Songklanakarin J. Sci. Technol.
45 (5), 585–590, Sep. – Oct. 2023

http://www.sjst.psu.ac.th

Original Article

$\vec{\mathcal{J}}\rho$ neutrosophic Δ_{F} theory in mobile ad hoc network

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Received: 25 August 2023; Accepted: 27 September 2023

Abstract

Applications of algebraic theory can be found in different logical fields, including materials science, hereditary qualities, design, as well as in hypothetical and applied math, including logarithmic calculation, cryptography, game hypothesis, and analysis of symphonies. Generally, binary operations are utilized to characterize the algebraic structure of a nonempty set. Group is a fundamental algebraic structure that frames the establishment of different structures in algebra. Further, the concept of a group has produced a lot of good ideas, such as subgroups, quotient groups, and simple groups, to divide larger groups into more understandable parts. Neutrosophic set is not only used in groups, but it is also applied in computing, network systems, processing systems, and computer technology. Neutrosophic logic is adopted in mobile ad hoc network (MANET). A mobile ad hoc network (MANET) is a wireless ad hoc network, and is often a self-configuring net of mobile devices with no requirement for added infrastructure. Further, we discuss the anti-fuzzy (Δ_{F}) subgroup. A Δ_{F} subgroup means a fuzzy subdivision of a category if the degree of membership of the combination of two components is larger than or equal to the minimum of their separate degrees. Moreover, in this study, we have examined the concepts of a implication-based ($\vec{\mathcal{J}}\rho$) neutrosophic Δ_{F} subgroup over a finite group and a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_{F} normal subgroup over a finite group. Finally, we demonstrate some of its fundamental properties.

Keywords: $\vec{\mathcal{J}}\rho$ fuzzy subgroup, $\vec{\mathcal{J}}\rho$- Δ_{F} subgroup, $\vec{\mathcal{J}}\rho$ neutrosophic Δ_{F} subgroup, $\vec{\mathcal{J}}\rho$ neutrosophic Δ_{F} normal subgroup, neutrosophic sets in mobile ad hoc network

1. Introduction

Rosenfeld (1971) presented the many concepts related to a fuzzy group: fuzzy subgroupoids and ideals, the lattices of fuzzy subgroupoids and ideals, homomorphism, and fuzzy subgroups. Mingsheng Ying (1991) discussed the neighbourhood structure of a point and the convergence of nets and filters in the new framework. Smarandache (1999) introduced many ideas in Neutrosophy: Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability, and Neutrosophic Statistics. Yuan, Zhang, and Ren (2003) defined the concept of $\vec{\mathcal{J}}\rho$ fuzzy subgroup and also discussed the relations between two fuzzy subgroups. Salama, El-Ghareeb, Manie, and Smarandache (2014) created a software for Excel that may be used to calculate neutrosophic data. Salama, Smarandache, and Eisa (2014) proposed a methodology to use neutrosophic theory in image processing. Salama, Haitham, Manie, and Lotfy (2014) developed a new way to examine the data collected from social networks during learning programmes applying the concept of neutrosophic sets. Çetkin and Aygün (2015) examined the group structure of single valued neutrosophic sets. Selvarathi and Spinneli (2015) introduced the notion of $\vec{\mathcal{J}}\rho$ fuzzy normal subgroup of a group. Jency and Arockiarani (2016) introduced the concept of fuzzy neutrosophic subgroupoids using fuzzy neutrosophic products. Mondal, Pramanik, and Giri (2016) demonstrated the function of single valued neutrosophic set logic in data mining. Selvarathi and Spinneli (2018) described the idea of $\vec{\mathcal{J}}\rho$ intuitionistic fuzzy subgroup and $\vec{\mathcal{J}}\rho$ intutitionistic Δ_{F} normal subgroup of a group. Broumi Said *et al*. (2019) examined information processing in MANET. Elhassouny, Idbrahim, and Smarandache (2019) examined additional contributions of machine learning algorithms with single-valued neutrosophic numbers in modelling incorrect information. In this paper, we develop the idea of a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_{F} subgroup over a finite group and explore it, presenting some fundamental properties.

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2. Preliminaries

Definition 2.1 Rosenfeld (1971), Let S be a group then μ: S → [0, 1] is called as fuzzy subgroup if

(i) $\mu(\kappa\varrho) \geq \min(\mu(\kappa), \mu(\varrho))$ for all $\kappa, \varrho \in S$

(ii) $\mu(\kappa^{-1}) \geq \mu(\kappa)$ for all $\kappa \in S$.

Assume that $(\Re, *)$ is a finite group, and that K is the universe of discourse. The [α] symbol is used in fuzzy logic to represent the truth value of a fuzzy proposition, α. The corresponding set of theoretical notations for fuzzy logic are used in this paper as follows.

$(\kappa \in A) = A(\kappa)$,

$(\alpha \wedge \beta) = \min\{[\alpha],[\beta]\}$,

$(\alpha \rightarrow \beta) = 1- \alpha + \alpha\beta$,

$(\forall\kappa\alpha(\kappa)) = \inf_{\kappa\in K}[\alpha(\kappa)]$,

$(\exists\kappa\alpha(\kappa)) = \sup_{\kappa\in K}[\alpha(\kappa)]$,

$\vec{\mathfrak{J}}\alpha$ if and only if [α] =1 for all valuations.

Implication that is used here is the Reichenbach implication operator.

Definition 2.2 Selvarathi and Spinneli (2018), Let $\Re$ be a group. A fuzzy subset μ of $\Re$ is called a Δ_F subgroup of $\Re$ if for $\kappa, \varrho \in \Re$

(i) $\mu(\kappa\varrho) \leq \mu(\kappa)\, \mu(\varrho)$

(ii) $\mu(\kappa^{-1}) \leq \mu(\kappa)$

Definition 2.3 Selvarathi and Spinneli (2015), If a fuzzy subset A of a group $\Re$ satisfies

(i) $\vec{\mathfrak{J}}(\kappa \in A) \wedge (\varrho \in A) \rightarrow (\kappa\varrho \in A)$ for any $\kappa, \varrho \in \Re$

(ii) $\vec{\mathfrak{J}}(\kappa \in A) \rightarrow (\kappa^{-1} \in A)$ for any $\kappa \in \Re$

then A is called a fuzzifying subgroup of $\Re$. The concept of ρ – tautology was introduced by Ying, i.e., $\vec{\mathfrak{J}}\rho(\alpha)$ if and only if $(\alpha) \geq \rho$ for all valuations

Definition 2.4 Selvarathi and Spinneli (2015), Let A be a fuzzy subset of a finite group $\Re$ and $\rho \in (0, 1]$ is a fixed number. If for any $\kappa, \varrho \in \Re$,

(i) $\vec{\mathfrak{J}}\rho(\kappa\varrho \in A) \rightarrow ((\kappa \in A) \vee (\varrho \in A))$ and

(ii) $\vec{\mathfrak{J}}\rho(\kappa^{-1} \in A) \rightarrow (\kappa \in A)$.

Then A is called a $\vec{\mathfrak{J}}\rho$ - Δ_F subgroup of $\Re$.

2.5 Neutrosophic sets in mobile ad hoc network:
Broumi Said *et al*. (2019)

The Mobile Ad-Hoc Network (MANET) is a wireless, independent group of mobile node networks with no infrastructure (Smartphones, Laptops, iPads, PDAs, etc.). For the exchange of data packets upon joining and departing each node on an as-needed basis, the network is self-configured to reconstruct its topology and routing table information. Here, Figure 1 illustrates the use of the mathematical model of neutrosophic sets to deal with information processing in a Wi-Fi connection. As seen in Figure 1, in the context of MANET the first region is found inside the network, the second is ambiguous or indeterminate, and the third is found outside the network.

The mentioned three regions for information processing in MANET based on Figure 1 are as follows.

Region 1: The first region is located indoor the MANET, which serve as the information acceptance (or) truth zone.

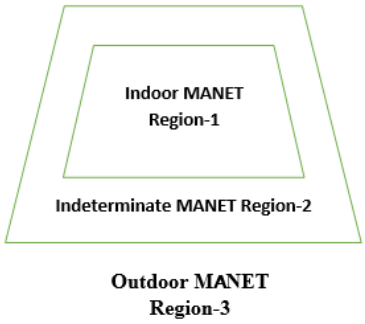

Figure 1 Communication region according to MANET

Region 2: The MANET's second region, which is between the first and third region, depicts unreachable and indeterminate information.

Region 3: The third region is located outdoor the MANET, which is the non-membership region, sometimes referred to as the out-of-coverage area.

3. $\vec{\mathfrak{J}}\rho$ Neutrosophic Δ_F Subgroup over a Finite Group

Definition 3.1 Let $(\Re, *)$ be a finite group. A neutrosophic fuzzy set $A = (A^T, A^I, A^F)$ of a finite group $\Re$ is called a $\vec{\mathfrak{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\Re$, if it satisfies for any $\kappa, \varrho \in \Re$,

(i) $\vec{\mathfrak{J}}\rho(\kappa\varrho \in A) \rightarrow ((\kappa \in A) \vee (\varrho \in A))$

i.e.,

$\vec{\mathfrak{J}}\rho(\kappa\varrho \in A^T) \rightarrow ((\kappa \in A^T) \vee (\varrho \in A^T))$

$\vec{\mathfrak{J}}\rho(\kappa\varrho \in A^I) \rightarrow ((\kappa \in A^I) \vee (\varrho \in A^I))$

$\vec{\mathfrak{J}}\rho((\kappa \in A^F) \wedge (\varrho \in A^F)) \rightarrow (\kappa\varrho \in A^F)$

(ii) $\vec{\mathfrak{J}}\rho(\kappa^{-1} \in A) \rightarrow (\kappa \in A)$

i.e.,

$\vec{\mathfrak{J}}\rho(\kappa^{-1} \in A^T) \rightarrow (\kappa \in A^T)$

$\vec{\mathfrak{J}}\rho(\kappa^{-1} \in A^I) \rightarrow (\kappa \in A^I)$

$\vec{\mathfrak{J}}\rho(\kappa \in A^F) \rightarrow (\kappa^{-1} \in A^F)$

Where $(\kappa \in A^T)$ denotes the truth membership value, $(\kappa \in A^I)$ denotes the indeterminacy membership value and $(\kappa \in A^F)$ denotes the falsity membership value such that, $0 \leq (\kappa \in A^T) + (\kappa \in A^I) + (\kappa \in A^F) \leq 3$.

Example 3.2 $\Re = \{0,1,2,3\}$ is a finite group of order 4 with respect to addition modulo 4. Cayley closure table is given as Table 1.

Table 1. Closure table

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Consider the neutrosophic fuzzy set A: $\Re \rightarrow [0, 1] \times [0,1] \times [0, 1]$ defined by

$(0 \in A) = (0.015, 0.035, 0.620)$: $(1 \in A) = (0.110, 0.135, 0.515)$
$(2 \in A) = (0.115, 0.150, 0.415)$: $(3 \in A) = (0.120, 0.145, 0.350)$

Given below are Tables 2, 3 and 4 of values for ∨ and ∧ of truth membership, indeterminacy membership, and falsity membership.

Table 2. Truth table

∨	[0]	[1]	[2]	[3]
[0]	0.015	0.110	0.115	0.120
[1]	0.110	0.110	0.115	0.120
[2]	0.115	0.115	0.115	0.120
[3]	0.120	0.120	0.120	0.120

Table 3. Indeterminate table

∨	[0]	[1]	[2]	[3]
[0]	0.035	0.135	0.150	0.145
[1]	0.135	0.135	0.150	0.145
[2]	0.150	0.150	0.150	0.150
[3]	0.145	0.145	0.150	0.145

Table 4. Falsity table

∧	[0]	[1]	[2]	[3]
[0]	0.620	0.515	0.415	0.350
[1]	0.515	0.515	0.415	0.350
[2]	0.415	0.415	0.415	0.350
[3]	0.350	0.350	0.350	0.350

With $\rho = 0.3$ and the implication operator being that of Reichenbach, then $A = (A^T, A^I, A^F)$ is a $\vec{J}\rho$ neutrosophic Δ_F subgroup over a finite group.

Theorem 3.3 Let $A = (A^T, A^I, A^F)$ be a $\vec{J}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$. Then for all $\kappa \in \mathfrak{R}$ we have

(i) $\vec{J}\rho(e \in A) \to (\kappa \in A)$
i.e.,
$\vec{J}\rho(e \in A^T) \to (\kappa \in A^T)$
$\vec{J}\rho(e \in A^I) \to (\kappa \in A^I)$
$\vec{J}\rho(\kappa \in A^F) \to (e \in A^F)$

(ii) $\vec{J}\rho\ (\kappa \in A) \to (\kappa^{-1} \in A)$
i.e.,
$\vec{J}\rho(\kappa \in A^T) \to (\kappa^{-1} \in A^T)$
$\vec{J}\rho(\kappa \in A^I) \to (\kappa^{-1} \in A^I)$
$\vec{J}\rho(\kappa^{-1} \in A^F) \to (\kappa \in A^F)$

Proof.

(i) $\vec{J}\rho(e \in A^T) \to (\kappa\kappa^{-1} \in A^T)$ since $\vec{J}\rho\ (\kappa^{-1} \in A^T) \to (\kappa \in A^T)$
$\to ((\kappa \in A^T) \vee (\kappa^{-1} \in A^T))$
$\to ((\kappa \in A^T) \vee (\kappa \in A^T))$
$\to (\kappa \in A^T)$

Therefore, $\vec{J}\rho(e \in A^T) \to (\kappa \in A^T)$ for all $\kappa \in \mathfrak{R}$

$\vec{J}\rho(e \in A^I) \to (\kappa\kappa^{-1} \in A^I)$ since $\vec{J}\rho\ (\kappa^{-1} \in A^I) \to (\kappa \in A^I)$
$\to ((\kappa \in A^I) \vee (\kappa^{-1} \in A^I))$
$\to ((\kappa \in A^I) \vee (\kappa \in A^I))$
$\to (\kappa \in A^I)$

Therefore, $\vec{J}\rho(e \in A^I) \to (\kappa \in A^I)$ for all $\kappa \in \mathfrak{R}$

$\vec{J}\rho(\kappa \in A^F) \to (\kappa \in A^F) \wedge (\kappa^{-1} \in A^F)$ since $\vec{J}\rho(\kappa \in A^F) \to (\kappa^{-1} \in A^F)$
$\to (\kappa\kappa^{-1} \in A^F)$
$\to (e \in A^F)$

Therefore, $\vec{J}\rho(\kappa \in A^F) \to (e \in A^F)$ for all $\kappa \in \mathfrak{R}$.

(ii) $\vec{J}\rho(\kappa \in A^T) \to (e \in A^T) \vee (\kappa^{-1} \in A^T)$
$\to (\kappa^{-1} \in A^T)$

Therefore, $\vec{J}\rho(\kappa \in A^T) \to (\kappa^{-1} \in A^T)$ for all $\kappa \in \mathfrak{R}$

$\vec{J}\rho(\kappa \in A^I) \to (e \in A^I) \vee (\kappa^{-1} \in A^I)$
$\to (\kappa^{-1} \in A^I)$

Therefore, $\vec{J}\rho(\kappa \in A^I) \to (\kappa^{-1} \in A^I)$ for all $\kappa \in \mathfrak{R}$

$\vec{J}\rho(\kappa^{-1} \in A^F) \to (e \in A^F)$
$\to (\kappa\kappa^{-1} \in A^F)$
$\to (\kappa \in A^F) \wedge (\kappa^{-1} \in A^F)$
$\to (\kappa \in A^F)$

Therefore, $\vec{J}\rho(\kappa^{-1} \in A^F) \to (\kappa \in A^F)$ for all $\kappa \in \mathfrak{R}$.

Theorem 3.4 Let (A^T, A^I, A^F) be a $\vec{J}\rho$ neutrosophic fuzzy set. Then A is a $\vec{J}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$, if and only if for all $\kappa, \varrho \in \mathfrak{R}$.

$\vec{J}\rho(\kappa\varrho^{-1} \in A) \to ((\kappa \in A) \vee (\varrho \in A))$
i.e.,
$\vec{J}\rho(\kappa\varrho^{-1} \in A^T) \to ((\kappa \in A^T) \vee (\varrho \in A^T))$
$\vec{J}\rho(\kappa\varrho^{-1} \in A^I) \to ((\kappa \in A^I) \vee (\varrho \in A^I))$
$\vec{J}\rho\ ((\kappa \in A^F) \wedge (\varrho \in A^F)) \to (\kappa\varrho^{-1} \in A^F)$

Proof. Let $A = (A^T, A^I, A^F)$ be a $\vec{J}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$.

$\vec{J}\rho(\kappa\varrho^{-1} \in A^T) \to ((\kappa \in A^T) \vee (\varrho^{-1} \in A^T))$
$\to ((\kappa \in A^T) \vee (\varrho \in A^T))$ since $\vec{J}\rho(\varrho^{-1} \in A^T) \to (\varrho \in A^T)$

$\vec{J}\rho(\kappa\varrho^{-1} \in A^I) \to ((\kappa \in A^I) \vee (\varrho^{-1} \in A^I))$
$\to ((\kappa \in A^I) \vee (\varrho \in A^I)$ since $\vec{J}\rho(\varrho^{-1} \in A^I) \to (\varrho \in A^I)$

$\vec{J}\rho((\kappa \in A^F) \wedge (\varrho \in A^F)) \to ((\kappa \in A^F) \wedge (\varrho^{-1} \in A^F)$
$\to (\kappa\varrho^{-1} \in A^F)$ since $\vec{J}\rho(\kappa \in A^F) \to (\kappa^{-1} \in A^F)$

Conversely,

$\vec{J}\rho(\kappa\varrho^{-1} \in A^T) \to ((\kappa \in A^T) \vee (\varrho \in A^T))$ (3.1)
$\vec{J}\rho(\kappa\varrho^{-1} \in A^I) \to ((\kappa \in A^I) \vee (\varrho \in A^I))$ (3.2)
$\vec{J}\rho\ ((\kappa \in A^F) \wedge (\varrho \in A^F)) \to (\kappa\varrho^{-1} \in A^F)$ (3.3)

Let $\varrho=\kappa$ in (3.1)

$\vec{J}\rho\ (\kappa\varrho^{-1} \in A^T) \to ((\kappa \in A^T) \vee (\varrho \in A^T))$
$\vec{J}\rho\ (\kappa\kappa^{-1} \in A^T) \to ((\kappa \in A^T) \vee (\kappa \in A^T))$
$\vec{J}\rho\ (e \in A^T) \to (\kappa \in A^T)$

$\vec{J}\rho(\kappa^{-1} \in A^T) \to (e\kappa^{-1} \in A^T)$
$\to (e \in A^T) \vee (\kappa \in A^T)$
$\to (\kappa \in A^T)$

Therefore, $\vec{J}\rho(\kappa^{-1} \in A^T) \to (\kappa \in A^T)$

$\vec{J}\rho(\kappa\varrho \in A^T) \to (\kappa(\varrho^{-1})^{-1} \in A^T)$
$\to ((\kappa \in A^T) \vee (\varrho^{-1} \in A^T))$
$\to ((\kappa \in A^T) \vee (\varrho \in A^T))$

Therefore, $\vec{J}\rho(\kappa\varrho \in A^T) \to ((\kappa \in A^T) \vee (\varrho \in A^T))$

Let $\varrho = \kappa$ in (3.2)

$\vec{J}\rho\ (\kappa\varrho^{-1} \in A^I) \to ((\kappa \in A^I) \vee (\varrho \in A^I))$
$\vec{J}\rho\ (\kappa\kappa^{-1} \in A^I) \to ((\kappa \in A^I) \vee (\kappa \in A^I))$
$\vec{J}\rho\ (e \in A^I) \to (\kappa \in A^I)$

$\vec{\mathcal{J}}\rho(\kappa^{-1} \in A^I) \rightarrow (e\kappa^{-1} \in A^I)$
$\rightarrow (e \in A^I) \vee (\kappa \in A^I)$
$\rightarrow (\kappa \in A^I)$
Therefore, $\vec{\mathcal{J}}\rho(\kappa^{-1} \in A^I) \rightarrow (\kappa \in A^I)$
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow (\kappa(\varrho^{-1})^{-1} \in A^I)$
$\rightarrow ((\kappa \in A^I) \vee (\varrho^{-1} \in A^I))$
$\rightarrow ((\kappa \in A^I) \vee (\varrho \in A^I))$
Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow ((\kappa \in A^I) \vee (\varrho \in A^I))$
Let ϱ in (3.3)
$\vec{\mathcal{J}}\rho\ ((\kappa \in A^F) \wedge (\varrho \in A^F)) \rightarrow (\kappa \in A^F) \wedge (\kappa \in A^F)$
$\rightarrow (\kappa\kappa^{-1} \in A^F)$
$\rightarrow (e \in A^F)$
Therefore, $\vec{\mathcal{J}}\rho((\kappa \in A^F) \wedge (\varrho \in A^F)) \rightarrow (e \in A^F)$
$\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow ((e \in A^F) \wedge (\varrho \in A^F))$
$\rightarrow (e\varrho^{-1} \in A^F)$
$\rightarrow (\varrho^{-1} \in A^F)$
Therefore, $\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow (\varrho^{-1} \in A^F)$
$\vec{\mathcal{J}}\rho((\kappa \in A^F) \wedge (\varrho \in A^F)) \rightarrow ((\kappa \in A^F) \wedge (\varrho^{-1} \in A^F))$
$\rightarrow (\kappa(\varrho^{-1})^{-1} \in A^F)$
$\rightarrow (\kappa\varrho \in A^F)$
Therefore, $\vec{\mathcal{J}}\rho((\kappa \in A^F) \wedge (\varrho \in A^F)) \rightarrow (\kappa\varrho \in A^F)$
Thus, A is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$.

Theorem 3.5 Let $A_1 = (A_1^T, A_1^I, A_1^F)$, $A_2 = (A_2^T, A_2^I, A_2^F)$ $\cdots$ $A_n = (A_n^T, A_n^I, A_n^F)$ be n $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroups over a finite group $\mathfrak{R}$. Then $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$.

Proof. $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$ if it is enough to prove by previous theorem.
$\vec{\mathcal{J}}\rho(\kappa\varrho^{-1} \in A_1 \cap A_2 \cap \cdots \cap A_n) \rightarrow (\kappa \in A_1 \cap A_2 \cap \cdots \cap A_n) \vee (\varrho \in A_1 \cap A_2 \cap \cdots \cap A_n)$ for all $\kappa, \varrho \in \mathfrak{R}$.

(i) $\vec{\mathcal{J}}\rho(\kappa\varrho^{-1} \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$
$\rightarrow (\kappa\varrho^{-1} \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$ and $(\kappa\varrho^{-1} \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$
$\rightarrow ((\kappa \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T) \vee (\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T))$ and $((\kappa \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T) \vee (\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T))$
$\rightarrow ((\kappa \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$ and $(\kappa \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)) \vee ((\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$ and $(\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T))$
$\rightarrow ((\kappa \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T) \vee (\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T))$

(ii) $\vec{\mathcal{J}}\rho(\kappa\varrho^{-1} \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$
$\rightarrow (\kappa\varrho^{-1} \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$ and $(\kappa\varrho^{-1} \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$
$\rightarrow ((\kappa \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I) \vee (\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I))$ and $((\kappa \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I) \vee (\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I))$
$\rightarrow ((\kappa \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$ and $(\kappa \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)) \vee ((\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$ and $(\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I))$
$\rightarrow ((\kappa \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I) \vee (\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I))$

(iii) $\vec{\mathcal{J}}\rho((\kappa \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F) \wedge (\varrho \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F))$
$\rightarrow ((\kappa \in A_1^F)$ and $(\kappa \in A_2^F)$ and $\cdots$ and $(\kappa \in A_n^F)) \wedge ((\varrho \in A_1^F)$ and $(\varrho \in A_2^F)$ and $\cdots$ and $(\varrho \in A_n^F)$
$\rightarrow ((\kappa \in A_1^F) \wedge (\varrho \in A_1^F))$ and $((\kappa \in A_2^F) \wedge (\varrho \in A_2^F))$ and $\cdots$ and $((\kappa \in A_n^F) \wedge (\varrho \in A_n^F))$
$\rightarrow (\kappa\varrho^{-1} \in A_1^F)$ and $(\kappa\varrho^{-1} \in A_2^F)$ and $\cdots$ and $(\kappa\varrho^{-1} \in A_n^F)$
$\rightarrow (\kappa\varrho^{-1} \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F)$
Therefore, $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$.

Definition 3.6 An $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup $A = (A^T, A^I, A^F)$ of $\mathfrak{R}$ is called a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group $\mathfrak{R}$ if
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A) \rightarrow (\varrho\kappa \in A)$ for all $\kappa, \varrho \in \mathfrak{R}$
i.e.,
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^T) \rightarrow (\varrho\kappa \in A^T)$
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow (\varrho\kappa \in A^I)$
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^F) \rightarrow (\varrho\kappa \in A^F)$

Theorem 3.7 Let $A = (A^T, A^I, A^F)$ of $\mathfrak{R}$ be a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group $\mathfrak{R}$. Then the following conditions are equivalent.
(i) $\vec{\mathcal{J}}\rho(\kappa\varrho \in A) \rightarrow (\varrho\kappa \in A)$ for all $\kappa, \varrho \in \mathfrak{R}$
(ii) $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A) \rightarrow (\varrho \in A)$ for all $\kappa, \varrho \in \mathfrak{R}$

Proof. Let $\kappa, \varrho \in \mathfrak{R}$
To prove: (i) ⇒ (ii)
Assume that: $\vec{\mathcal{J}}\rho(\kappa\varrho \in A) \rightarrow (\varrho\kappa \in A)$
i.e.,
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^T) \rightarrow (\varrho\kappa \in A^T)$
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow (\varrho\kappa \in A^I)$
$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^F) \rightarrow (\varrho\kappa \in A^F)$
To prove: $\vec{\mathcal{J}}\rho\ (\kappa\varrho\kappa^{-1} \in A) \rightarrow (\varrho \in A)$ for all $\kappa, \varrho \in \mathfrak{R}$
i.e.,
$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^T) \rightarrow (\varrho \in A^T)$ for all $\kappa, \varrho \in \mathfrak{R}$
$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^I) \rightarrow (\varrho \in A^I)$ for all $\kappa, \varrho \in \mathfrak{R}$
$\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow (\kappa\varrho\kappa^{-1} \in A^F)$ for all $\kappa, \varrho \in \mathfrak{R}$
(i) $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^T) \rightarrow (\kappa\varrho(\kappa^{-1}) \in A^T)$
$\rightarrow (\kappa^{-1}\kappa\varrho \in A^T)$ by using (i)
$\rightarrow (\varrho \in A^T)$
Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^T) \rightarrow (\varrho \in A^T)$
(ii) $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^I) \rightarrow (\kappa\varrho(\kappa^{-1}) \in A^I)$
$\rightarrow (\kappa^{-1}\kappa\varrho \in A^I)$ by using (i)
$\rightarrow (\varrho \in A^I)$
Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^I) \rightarrow (\varrho \in A^I)$
(iii) $\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow (\kappa^{-1}\kappa\varrho \in A^F)$ by using (i)
$\rightarrow (\kappa\varrho\kappa^{-1} \in A^F)$
Therefore, $\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow (\kappa\varrho\kappa^{-1} \in A^F)$
Let $\kappa, \varrho \in \mathfrak{R}$
To prove: (ii) ⇒ (i)
Assume that: $\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A) \rightarrow (\varrho \in A)$ for all $\kappa, \varrho \in \mathfrak{R}$
i.e.,
$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^T) \rightarrow (\varrho \in A^T)$ for all $\kappa, \varrho \in \mathfrak{R}$
$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A^I) \rightarrow (\varrho \in A^I)$ for all $\kappa, \varrho \in \mathfrak{R}$

$\vec{\mathcal{J}}\rho(\varrho \in A^F) \rightarrow (\kappa\varrho\kappa^{-1} \in A^F)$ for all $\kappa, \varrho \in \mathfrak{R}$

To prove: $\vec{\mathcal{J}}\rho(\kappa\varrho \in A) \rightarrow (\varrho\kappa \in A)$

i.e.,

$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^T) \rightarrow (\varrho\kappa \in A^T)$

$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow (\varrho\kappa \in A^I)$

$\vec{\mathcal{J}}\rho(\kappa\varrho \in A^F) \rightarrow (\varrho\kappa \in A^F)$

(i) $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^T) \rightarrow ((\kappa\varrho)(\kappa\kappa^{-1}) \in A^T)$

$\rightarrow (\kappa(\varrho\kappa)\kappa^{-1} \in A^T)$

$\rightarrow (\varrho\kappa \in A^T)$ by using (ii)

Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^T) \rightarrow (\varrho\kappa \in A^T)$

(ii) $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow ((\kappa\varrho)(\kappa\kappa^{-1}) \in A^I)$

$\rightarrow (\kappa(\varrho\kappa)\kappa^{-1} \in A^I)$

$\rightarrow (\varrho\kappa \in A^I)$ by using(ii)

Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^I) \rightarrow (\varrho\kappa \in A^I)$

(iii) $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^F) \rightarrow (\kappa^{-1}(\kappa\varrho)(\kappa^{-1})^{-1} \in A^F)$

$\rightarrow (\kappa^{-1}(\kappa\varrho)\kappa \in A^F)$

$\rightarrow (\kappa^{-1}\kappa(\varrho\kappa) \in A^F)$

$\rightarrow (\varrho\kappa \in A^F)$ by using (ii)

Therefore, $\vec{\mathcal{J}}\rho(\kappa\varrho \in A^F) \rightarrow (\varrho\kappa \in A^F)$

Theorem 3.8 Let $A_1 = (A_1^T, A_1^I, A_1^F)$, $A_2 = (A_2^T, A_2^I, A_2^F)$ $\cdots$ $A_n = (A_n^T, A_n^I, A_n^F)$ be n $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroups over a finite group $\mathfrak{R}$. Then $A_1 \cap A_2 \cap \cdots A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group $\mathfrak{R}$.

Proof. By theorem 3.5, $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group $\mathfrak{R}$. Then

$\vec{\mathcal{J}}\rho(\varrho \in A_1 \cap A_2 \cap \cdots \cap A_n) \rightarrow (\kappa\varrho\kappa^{-1} \in A_1 \cap A_2 \cap \cdots \cap A_n)$

i.e.,

$\vec{\mathcal{J}}\rho(\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T) \rightarrow (\kappa\varrho\kappa^{-1} \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$

$\vec{\mathcal{J}}\rho(\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I) \rightarrow (\kappa\varrho\kappa^{-1} \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$

$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F) \rightarrow (\varrho \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F)$

So, by the definition of the intersection,

$\vec{\mathcal{J}}\rho(\varrho \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$

$\rightarrow (\varrho \in A_1^T)$ and $(\varrho \in A_2^T)$ and $\cdots$ and $(\varrho \in A_n^T)$

$\rightarrow (\kappa\varrho\kappa^{-1} \in A_1^T)$ and $(\kappa\varrho\kappa^{-1} \in A_2^T)$ and $\cdots$ and $(\kappa\varrho\kappa^{-1} \in A_n^T)$

$\rightarrow (\kappa\varrho\kappa^{-1} \in A_1^T \cap A_2^T \cap \cdots \cap A_n^T)$

$\vec{\mathcal{J}}\rho(\varrho \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$

$\rightarrow (\varrho \in A_1^I)$ and $(\varrho \in A_2^I)$ and $\cdots$ and $(\varrho \in A_n^I)$

$\rightarrow (\kappa\varrho\kappa^{-1} \in A_1^I)$ and $(\kappa\varrho\kappa^{-1} \in A_2^I)$ and $\cdots$ and $(\kappa\varrho\kappa^{-1} \in A_n^I)$

$\rightarrow (\kappa\varrho\kappa^{-1} \in A_1^I \cap A_2^I \cap \cdots \cap A_n^I)$

$\vec{\mathcal{J}}\rho(\kappa\varrho\kappa^{-1} \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F)$

$\rightarrow (\kappa\varrho\kappa^{-1} \in A_1^F)$ and $(\kappa\varrho\kappa^{-1} \in A_2^F)$ and $\cdots$ and $(\kappa\varrho\kappa^{-1} \in A_n^F)$

$\rightarrow (\varrho \in A_1^F)$ and $(\varrho \in A_2^F)$ and $\cdots$ and $(\varrho \in A_n^F)$

$\rightarrow (\varrho \in A_1^F \cap A_2^F \cap \cdots \cap A_n^F)$

By theorem 3.7, $A_1 \cap A_2 \cap \cdots \cap A_n$ is a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group $\mathfrak{R}$.

4. Conclusions

Neutrosophic set is very interesting to study and research because neutrosophic sets are used in different areas such as computer systems, computer technology, and decision-making tools. Not only that, but they are also used in developing neutrosophic relational databases, neutrosophic images, and e - learning. In this work, the definition of a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group and a $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group has been provided. Along these, we have reported on the fundamental characteristics of the $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F subgroup over a finite group and the $\vec{\mathcal{J}}\rho$ neutrosophic Δ_F normal subgroup over a finite group. This theory can be applied to optimize the MANET by reducing the significant issues.

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