw material cost and the marketing cost.

2. Literature Review

The study of JELS models has been done by many scholars during last decade. Goyal (1976) was probably the first scholars who considered the joint optimization problem which consists of a single vendor and a single buyer. Later, Banerjee (1986) developed a vendor-buyer model for product with lot-for-lot shipment policy and assumed a finite production rate. Afterwards, Goyal (1988) developed Banerjee (1986) model by assuming equal-sized shipment rather than lot-for-lot. Hill (1997) relaxed the assumption of equal shipments and proposed unequal shipments policy in which successive shipments are increasing by a geometric growth factor. The model of Goyal (1976) are then developed to various conditions such as, stochastic demand (Ben-Daya & Hariga, 2004; Hsiao, 2008), adjusted production rate (Glock, 2010, 2011; Song *et al.,* 2013), and discounts (Arcelus *et al.,* 2007; Heydari, 2014; Viswanathan, 2009).

Recently, scholars have developed JELS models by incorporating imperfect production process. Porteus (1986) was the first to introduce the concept of imperfect production process into the inventory model. Researchers such as Rosenblatt and Lee (1987), Schwaller (1988), Ben-Daya and Hariga (2000) and other references have also studied imperfect production in inventory model.Then the topic of imperfect production is considered in vendor-buyer model. Jauhari *et al.* (2014) developed a JELS model with defective items and unequal shipment policy and investigated the impact of carbon emission cost on the model. Li and Chen (2015) proposed manufacturer-retailer model with imperfect production process and unequal shipment policy. Singh *et al.* (2014) and Kundu and Chakrabarti (2015) studied the impact of defective items on three-stage inventory models, while other reseachers, including Lin (2010), Lin (2013) and Dey and Giri (2014) analysed a JELS model with stochastic demand and imperfect production process.

Further, the assumption of perfect inspection which has been used by the above mentioned papers, was then relaxed into a situation in which the inspection is affected by human error. Raouf *et al.* (1983) was the first to study human error in inspection process. Hsu and Hsu (2012) developed an integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors. Konstantaras *et al.* (2012) investigated the learning effect in inspection process. Widianto *et al.* (2014) developed a single-vendor single-buyer model with inspection errors and different shipment policies. Priyan and Uthayakumar (In press) proposed a probabilistic defective vendor-buyer model with inspection errors and variable setup cost.

Production cost has also been discussed in inventory models. Several researchers have developed inventory models for a single-stage system or multi-stage system under constant or variable production rate. Bhunia and Maiti (1997), Mandal and Phaujder (1989) and Misra (1975) incorporated a constant production cost in their inventory models. Then, the researchers including Glock (2012b) and Khouja and Mehrez (1994) proposed inventory models with adjusted production rate or assuming that the production cost is variable.

From the above literature review, we can conclude that the JELS model has been discussed widely by researchers. Moreover, the defective items, inspection errors and price-sensitive demand have also been studied. However, defective items, inspection errors, price-sensitive demand, backorders and variable production cost have not yet been investigated in combination in the JELS models. Thus, here, we develop a JELS model considering the above problems and study the interdependencies of the above problems in our proposed model. For better understanding,a comparison of the model with some of the related papers in the inventory literature is provided in Table 1.

3. Notations and Assumptions

To develop the model, we use the following notations:

3.1 Notations

D	market demand
P	production rate
ρ	ratio of the demand market to the production rate ($\rho=D/P$)
Q	buyer's delivery quantity
b	buyer's backorder quantity
n	number of shipments
γ	defective rate
x	inspection rate
S	buyer's unit selling price
v	vendor's discounted selling price for defective product
$f(P)$	unit production cost
F	buyer's transportation cost per shipment
A	advertisement cost
w	vendor's unit selling price
m_u	mark-up
π	backorder cost per unit item
S_v	vendor's setup cost
S_b	buyer's order cost per order
C_{rw}	raw material cost
L	labor charges
C_i	buyer's inspection cost
C_{av}	vendor's cost of a post-sales defective item
C_{ab}	buyer's cost of a post-sales defective item
C_r	the cost of rejecting a non-defective item
H_v	vendor's holding cost per unit product per unit time
H_b	buyer's holding cost per unit product per unit time
B_1	number of items that are classified as defective in each delivery of Q units

Table 1. A comparison of the proposed model with some related published works

Paper	Demand	Production	Inspection error	Backorder	Variable production cost
Mondal *et al.* (2009)	Price sensitive demand	Imperfect	No	No	Yes
Rad *et al.* (2014)	Price sensitive demand	Imperfect	No	Yes	No
Jauhari (2016)	Stochastic	Imperfect	Yes	Yes	No
Hsu and Hsu (2012)	Deterministic	Imperfect	Yes	No	No
Chen and Kang (2010)	Price sensitive demand	Perfect	No	No	No
Glock (2012)	Stochastic	Perfect	No	Yes	Yes
Glock (2010)	Deterministic	Perfect	No	No	Yes
Proposed model	Price sensitive demand	Imperfect	Yes	Yes	Yes

B_2 number of items that are returned from the market in each delivery of Q units

e_1 probability of a type I error (classifying a non-defective item as defective)

e_2 probability of a type II error (classifying a defective item as non-defective)

T the cycle time

3.2 Assumptions

a. We consider an integrated inventory model for supply chain system consisting of single buyer and single vendor with a single product.

b. The demand rate in buyer side is a function of selling price and advertisement cost with $D = A^{\sigma}(\alpha - \beta S)$ where α, β, and $\sigma \geq 0$.

c. The vendor's unit production cost consists of raw material cost Cr, advertisement cost A, and labor cost L, which is $f(P) = C_{\mathrm{rw}} + A + \frac{L}{P^{\lambda 1}} + KP^{\lambda 2}$, where C_{rw}, L, K are non-negative real numbers to be chosen to provide the best fit for estimated unit cost function. λ_1 and λ_2 are also chosen to provide the feasible solution of the proposed model.This type of production cost was also used by Mondal *et al.* (2009).

d. Vendor's unit selling price is formulated by a mark-up over the vendor's unit production cost, $w = m_u f(P)$, $m_u > 1$, where m_u is the mark-up. A markup pricing option is used primarily because it is easy to calculate and requires little information. Information on demand and costs is not easily available. However, this information is necessary to generate accurate estimates of marginal costs and revenues. Moreover, the process of obtaining this additional information is expensive.

e. For each shipment, the buyer receives a lot which contains γ percent of defective items with the probability density function $f(\gamma)$.

f. The inspection process is imperfect. The probability of classifying a non-defective item as a defective is e_1 with the probability function $f(e_1)$. The probability of classifying a defective item as a non-defective is e_2 with the probability function $f(e_2)$.

g. The buyer will return all items classified as defective and those returned from the customer to the vendor at the end of the 100% screening process. The vendor will pay the buyer with full price to the buyer and sell the returned items at a discounted price to secondary market.

h. End costumer who buys the defective items will detect the quality problem and return them to the buyer and receive a good item for replace. Both the vendor and the buyer incur a post-sale failure cost for the items returned from the market.

i. Backorders are satisfied from the next incoming shipments before the items are screened, and that items which are used to satisfy backorders are defect-free.

4. Model Development

The cost incurred by the buyer consists of buying cost, ordering cost, transportation cost, inspection cost, post-sale failure cost, backorder cost, and holding cost. Figure. 1 shows buyer's inventory level in each ordering cycle. By definition, B_1, B_2, $t$are calculated as follows:

$$B_1 = Q(1-\gamma)e_1 + Q\gamma(1-e_2) \quad (1)$$

$$B_2 = Q\gamma e_2 \quad (2)$$

$$t = D/x \quad (3)$$

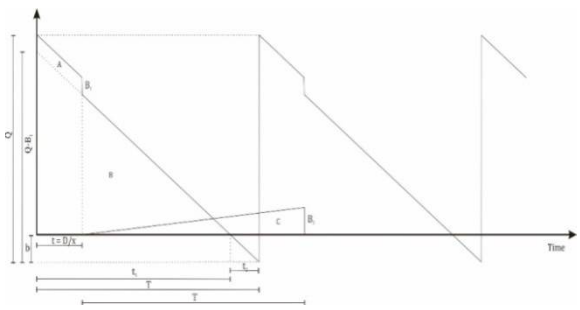

Figure 1. Buyer's Inventory Profile

Market demand is composed of 2 kinds of demand, the regular demand and the demand to replace the returned items. Let D' be the effective demand, then we have $D' = D + B_2/T$. By definition, the cycle length of each delivery of size Q is $T = (Q - B_1)/D'$. Subtituting $D' = D + B_2/T$ and solving the equation, we will have the following equation:

$$T = \frac{(Q - B_1 - B_2)}{A^{\sigma}(\alpha - \beta S)} = \frac{Q(1-\gamma)(1-e_1)}{A^{\sigma}(\alpha - \beta S)} \quad (4)$$

The inventory for defective items per delivery cycle can be formulated by considering the rectangle A in Figure 1, that is:

$$A = \frac{Q^2}{x}[(1-\gamma)e_1 + \gamma(1-e_2)] \quad (5)$$

The inventory for good items per delivery cycle can be formulated based on the triangle B in Figure 1, that is:

$$B = \frac{(Q.N-b)^2}{2A^{\sigma}(\alpha-\beta S)} \tag{6}$$

where $N = (1-(1-\gamma)e_1 - \gamma(1-e_2))$

By considering the triangle C in Figure 1, the total items returned from the market in one delivery cycle is given as follows:

$$C = \frac{Q^2}{2A^{\sigma}(\alpha-\beta S)}(1-\gamma)(1-e_1)\gamma e_2 \tag{7}$$

Therefore, the holding cost for buyer per production cycle can be formulated by adding equations (5), (6), and (7) which is

$$HC_b = \frac{nH_b}{2}\left(\frac{2Q^2((1-\gamma)e_1+\gamma(1-e_2))}{x} + \frac{\left(Q(1-(1-\gamma)e_1-\gamma(1-e_2))-b\right)^2}{A^{\sigma}(\alpha-\beta S)} + \frac{Q^2}{A^{\sigma}(\alpha-\beta S)}(1-\gamma)(1-e_1)\gamma e_2\right. \tag{8}$$

After adding the purchasing cost, ordering cost, transportation cost, inspection cost, post-sale failure cost, and backorder cost, the expected total cost for the buyer per production cycle is given by:

$$ETC_b = S_b + nC_iQ + nF + nwQ + n\pi\frac{b^2}{A^{\sigma}(\alpha-\beta S)} + nC_{\text{ab}}Q\gamma e_2 + \frac{nH_b}{2}\left(\frac{2Q^2((1-\gamma)e_1+\gamma(1-e_2))}{x} + \right.$$
$$\left.\frac{\left(Q(1-(1-\gamma)e_1-\gamma(1-e_2))-b\right)^2}{A^{\sigma}(\alpha-\beta S)} + \frac{Q^2}{A^{\sigma}(\alpha-\beta S)}(1-\gamma)(1-e_1)\gamma e_2\right) \tag{9}$$

The expected total cost for the vendor consists of production cost, setup cost, Type I and Type II inspection errors costs, and holding cost. During production process, the vendor produces a lot of *nQ* and delivers those to the buyer for each *T* period. Therefore, the vendor's inventory level can be determined by subtracting accumulative delivery from vendor's accumulated inventory. The holding cost for vendor per production cycle is given as follows:

$$HC_v = H_v\left(\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2A^{\sigma}(\alpha-\beta S)}\right) \tag{10}$$

The expected total cost for vendor per production cycle is given by

$$ETC_v = nQ\left(C_{\text{rw}} + A + \frac{L}{P^{\lambda 1}} + KP^{\lambda 2}\right) + S_v + nC_rQ(1-\gamma)e_1 + nC_{\text{av}}Q\gamma e_2 + H_v\left(\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2A^{\sigma}(\alpha-\beta S)}\right) \tag{11}$$

Therefore the expected joint total cost for supply chain system is

$$EJTC(n,Q,b,S) = nQ\left(C_{\text{rw}} + A + \frac{L}{P^{\lambda 1}} + KP^{\lambda 2}\right) + nwQ + S_v + S_b + nC_iQ + nF + nC_rQ(1-\gamma)e_1 + nC_{av}Q\gamma e_2 +$$
$$nC_{ab}Q\gamma e_2 + n\pi\frac{b^2}{A^{\sigma}(\alpha-\beta S)} + H_v\left(\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2A^{\sigma}(\alpha-\beta S)}\right) + \frac{nH_b}{2}\left(\frac{2Q^2((1-\gamma)e_1+\gamma(1-e_2))}{x} + \frac{\left(Q(1-(1-\gamma)e_1-\gamma(1-e_2))-b\right)^2}{A^{\sigma}(\alpha-\beta S)} + \right.$$
$$\left.\frac{Q^2}{A^{\sigma}(\alpha-\beta S)}(1-\gamma)(1-e_1)\gamma e_2\right) \tag{12}$$

The revenue of vendor per production cycle, which is obtained from selling *nQ* products to buyer with price *w* and selling defective product to secondary market with a discounted price *v*, is given by equation (13). While the revenue of buyer per production cycle, which is obtained from selling *nQ* products to end customers with price *S*, is provided by equation (14).

$$TR_v = nwQ + nvQ(1-\gamma)e_1 \tag{13}$$

$$TR_b = nSQ(1-\gamma)(1-e_1) \tag{14}$$

Thus, the expected joint total profit supply chain system can be formulated as follows:

$$JTP(n,Q,b,S) = nvQ(1-\gamma)e_1 + nSQ(1-\gamma)(1-e_1)(nQ\left(C_{\text{rw}} + A + \frac{L}{P^{\lambda 1}} + KP^{\lambda 2}\right) + S_v + S_b + nC_iQ + nF$$
$$+nC_rQ(1-\gamma)e_1 + nC_{av}Q\gamma e_2 + nC_{ab}Q\gamma e_2 + n\pi\frac{b^2}{A^{\sigma}(\alpha-\beta S)}H_v\left(\frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + \frac{n(n-1)Q^2(1-\gamma)(1-e_1)}{2A^{\sigma}(\alpha-\beta S)}\right)$$

$$+\frac{nH_b}{2}\left(\begin{array}{c}\frac{2Q^2((1-\gamma)e_1+\gamma(1-e_2))}{x}+\frac{(Q(1-(1-\gamma)e_1-\gamma(1-e_2))-b)^2}{A^\sigma(\alpha-\beta S)}\\ +\frac{Q^2}{A^\sigma(\alpha-\beta S)}(1-\gamma)(1-e_1)\gamma e_2\end{array}\right)) \quad (15)$$

Since the production cycle is $T_c = n\frac{Q(1-\gamma)(1-e_1)}{A^\sigma(\alpha-\beta S)}$, one has: $E[T_c] = n\frac{Q(1-E[\gamma])(1-E[e_1])}{A^\sigma(\alpha-\beta S)}$ (16)

By using renewal-reward theorem, the expected joint total profit of the vendor and the buyer is

$$EJTP(n,Q,b,S) = \frac{E[EJTP(n,Q,S,b)]}{E[T_c]} \quad (17)$$

$$EJTP(n,Q,b,S) = \frac{A^\sigma(\alpha-\beta S)(S(1-E[\gamma])(1-E[e_1])+v(1-E[\gamma])E[e_1])}{(1-E[\gamma])(1-E[e_1])} - \frac{A^\sigma(\alpha-\beta S)(S_b+S_v+nF)}{nQ(1-E[\gamma])(1-E[e_1])}$$

$$-\frac{A^\sigma(\alpha-\beta S)((C_{\text{rw}}+A+\frac{L}{P^{\lambda 1}}+KP^{\lambda 2})+C_i+C_r(1-E[\gamma])E[e_1])+C_{ab}\gamma E[e_2]+C_{av}\gamma E[e_2]}{(1-E[\gamma])(1-E[e_1])} - \frac{\pi b^2}{Q(1-E[\gamma])(1-[e_1])}$$

$$-H_b\left(\frac{QA^\sigma(\alpha-\beta S)E[M]}{x(1-E[\gamma])(1-E[e_1])}+\frac{(QE[N]-b)^2}{2Q(1-E[\gamma])(1-E[e_1])}+\frac{QE[\gamma]E[e_2]}{2}\right) - H_vQ(\frac{\rho}{(1-E[\gamma])(1-E[e_1])}-\frac{n\rho}{2(1-E[\gamma])(1-E[e_1])}+\frac{n-1}{2}) \quad (18)$$

With

$$M = (1-\gamma)e_1 + \gamma(1-e_2) \quad (19)$$

$$N = 1-(1-\gamma)e_1 - \gamma(1-e_2) \quad (20)$$

and

$$E[M] = (1-E[\gamma])E[e_1] + E[\gamma](1-E[e_2]) \quad (21)$$

$$E[N] = 1-(1-E[\gamma])E[e_1] - E[\gamma](1-E[e_2]) \quad (22)$$

5. Solution Methodology

The maximum value of $EJTP(n,Q,b,S)$ occurs at the point (Q,b,S) which satisfies $\frac{\partial EJTP(n,Q,b,S)}{\partial Q}=0$, $\frac{\partial EJTP(n,Q,b,S)}{\partial b}=0$ and $\frac{\partial EJTP(n,Q,b,S)}{\partial S}=0$ simultaneuosly. To find the solution of the above problem, we investigate the first partial derivative of $EJTP(n,Q,b,S)$ with respect to *Q*, *b*, and *S*, which are given by equations (23), (24) and (25).

$$\frac{\partial EJTP(n,Q,S,b)}{\partial Q} = \frac{A^\sigma(\alpha-\beta S)(S_b+S_v+nF)}{nQ^2(1-E[\gamma])(1-E[e_1])} + \frac{\pi b^2}{Q^2(1-E[\gamma])(1-E[e_1])}$$
$$-\frac{H_bA^\sigma(\alpha-\beta S)M}{x(1-E[\gamma])(1-E[e_1])} - \frac{H_bN^2}{2(1-E[\gamma])(1-E[e_1])}$$
$$+\frac{H_bb^2}{2Q^2(1-E[\gamma])(1-E[e_1])} - \frac{H_bE[\gamma]E[e_2]}{2}$$
$$-\frac{H_v\rho}{(1-E[\gamma])(1-E[e_1])} + \frac{H_vn\rho}{2(1-E[\gamma])(1-E[e_1])} - \frac{H_v(n-1)}{2} \quad (23)$$

$$\frac{\partial EJTP(n,Q,S,b)}{\partial S} = A\alpha - 2A\beta S - \frac{A\beta v(1-E[\gamma])E[e_1]}{(1-E[\gamma])(1-E[e_1])} + \frac{A\beta(S_b+S_v+nF)}{nQ(1-E[\gamma])(1-E[e_1])}$$
$$+\frac{A\beta\left((C_{\text{rw}}+A+\frac{L}{P^{\lambda 1}}+KP^{\lambda 2})+C_i+C_r(1-E[\gamma])E[e_1]+C_{\text{ab}}E[\gamma]E[e_1]+C_{\text{av}}E[\gamma]E[e_1]\right)}{(1-E[\gamma])(1-E[e_1])} + \frac{H_bQAE[M]\beta}{(1-E[\gamma])(1-E[e_1])} \quad (24)$$

$$\frac{\partial EJTP(n,Q,S,b)}{\partial b} = -\frac{2\pi b}{Q(1-E[\gamma])(1-E[e_1])} + \frac{H_bE[N]}{(1-E[\gamma])(1-E[e_1])} - \frac{H_bb}{Q(1-E[\gamma])(1-E[e_1])} \quad (25)$$

By setting equation (23), equation (24), and equation (25) equals to zero, rearranging and simplifying, the optimal shipment quantity, selling price and backorder are given by equation (26), equation (28) and equation (29), respectively.

$$Q^* = \sqrt{\frac{2A^{\sigma}(\alpha-\beta S)(S_b+S_v+nF)+2n\pi b^2+nH_b b^2}{2n(1-E[\gamma])(1-E[e_1])Z}} \quad (26)$$

with

$$Z = \left(\frac{H_b D E[M]}{x(1-E[\gamma])(1-E[e_1])}+\frac{H_b E[N]^2}{2(1-E[\gamma])(1-E[e_1])}+\frac{H_b E[\gamma]E[e_1]}{2}+\frac{H_v\rho}{(1-E[\gamma])(1-E[e_1])}-\frac{H_v n\rho}{2(1-E[\gamma])(1-E[e_1])}+\frac{H_v(n-1)}{2}\right) \quad (27)$$

$$S^* = \frac{1}{2}\Big(\frac{\alpha}{\beta}-\frac{v(1-E[\gamma])E[e_1]}{(1-E[\gamma])(1-E[e_1])}+\frac{S_b+S_v+nF}{nQ(1-E[\gamma])(1-E[e_1])}$$
$$+\frac{\left(C_{rw}+A+\frac{L}{P^{\lambda 1}}+KP^{\lambda 2}\right)+C_i+C_r(1-E[\gamma])E[e_1]+C_{ab}E[\gamma]E[e_1]+C_{av}E[\gamma]E[e_1]}{(1-E[\gamma])(1-E[e_1])}+\frac{H_b Q E[M]}{(1-E[\gamma])(1-E[e_1])}\Big) \quad (28)$$

$$b^* = \frac{H_b Q E[N]}{2\pi+H_b} \quad (29)$$

Proposition 1. For fixed n, the Hessian Matrix for *EJTP(n,Q,S,b)*is negative definite at point (Q^*,S^*,b^*).

Proof. See Appendix**.**

Considering the above equations, it is obvious that the key parameters (Q, S and b) are not independent of each other. For example, to find Q we need to calculate b, which in turn is a prerequisite for determining Q. Therefore, to obtain the solution of the proposed model we adopt the basic idea of the algorithm proposed by Ben-Daya and Hariga (2004). The solution procedure of the proposed model is given as follows:

1. Set$n$ = 1and $EJTP(n\text{-}1,Q^{n-1}, S^{n-}1,bn^{-1})= -\infty$
2. Compute the value of Qusing the following equation

$$Q^* = \sqrt{\frac{2A^{\sigma}\alpha(S_b+S_v+nF)}{2n(1-E[\gamma])(1-E[e_1])Z}} \quad (30)$$

3. Compute the value of Susing equation (28)
4. Compute the value of b using equation (29)
5. Find Q from equation (26) using the previous values of S and b.
6. Repeat steps 3-5 until no change occurs in the values of Q, S, b.
7. Set $Q_n = Q$, $S_n = S$, $b_n = b$ and compute $EJTP(n,Q_n,S_n,b_n)$
8. If $EJTP(n,Q_n, S_n,b_n) \geq EJTP(n\text{-}1,Q_{n-1}, S_{n-1},b_{n-1})$, repeat steps 1-7 with $n = n + 1$. Otherwise go to step 9.
9. Compute$EJTP(n^*,Q_n^*,S_n^*,b_n^*) = EJTP(n\text{-}1,Q_{n-1}^*,S_{n-1}^*,b_{n-1}^*)$and the optimal solution is n^*,Q^*,b^*,S^*.

6. Numerical Example

In this section, we provide the values of the parameters involved in the proposed model to demonstrate the application of the model. A brief sensitivity analysis is performed by varying the values of the model's key parameters to show how the proposed model behaves and to draw some insight. The values of the parameters are listed below.

σ	:	0.01	m_u	:	1.2	C_i	:	\$5/unit/year
α	:	30	F	:	\$100/shipment	C_{ab}	:	\$200/unit/year
β	:	0.01	A	:	\$100	C_{av}	:	\$300/unit/year
ρ	:	0.8	S_b	:	\$50/order	C_r	:	\$100/unit/year
K	:	0.01	S_v	:	\$100/setup	π	:	\$15/unit
λ_1	:	0.2	x	:	600 units/year	H_v	:	\$10/unit/year
λ_2	:	0.01	C_{rw}	:	\$100/unit	H_b	:	\$20/unit/year
v	:	\$500/unit	L	:	\$1,000			

The probability density function of uniform distribution of the defect rate and inspection errors are provided as follows:

$$f(\gamma)=\begin{Bmatrix} \frac{1}{\mu}, 0 \le \gamma \le \mu \\ 0, \text{otherwise} \end{Bmatrix}$$

$$f(e_1)=\begin{Bmatrix} \frac{1}{\sigma}, 0 \le e_1 \le \sigma \\ 0, \text{otherwise} \end{Bmatrix}$$

$$f(e_2)=\begin{Bmatrix} \frac{1}{\tau}, 0 \le e_2 \le \tau \\ 0, \text{otherwise} \end{Bmatrix}$$

$$E[\gamma] = \int_0^{\mu} \gamma f(\gamma) d\gamma = \int_0^{\mu} \frac{\gamma}{\mu} d\gamma = \frac{\mu}{2}$$

$$E[e_1] = \int_0^{\sigma} e_1 f(e_1) de_1 = \int_0^{\sigma} \frac{e_1}{\sigma} de_1 = \frac{\sigma}{2}$$

$$E[e_2] = \int_0^{\tau} e_2 f(e_2) de_2 = \int_0^{\tau} \frac{e_2}{\sigma} de_2 = \frac{\tau}{2}$$

$\mu = \sigma = \tau = 0{,}04$, then we have:

$E[\gamma] = 0.02$

$E[e_1] = 0.02$

$E[e_2] = 0.02$

The proposed mathematical model developed in previous section is solved for the input parameters given above. The optimal values of number of deliveries, delivery quantity, buyer's selling price, backorder and the expected profit for supply chain system are n^*=4, Q^*= 13.04 units, S^*= \$1,911.3, b^*= 5.01 units and *EJTP* = \$12,181, respectively. The results of sensitivity analysis are summarized in the following subsections.

6.1 Sensitivity analysis for the defective rate (γ)

Table 2 shows the solutions for different value of defective rate, in a range from 0.01 to 0.1. If the defective rate increases, the expected joint total profit decreases. For example when the defective rate is increased from 0.01 to 0.02, the expected joint total profit decreases from \$12,283 to \$12,181. It is found that the average decrease in the expected joint total profit is 0.92%. If the defective rate increases, the costs related to defective products incurred by the buyer and the vendor, i.e post sale failure of defective product, cost of rejecting non-defective items, will also increase.One can also see that if the defective rate increases, the order quantity increases as well. The changes in defect rate give significant impacts to the buyer's selling price. If the defective rate is increased from 0.09 to 0.1, it is found that the buyer's selling price will increase from \$1,944.9 to \$1,950.1 (0.27 %).

6.2 Sensitivity analysis for the probability of type 1 inspection error (e_1)

Table 3 presents the influence of the probability of type I inspection error in the proposed model. The results are similar to the probability of defective item does on the model's solutions. It shows that when the probability of type I inspection error is gradually increased, the order quantity increases while the backorder and expected joint total profit decrease. We observe that when the probability of type I inspection error is increased from 0.02 to 0.03, the expected joint total profit decreases 0.42% while the buyer's selling price increases 0.12%. The average decrease in the expected joint total profit is 0.455% which is lower than the one in Table 2. Also, the average increase in selling price is 0.13% which is lower than that of in Table 2. This result indicates that the proposed model is more sensitive to the changes in defect rate than the changes in probability of type I inspection error.

Table 2. The impact of changes in γ on the behavior of proposed model

γ	*N*	*Q*	*b*	*S*	*D*	*EJTP*
0.01	4	12.92	5.01	1,906.90	11.45	12,283.00
0.02	4	13.04	5.01	1,911.30	11.40	12,181.00
0.03	4	13.17	5.01	1,915.80	11.35	12,078.00
0.04	4	13.30	5.01	1,920.40	11.31	11,973.00
0.05	4	13.43	5.01	1,925.10	11.26	11,866.00
0.06	5	12.82	4.73	1,929.80	11.21	11,757.00
0.07	5	12.97	4.74	1,934.70	11.15	11,646.00
0.08	5	13.13	4.74	1,939.70	11.10	11,534.00
0.09	5	13.30	4.75	1,944.90	11.05	11,419.00
0.10	6	12.94	4.57	1,950.10	10.99	11,302.00

Table 3. The impact of changes in e_1 on the behavior of proposed model

e_1	*n*	*Q*	*B*	*S*	*D*	*EJTP*
0.01	4	12.91	5.01	1,909.10	11.42	12,232.00
0.02	4	13.04	5.01	1,911.30	11.40	12,181.00
0.03	4	13.18	5.01	1,913.60	11.38	12,130.00
0.04	4	13.31	5.01	1,915.90	11.35	12,077.00
0.05	4	13.45	5.02	1,918.20	11.33	12,023.00
0.06	5	12.85	4.74	1,920.70	11.30	11,969.00
0.07	5	13.02	4.75	1,923.10	11.28	11,913.00
0.08	5	13.19	4.76	1,925.60	11.25	11,857.00
0.09	5	13.36	4.78	1,928.20	11.22	11,799.00
0.1	6	13.01	4.60	1,930.80	11.20	11,740.00

6.3 Sensitivity analysis for the probability of type II inspection error (e_2)

The numerical results presented in Table 4 show the effect of the probability of type II inspection error on the proposed model. It seems that the results are not similar to those in previous tables. The delivery quantity, buyer's selling price, demand and the backorders are relatively insensitive to the changes in the probability of the type II inspection error. However, the expected joint total profit is slightly influenced by the changes in the probability of type II inspection error.

Table 4. The impact of changes in e_2 on the behavior of proposed model

e_2	n	Q	B	S	D	$EJTP$
0.01	4	13.04	5.01	1,911.30	11.4	12,182.00
0.02	4	13.04	5.01	1,911.30	11.4	12,181.00
0.03	4	13.04	5.01	1,911.30	11.4	12,180.00
0.04	4	13.04	5.01	1,911.30	11.4	12,179.00
0.05	4	13.04	5.01	1,911.30	11.4	12,178.00
0.06	4	13.04	5.01	1,911.30	11.4	12,177.00
0.07	4	13.04	5.01	1,911.30	11.4	12,175.00
0.08	4	13.04	5.01	1,911.30	11.4	12,174.00
0.09	4	13.03	5.01	1,911.30	11.4	12,173.00
0.1	4	13.03	5.01	1,911.30	11.4	12,172.00

It is observed that if the probability of type II inspection error is increased from 0.04 to 0.05, the expected joint total profit decreases from $12,179 to 12,178. It is found that the decrease in the expected joint total profit affected by the changes in the probability of type II inspection error is much lower than the decrease that affected by the probability of type I inspection error.

6.4 Sensitivity analysis for the vendor's holding cost (H_v)

As can be seen from Table 5,the increase in the vendor's holding gives significant impact to the expected joint total profit. If the vendor's holding cost increases from $14 to $15, the expected joint total profit decreases from $12,151 to $12,144. In addition, the delivery quantity is slightly reduced as there is an increase in vendor's holding cost. When the vendor's holding cost is relatively higher, it is beneficial for the vendor to decrease the production batch which may lead to reducing total holding cost. Further, the backorders, buyer's selling price and demand are relatively insensitive to the changes in vendor's holding cost.

6.5 Sensitivity analysis for the buyer's holding cost (H_b)

Table 6 presents the impact of the changes in buyer's holding cost on the model's behavior. The results are similar to the vendor's holding cost does on model's solutions. If the buyer's holding cost is increased gradually, the expected joint total profit decreases due to the increase in buyer's holding cost. For example, if the buyer's holding cost is increased from $20 to $21, the expected joint total profit decreases from $12,181 to $12,179. Facing a higher holding cost, the system tends to decrease the delivery quantity. This is can be understood since reducing the size of delivery will prevent the system from having higher holding cost. In addition, the key parameters such as the buyer's selling price, the demand, are insensitive to the changes in buyer's holding cost.

Table 5. The impact of changes in H_v on the behavior of proposed model

H_v	n	Q	b	S	D	$EJTP$
10	4	13.04	5.01	1,911.30	11.40	12,181.00
11	4	12.69	4.88	1,911.50	11.40	12,174.00
12	4	12.36	4.75	1,911.60	11.40	12,166.00
13	4	12.05	4.63	1,911.80	11.40	12,159.00
14	4	11.76	4.52	1,911.90	11.39	12,151.00
15	4	11.49	4.42	1,912.10	11.39	12,144.00
16	4	11.24	4.32	1,912.20	11.39	12,138.00
17	4	11.00	4.23	1,912.40	11.39	12,131.00
18	3	11.80	4.54	1,912.50	11.39	12,124.00
19	3	11.58	4.45	1,912.60	11.39	12,118.00

Table 6. The impact of changes in H_b on the behavior of proposed model

H_b	n	Q	b	S	D	$EJTP$
20	4	13.04	5.01	1,911.30	11.40	12,181.00
21	4	12.97	5.13	1,911.30	11.40	12,179.00
22	4	12.91	5.25	1,911.40	11.40	12,177.00
23	4	12.84	5.35	1,911.40	11.40	12,175.00
24	4	12.78	5.46	1,911.40	11.40	12,174.00
25	4	12.71	5.55	1,911.50	11.40	12,172.00
26	4	12.65	5.64	1,911.50	11.40	12,170.00
27	4	12.58	5.73	1,911.50	11.40	12,169.00
28	4	12.52	5.81	1,911.60	11.40	12,167.00
29	4	12.46	5.88	1,911.60	11.40	12,166.00

7. Conclusions

In this paper, we propose a joint economic lot size model of a single-vendor single-buyer for a single product by considering imperfect product, inspection errors, backorders and price sensitive demand. We assume that the items received by the buyer contains some defective items which could be returned to the vendor and sold to the secondary market with discounted price. After receiving a lot from the vendor, the buyer will inspect all items in a lot to categorize the quality of the items. However, the inspection process is imperfect, thus the inspector may incorrectly classify the items. We consider two types of inspection errors, that is type I for the condition that if the inspector incorrectly classify non-defective item as defective and type II for the condition that if the inspector incorrectly classify a defective item as non-defective. In addition, the market demand is sensitive to the buyer's selling price. We seek to maximize the expected joint total profit by simultaneously determining the delivery quantity, number of backorder, number of deliveries and the buyer's selling price for good products. We suggest an iterative procedure to find the solution of the proposed model. Numerical example and sensitivity analysis are performed to know the application of the model and to investigate the impact of the changes of the defective rate, the probability of type I inspection error, the probability of type II inspection error, buyer's holding cost and vendor's holding cost on the model's solution. The results obtained from numerical examples show that the increases in all key parameters studied in this paper will lead to the decrease in the expected joint total profit. In addition, the impacts of the changes in defect rate to the model are similar to the probability type I inspection error on the proposed model. It is also observed that the proposed model is more sensitive to the changes in defect rate than the changes in probability of type I inspection error.

There are several future studies that can be done to extend the proposed model. In our proposed model we use mark-up pricing option for formulating vendor's selling price. However, the application of mark-up pricing option in the model has disadvantages, such as providing incentive for inefficiency, ignoring the role of customers and competitors, ignoring opportunity cost and using historical rather than replacement value. Thus, the model can be extended by considering other pricing strategies, such as competitor-based pricing and customer-based pricing. Another study can look into a more complex supply chain model such as single-vendor multi-buyers, multi-vendors multi-buyers, multi-suppliers single-vendor multi-buyers. Further, future studies can also be done by incorporating learning and forgetting effects on inventory model as in Glock and Jaber (2013).

References

Arcelus, F. J., Pakkala, T. P. M., & Srinivasan, G. (2007). Ordering and pricing policies for random discount offerings and permissible shortages. *International Journal of Operational Research, 2*, 400-413.

Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. *Decision Sciences, 17*, 292-311.

Bazan, E., Jaber, M. Y., Zanoni, S., & Zavanella, L. E. (2014). Vendor managed inventory (VMI) with consignment stock (CS) agreement for a two-level supply chain with an imperfect production process with/ without restoration interruptions. *International Journal of Production Economics, 157*, 289–301.

Ben-Daya, M., & Hariga, M. (2000). Economic lot scheduling problem with imperfect production processes. *Journal of Operational Research Society, 51*, 878-881.

Ben-Daya, M., & Hariga, M. (2004). Integrated single vendor single buyer model with stochastic demand and variable lead time. *International Journal of Production Economics, 92*, 75-80.

Bera, U. K., Mahapatra, N. K., & Maiti, M. (2009). An imperfect fuzzy production-inventory model over time horizon under effect of learning. *International Journal of Mathematics in Operational Research, 1*, 351–371.

Bhunia, A. K., & Maiti, M. (1997). An inventory model for decaying items with selling price, frequency of advertisement and linearly time-dependent demand withshortages. *IAPQR Transactions, 22*, 41-49.

Chen, L. H., & Kang, F. S. (2010). Integrated inventory models considering the two-level credit policy and a price-negotiation scheme. *European Journal of Operational Research, 205*, 47-58.

Dey, O., & Giri, B. C. (2014). Optimal vendor investment for reducing defect rate in a vendor-buyer integrated system with imperfect production process. *International Journal of Production Economics, 155*, 222–228.

Glock, C. H. (2010). Batch sizing with controllable production rates. *International Journal of Production Research, 48*, 5925-5942.

Glock, C. H. (2011). Batch sizing with controllable production rates in a multi-stage production system. *International Journal of Production Research, 49*, 6017-6039.

Glock, C. H. (2012a). The joint economic lot size problem: A review. *International Journal of Production Economics, 135*, 2, 671–686.

Glock, C. H. (2012b). Lead time reduction strategies in a single-vendor-single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand. *International Journal of Production Economics, 136*, 1, 37–44.

Glock, C. H., & Jaber, M. Y. (2013). A multi-stage production-inventory model with learning and forgetting effects, rework and scrap. *Computers and Industrial Engineering, 64*.708-720.

Goyal, S. K. (1976). An integrated inventory model for a single supplier – single customer problem. *International Journal of Production Research, 15*,107-111.

Goyal, S. K. (1988). A joint economic-lot-size model for purchaser and vendor : A comment, decision. *Sciences, 19*, 236-241.

Heydari, J. (2014). Supply chain coordination using time-based temporary price discounts. *Computers and Industrial Engineering, 75*, 96-101.

Hill, R. M. (1997). The single-vendor single-buyer integrated production-inventory model with a generalized policy. *European Journal of Operational Research, 97*, 493-499.

Hsiao, Y. C. (2008). A note on integrated single vendor single buyer model with stochastic demand and variable lead time. *International Journal of of Production Economics, 114*, 294-297.

Hsu, J. T, & Hsu, L. F. (2012). A note: Optimal inventory model for items with imperfect quality and shortage backordering. *International Journal of Industrial Engineering Computations, 3*, 939-948.

Jauhari, W. A. & Pujawan, I. N. (2014). Joint economic lot size JELS model for single-vendor single-buyer with variable production rate and partial backorder. *International Journal of Operational Research, 20*, 91-108.

Jauhari, W. A., Pamuji, A. S., & Rosyidi, C. N. (2014). Cooperative inventory model for vendor-buyer system with unequal-sized shipment, defective items and carbon emission cost. *International Journal of Logistics Systems and Management, 19*, 163-186.

Jauhari, W. A, Sofiana, A. Kurdhi, N. A., & Laksono, P. W. (2016). An integrated inventory model forsupplier-manufacturer-retailer systemwith imperfect quality and inspection errors. *International Journal of Logistics and Systems Management, 24*, 383-407.

Jindal, P., & Solanki, A. (2016). Integrated supply chain inventory model with quality improvement involving controllable lead time and backorder price discount. *International Journal of Industrial Engineering Computations, 7*, 463-480.

Khan, M., Jaber, M. Y., & Ahmad, A. R. (2014). An integrated supply chain model with errors in quality inspection and learning in production. *Omega, 42*, 16–24.

Khouja, M., & Mehrez, A. (1994). Economic production lot size model with variable production rate and imperfect quality. *Journal of the Operational Research Society, 45*, 1405-1417.

Kundu, S, & Chakrabarti. (2015). An integrated multi-stage supply chain inventory model with imperfect productionprocess. *International Journal of Industrial Engineering Computations, 6*, 565-580.

Konstantaras, I., Skouri, K., & Jaber, M. Y. (2012). Inventory models for imperfect quality items with shortages and learning in inspection. *Applied Mathematical Modelling, 36*, 5334-5343.

Li, Y., & Chen, Z. (2015). Optimal integrated production-inventory system for imperfect quality items considering unequal shipment policy. *International Journal of Industrial and Systems Engineering. 19*, 206-238

Lin, Y. J. (2010). A stochastic periodic review integrated inventory model involving defective items. Back-order price discount, and variable lead time. *A Quarterly Journal of Operations Research, 8*, 281-297.

Lin, H. J. (2013). An integrated supply chain inventory model with imperfect-quality items, controllable lead time and distribution-free demand. *Yugoslav Journal of Operations Research, 23*, 87–109.

Mandal, B. N., & Phaujder, S. (1989). An inventory model for deteriorating items and stock-dependent consumption rate. *Journal of Operational Research Society, 40*, 483-488.

Mondal, B., Bhunia, A. K., & Maiti, M. (2009). Inventory models for defective items incorporating marketing decisions with variable production cost. *Applied Mathematical Modelling, 33*, 2845-2852.

Misra, R. B. (1975). Optimum production lot size model for a system with deteriorating inventory. *International Journal of Production Research, 13*, 495-505.

Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction, *Operations Research, 34*, 137–144.

Priyan, S., & Uthayakumar, R. (In press). *International Transactions in Operational Research.*

Rad, M. A.,Khoshalhan, F., & Glock, C. H. (2014). Optimizing inventory and sales decisions in a two-stage supply chainwith imperfect production and backorders. *Computers and Industrial Engineering, 74*, 219-227.

Raouf, A., Jain, J. K., & Sathe, P. T. (1983). A cost-minimization model for multicharacteristic component inspection. *IIE Transsactions, 15*,187-194.

Rosenblatt, M. J., & Lee, H. L. (1986). Economic production cycles with imperfect production process. *IIE Transactions, 18*,48–55.

Schwaller, R. L. (1988). EOQ under inspection cost. *Production and Inventory Management, 29*, 22–24.

Singh, N., Vaish, B., & Singh, S. R. (2014). Coordination policy for a three echelon supply chain considering imperfect quality items. *International Journal of Industrial Engineering Computations, 5*, 589–602.

Song, H. M.,Yang, H., & Bensoussan, A. (2013). Optimizing production and inventory decisions in a supply chain with lot size, production rate and lead time interactions. *Applied Mathematics and Computation, 224*, 150-165.

Viswanathan, S. (2009). Coordination in vendor-buyer inventory systems: on price discounts, Stackelberg game and joint optimisation. *International Journal of Operational Research, 6*, 110-124.

Widianto, I. P.,Jauhari, W. A., & Rosyidi, C. N. (2014). Cooperative vendor-buyer inventory model with imperfect quality and inspection errors. *Lecture Notes in Engineering and Computer Science*, 1-5.

Yang, P. C., Chung, S. L., Wee, H. M., Zahara, E., & Peng, C. Y. (2013). Collaboration for a closed-loop deteriorating inventory supply chain with multi-retailer and price-sensitive demand. *International Journal of Production Economics, 143*, 557-566

Appendix

Proof of Proposition 1. For a given value of *n,* we first obtain the Hessian Matrix **H** as follows:

$$\mathbf{H}=\begin{bmatrix}\frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q^2} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q\partial S} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q\partial b}\\ \frac{\partial^2 EJTP(n,Q,S,b)}{\partial S\partial Q} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial S^2} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial S\partial b}\\ \frac{\partial^2 EJTP(n,Q,S,b)}{\partial b\partial Q} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial b\partial S} & \frac{\partial^2 EJTP(n,Q,S,b)}{\partial b^2}\end{bmatrix}$$

where

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q^2}=-\frac{2b^2\pi}{(1-e_1)Q^3(1-y)}-\frac{2A^\sigma(Fn+S_b+S_v)(\alpha-S\beta)}{nQ^3(1-y)^2}-H_b\left(\frac{(1-e_1(1-y)+(1-e_2)y)^2}{(1-e_1)Q(1-y)}+\frac{\left(-b+Q(1-e_1(1-y)+(1-e_2)y)\right)^2}{(1-e_1)Q^3(1-y)}\right)+\frac{2(1-e_1(1-y)+(1-e_2)y)(-b+Q(1-e_1(1-y)+(1-e_2)y))}{(1-e_1)Q^2(1-y)}\frac{\partial^2 EJTP(n,Q,S,b)}{\partial S^2}=-2A^\sigma\beta$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial b^2}=-\frac{H_b}{(1-e_1)Q(1-y)}-\frac{2\pi}{(1-e_1)Q(1-y)}$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q\partial S}=\frac{A^\sigma H_b(e_1(1-y)+(1-e_1)y)\beta}{(1-e_1)x(1-y)}-\frac{A^\sigma(Fn+S_b+S_v)\beta}{nQ^2(1-y)^2}$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q\partial b}=\frac{2b\pi}{(1-e_1)Q^2(1-y)}-H_b(-\frac{1-e_1(1-y)+(1-e_2)y}{(1-e_1)Q(1-y)}+\frac{-b+Q(1-e_1(1-y)+(1-e_2)y)}{(1-e_1)Q^2(1-y)})$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial Q\partial b}=\frac{2b\pi}{(1-e_1)Q^2(1-y)}-H_b(-\frac{1-e_1(1-y)+(1-e_2)y}{(1-e_1)Q(1-y)}+\frac{-b+Q(1-e_1(1-y)+(1-e_2)y)}{(1-e_1)Q^2(1-y)})$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial S\partial Q}=\frac{A^\sigma H_b(e_1(1-y)+(1-e_1)y)\beta}{(1-e_1)x(1-y)}-\frac{A^\sigma(Fn+S_b+S_v)\beta}{nQ^2(1-y)^2}$$

$$\frac{\partial EJTP(n,Q,S,b)}{\partial S\partial b}=0$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial b\partial Q}=\frac{2b\pi}{(1-e_1)Q^2(1-y)}+\frac{H_b(1-e_1(1-y)+(1-e_2)y)}{(1-e_1)Q(1-y)}-\frac{Hb(-b+Q(1-e_1(1-y)+(1-e_2)y))}{(1-e_1)Q^2(1-y)}$$

$$\frac{\partial^2 EJTP(n,Q,S,b)}{\partial b\partial S}=0$$

For a given value of *n*, since there are three decision variables **(***Q*,*S*,*b***)**, the sign of the last two principal minor determinants of **H** at point**(***Q**,*S**,*b****)** will be examined**.** Checking the sign of the second and third principal minor determinants of **H** at point **(***Q**,*S**,*b****)**, we obtain the following equations

$$|H_{22}|=\frac{-\beta A^\sigma(\beta A^\sigma)\left(\frac{H_b(y-1)(e_1(2y-1)-y)}{(e_1-1)x}+\frac{Fn+S_b+\mathrm{Sv}}{nQ^2}\right)^2-\frac{2(y-1)^2(-2(e_1-1)A^\sigma(\alpha-\beta S)(Fn+S_b+S_v)-b^2(H_b+2\pi)n(y-1))}{(e_1-1)nQ^3})}{(1-y)^4}<0$$

$$|H_{33}|=-\frac{4A^{2\sigma}(H_b+2\pi)(Fn+S_b+S_v)\beta(S\beta-\alpha)}{(1-e_1)nQ^4(1-y)^3}<0$$

Since the sign of $|H_{22}|$ and $|H_{33}|$ is all negative, therefore, the solution point **(***Q**,*S**,*b****)** satisfies the sufficient condition for the above maximising problem**.**