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Original Article

Effects of physical determinants in an unsteady blood flow in a stenosed artery

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Abstract

This article presents a mathematical model for the impact of physical determinants in unsteady flow of blood in an artery that has stenosis. Continuity and momentum nonlinear partial differential equations (PDEs) are obtained with consideration that the blood behaves as a Newtonian fluid. The explicit finite difference method (EFDM) is applied to solve the non-dimensional equations numerically subject to initial and boundary conditions. The effects of various physical parameters such as velocity of a moving particle, particle concentration, pressure gradient and stenotic height on blood velocity profile and blood flow rate have been examined and are presented graphically. The study finds that the increase of velocity of a moving particle, particle concentration and stenotic height diminish the blood velocity profile, but they increase with the increase of pressure gradient. Increases in velocity of a moving particle, particle concentration and stenotic height diminish the blood flow rate, but an increase in pressure gradient enhances the blood flow rate.

Keywords: explicit finite differences method, velocity of a moving particle, particle concentration, pressure gradient, stenotic height, flow rate

1. Introduction

Stenosis in the arteries of humans significantly impacts the flow of blood. This disease arises from buildup of substances or particles, such as cholesterols and fats within the arterial walls, leading to serious disorders in the circulatory system (Abdullah & Amin, 2010; Imoro, Etwire, C, & Musah, 2024). Since many individuals experience various health issues or some form of disorder such as joint pain, strokes, and heart attack due to stenosis associated with impaired blood flow at some point in their lives, special attention is required by the problem. Despite several efforts to develop mathematical models and fluid simulations of blood flow in a stenosed artery that have been made, the effects a moving particle in connection with other physical determinants such as particle concentration, stenotic height and negative pressure gradient on velocity profile and flow rate of blood have not been reported. For instance, Shankar and Siva (2024) investigated the behavior of blood in a narrowed artery affected by a magnetic field, while also considering the roles of mass transfer and heat. They

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considered the blood to act like a Newtonian fluid and observed the consequences of different parameters on its behavior. The findings revealed that an increase in the Reynolds number corresponds to greater turbulence in the blood flow near the downstream region of cholesterol or plaque deposition. Bunonyo, Israel-Cookey, and Amos (2018) considered heat and magnetic field to model blood flow through an affected part of an artery and results revealed that velocity of the blood can be controlled by adjustment of magnetic field strength and other parameters, and the results presented were interesting to surgeons for keeping the rate of blood flow at the chosen level for medical operation. Hussain, Sarwar, Rehman, Akbar, Gamaoun, and Coban (2022) performed analysis on heat transfer and the effects of nanoparticles on blood flow in arterial stenosis. They considered fluid physical determinants such as temperature, pressure, and velocity through arterial stenosis. The results obtained reveal that velocity changes in the affected part and decreases prior to and post the stenotic area. Geeta and Siddiqui (2016) analyzed the flow of blood in an unsteady state through the artery that has stenosis with slip velocity, and found that the rate of volumetric flow reduces with increasing of stenosis height but decreases with time. Mirza, Abdulhameed, and Shafie (2017) investigated the blood flow together with magnetic particles in a stenosed artery with consideration of Newtonian flow and a slip velocity. The results obtained show that the magnetic particle and blood velocities can be managed by changing the magnetic parameter, and the blood velocity had a strong variation within an artery that has become stenosed. The non-Newtonian fluid flowing through an angled tapered artery suspended with magnetic particles was reported by Padma , Tamil Selvi, and Ponalagusamy (2019), who observed that electro-kinetic number increased the velocities of the fluid and particle. These results (Bunonyo, Israel-Cookey, & Amos, 2018; Geeta & Siddiqui, 2016; Mirza, Abdulhameed, & Shafie, 2017; Padma , Tamil Selvi, & Ponalagusamy, 2019) play a vital function in diagnosing cardiovascular diseases. Priyadharshini and Ponalagusamy (2019) investigated the impact of an external magnetic field on the pulsatile flow of blood through stenosed artery carrying magnetic nanoparticles. The results obtained show that stenotic height, mass, particle concentration and magnetic field parameters are important factors in analyzing the blood flow through a stenosed artery while carrying particles, and could be designed a setup *in vitro* to forecast the use of magnetic drug in blocked blood vessels. Sharma, Sharma, Gaur, and Mishra (2015) investigated the effects of magnetic field on rheological models of blood. They highlighted that magnetic field could be employed to regulate the blood flow, which was beneficial for hypertension treatment. Varshney, Katiyar, and Kumar (2010) studied the flow of blood in a clogged artery with the presence of magnetic field. They found that the presence of stenosis and magnetic field would affect the flow characteristics. Kaewbumrung, Orankitjaroen, Boonkrong, Nuntadilok, and Wiwatanapataphee (2018) discussed a model for blood flow of dispersed particle in the left coronary artery, they considered flow of blood to be incompressible non-Newtonian fluid, turbulent and governed according to the Navier-Stokes formulas with Reynolds average. They also considered moving particles are spherical in shape having the same density as the density of blood. The rotational and translation motions were administered by Newtonian flow. The results showed that as the mark of stenosis increased, it developed wall shear stresses and pressure drop, thus restricting proper blood flow. Shit & Roy (2011) examined the effects of a magnetic field and externally applied body acceleration on pulsatile blood flow in a stenotic arterial segment. The governing non-linear equations of the flow were numerically solved by employing the finite difference technique through appropriate coordinate transformation. They developed a mathematical model by considering micro-particles suspended in blood. The results revealed that a magnetic field applied into blood flow caused particles to move parallel to the axis of the magnetic field and created additional viscosity, which consequently reduced the blood velocity. Reduction of flow velocity causes improper blood flow. Uddin *et al*. (2019) studied magnetic particles with electromagnetic to control the flow of Newtonian fluid in the stenosed area. The results showed that, when the stenotic height reached its maximum, the impact on blood and magnetic particles increased, but when a magnetic field was perpendicular the impact decreased, and the particle velocity was found to be almost equal to the blood velocity but magnetic particles moved slower than blood. Also, they reported that strong and sufficient application of magnetic field to the stenosis can regulate the flow of blood, which is a crucial fact in clinical diagnosis and management of treating hypertension, atherosclerosis and other cardiovascular diseases. Sankar, Goh, and Ismail (2010) analyzed the erratic blood flow in a stenosed tapered narrow artery, and they found that flow rate decreases with stenotic height. These observations were also obtained by Chakravarty and Mandal (2000), Gujral and Singh (2020). Jimoh (2020) investigated the effects of using a third grade model with unsteadiness on flow of blood in a stenosed artery with slip condition. The results revealed that slip velocity and shear increase in proportion with the velocity profile and the flow rate, but for both steady and unsteady blood flow models, they diminish with flow resistance. Riahi and Roy (2012) investigated unsteady blood flow in an artery with an overlapping stenosis. Srikanth, Ramana, Jain and Kale (2015) studied the effects of catheter and slip velocity on blood flow through a tapered ω-shaped stenosed artery using the unsteady polar fluid model. Ellahi, Shafiq, and Nadeem (2013) studied analytical solution of unsteady Jeffery fluid of blood flow through stenosed artery with permeable walls. Javadzadegan, Esmaeili, Majidi, and Fakhimghanbarzadeh (2009) examined the viscoelastic and viscous fluid flow in a narrow tube. They displayed unsteady numerical results for blood flow in a stenosed artery, and reported that the characteristics of blood flow rate are affected by non-Newtonian rheology. Also, Shahed, Ali, Saha, and Akhter (2014) studied the unsteady flow field of arterial stenosis numerically. Jamil *et al*. (2018) studied the unsteady blood flow with nanoparticle through a stenosed artery with the presence of periodic body acceleration. The study revealed that the speed of blood increased with the slip velocity. It also diminishes with yield stress. Based on the existing literature, the current study differs from previous research by investigating the effects of a moving particle in relation to particle concentration, pressure gradient, and stenotic height in blow flow in stenosed artery on velocity profile and flow rate of the blood. It is a mathematical model to find the solutions to real-life problems, for developing technology and improving the existing models and exploring the effects of emerging parameters in the flow system to control the treatment of stenosis. In regard to this research article, mathematical modeling of blood flow dynamics in a stenosed

artery varies due to complexity and distinctiveness of the solution obtained. In this case, a one-dimensional model is used and more details are discussed according to the structure of the paper. In section 2, we illustrate the flow geometry, formulate the mathematical model, stability of EFDM and determine numerical solutions for the effects of flow determinants according to appropriate assumptions. Section 3 presents the numerical simulation, discussion and interpretation of physical meaning graphically. Section 4 presents a conclusion and recommendation.

2. Materials and Methods

2.1 Model formulation

The physical domain of the problem is displayed in Figure 1 where $\bar{z}$ - axis be taken along the axis of symmetry, and direction for corresponding velocities u and v (u is the velocity of blood and v is the velocity of a moving particle), $H(\bar{z})$ is the radius of geometry of the stenosed region (constricted region); a is the radius of the artery in the absence of stenosis; z_0 is the axial length of stenosis, r is the radial coordinate, and δ is the stenotic height.

The model is formulated by considering the following assumptions.

The stenosis develops in the artery wall, and an artery is taken as a very thin long cylindrical vessel because of minimizing complex pattern of flow. Blood behaves as a viscous Newtonian fluid at normal or low shear rates, exhibiting a relatively constant viscosity. This assumption simplifies mathematical modeling in engineering and medical applications, particularly when precise predictions are not critical. The flow is characterized as unsteady, laminar (meaning that the shear rates do not vary drastically), fully developed, unidimensional, incompressible, and axisymmetric. The blood is viscous Newtonian at normal shear rates or low shear rate, and blood's behavior approximates that of a Newtonian fluid, exhibiting a relatively constant viscosity. This approximation simplifies application of mathematical modeling in engineering and medical applications, especially when precise predictions are not critical the flow is unsteady, laminar, (that is the shear rates do not vary drastically) fully developed, unidimensional, incompressible, and axisymmetric; the moving particle is flowing along the axis of a stenosed artery and is uniformly in shape throughout the blood and it is influenced by blood flow but is not filtered or destroyed within the blood stream; instead it moves. Also, both blood and moving particle are flowing in the axial direction (z-axis) through the stenosis. Therefore, the study considers the velocity of a moving particle and other materialized determinants. With these assumptions, we define the geometry equation of the stenosis as given in (2.1).

$$H(\bar{z}) = \begin{cases} a - \delta\left(1 + \cos\frac{\pi\bar{z}}{z_0}\right) & -z_0 \le \bar{z} \le z_0 \\ a & otherwise \end{cases}. \tag{2.1}$$

As well as the continuity and momentum equations controlling the flow:

$$\frac{d\rho}{dt} + \nabla.(\rho u) = 0 \tag{2.2}$$

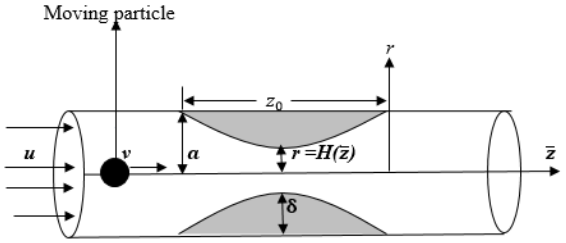

Figure 1. Problem geometry

$$\frac{du}{dt} = -\nabla.p + [\nabla.\tau] + f \tag{2.3}$$

Due to assumptions made, and computational process, equations (2.2) and (2.3) have been simplified with the following initial and boundary conditions as:

$$\frac{\partial u}{\partial z} = 0, \tag{2.4}$$

Because the velocity in the radial direction is negligible due to assumptions made.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{z}} + \frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\bar{\mu}\frac{\partial u}{\partial r}\right) + \frac{D}{\rho}(v - u), \tag{2.5}$$

where ρ is the density, $\bar{P}$ is the pressure, $\bar{\mu}$ is the dynamic viscosity, t is the time, and D is the Stokes' constant.

$$\frac{\partial u(r,t)}{\partial r} = 0 \text{ at } r = 0, \tag{2.6}$$

$$u(r,t) = v(r,t) = 0, \text{ at } r = H(\bar{z}), \tag{2.7}$$

$$u(r,t) = v(r.t) = 0 \text{ at } t = 0. \tag{2.8}$$

The volumetric rate of flow after considering a small cylindrical ring element inside an artery is given as:

$$Q = \int_0^{H(\bar{z})} 2\pi u r dr, \tag{2.9}$$

where $2\pi r dr$ is an area of the cross section the blood passing through an artery.

Introducing the dimensionless quantities from Equation (2.10) to Equations (2.1), (2.4) - (2.9)

$$w = \frac{ua}{\vartheta_0},\ w_p = \frac{va}{\vartheta_0},\quad t^* = \frac{\vartheta_0 t}{a^2},\ \vartheta_0 = \frac{\mu_0}{\rho},\ P = \frac{a^3\bar{P}}{\vartheta_0^2\rho z_0},\ \mu = \frac{\bar{\mu}}{\mu_0},\ A = \frac{\partial P}{\partial Z}, z = \frac{\bar{z}}{z_0},\ e = \frac{\delta}{a},\ R = \frac{Da^2}{\mu_0},\ \gamma = \frac{r}{H}. \tag{2.11}$$

after dropping * are reduced to:

$$\frac{\partial w}{\partial z} = 0, \tag{2.12}$$

$$\frac{\partial w}{\partial t} = -A + \frac{\mu}{h^2}\left(\frac{\partial^2 w}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial w}{\partial \gamma}\right) + R\left(w_p - w\right): h(z) = \begin{cases} 1 - e(1 + \cos\pi z) & -1 \le z \le 1 \\ 1 & otherwise \end{cases}, \tag{2.13}$$

$$\frac{\partial w(\gamma,t)}{\partial \gamma} = 0, \text{ at } \gamma = 0, \tag{2.14}$$

$$w(\gamma,t) = 0,\ w_p(\gamma,t) = 0, \text{ at } \gamma = 1, \tag{2.15}$$

$$w(\gamma,t) = 0,\ w_p(\gamma,t) = 0, \text{ at } t = 0, \tag{2.16}$$

$$Q = 2\pi h^2 \int_0^1 \gamma w d\gamma. \tag{2.17}$$

The parameters appearing in Eqns. (2.13) – (2.17) are pressure gradient (A), dynamic viscosity (μ), radius of an affected part of an artery (h), particle concentration (R), velocity of a moving particle (w_p), velocity of blood (w), stenotic height (e), coordinate transformation (γ) and volumetric flow rate (Q). Since Eqns. (2.14) – (2.16) are dimensionless initial and boundary conditions subject to the model equations, the first condition (2.14) is imposed due the symmetrical flow with respect to central plane, second condition (2.15) signifies no-slip at the wall, and the third condition (2.16) indicates that the flow starts from rest.

2.2 Method of solution

The non-dimensional model Equation (2.13) forms the initial boundary value problem (IBVP) whose solution is sought. The solution is approximated numerically using the explicit finite differences technique. This method has been used extensively by many researchers in computational fluid dynamics and biomedical engineering sciences studies and confirmed to be a fine algorithm. It is easy to derive, program, converges, takes little storage, quickly executes and stable with some conditions. In this method, the radial coordinates (γ) are discretized uniformly spaced in the interval $0 \le \gamma \le l$ as $\gamma_i = (i-1)\Delta\gamma$, $i = 1,2,\dots,N$, where N is the total number of spatial nodes including those on the boundaries with spacing $\Delta\gamma = \frac{l}{N-1}$. Similarly, t is discretized on $0 \le t \le t_{max}$ as $t_k = (k-1)\Delta t, k = 1,2,\dots M$; where M is the number of time steps and Δt is the time step size expressed as $\Delta t = \frac{t_{max}}{M-1}$. Its numerical approximation solution is then described by replacing continuous partial derivatives of the partial differential equations (PDEs) into discrete algebraic difference quotients. Derivatives are discretized by using difference quotients for the time (t) variable and centered difference quotients for the spatial variable. Then we have following:

$$\frac{\partial w}{\partial t} = \frac{w_i^{k+1} - w_i^k}{\Delta t}, \quad \frac{\partial w}{\partial \gamma} = \frac{w_{i+1}^k - w_{i-1}^k}{2\Delta\gamma}, \quad \frac{\partial^2 w}{\partial \gamma^2} = \frac{w_{i+1}^k - 2w_i^k + w_{i-1}^k}{(\Delta\gamma)^2},\ w = w_i^k \ . \tag{2.18}$$

where the superscript k denotes a counter on the t variable and the subscript i denotes a counter on the γ variable. Inserting finite difference Equation (2.18) to Equation (2.13), the discretized equation becomes:

$$w_i^{k+1} = \frac{\mu S}{h^2}\left[\left(1 - \frac{\Delta\gamma}{2\gamma}\right) w_{i-1}^k + \left(1 + \frac{\Delta\gamma}{2\gamma}\right) w_{i+1}^k\right] + \left(1 - \frac{2\mu S}{h^2} - R\Delta t\right) w_i^k + \Delta t(Rw_p - A), \tag{2.19}$$

where w_i^k is an approximation solution, and discretized initial and boundary conditions;

$$w_{i+1}^k = w_{i-1}^k, \quad \text{at } \gamma = 0; \quad w_{N-1}^k = 0 \text{ , at } \gamma = 1, \tag{2.20}$$

$$w_i^1 = w_p = 0 \text{ at } t = 0, \tag{2.21}$$

In view of Equation (2.16), having the solution of blood velocity (w_i^k), one can obtain the discretized form of rate of flow given by:

$$Q_i^k = 2\pi\left(h_i^k\right)^2 \int_0^1 \gamma w_i^k d\gamma. \tag{2.22}$$

2.3 Stability of the solution

The EFDM is only conditionally stable depending on time increment (Δt) and step size ($\Delta\gamma$).Thus, the fixed values of $\Delta t = 0.0001$ for time and $\Delta\gamma = 0.025$ for step size have been used. (Abdullah & Amin, 2010; Varshney , Katiyar, & Kumar , 2010 and Shit & Roy, 2011) proved that these values are suitable for stability and convergence. In this case, it can be shown that the EFDM both converges and is stable if $s = \frac{\Delta t}{(\Delta\gamma)^2} \le \frac{1}{2}$ or $\Delta t \le \frac{(\Delta\gamma)^2}{2}$ (according to Von Neumann stability criteria), meaning that to match the solution we need time not faster than $\frac{(\Delta\gamma)^2}{2}$. Thus, from equation (2.16) the coefficients of w_{i-1}^k, w_{i+1}^k and w_i^k must be non-negative, hence for stability it must satisfy these conditions.

Finally, Matlab codes are used to simulate and plot graphs. Analysis of the results for the velocity profile and volumetric flow rate are done for various parameters such as stenotic height, a moving particle, pressure gradient and particle concentration.

3. Results and Discussion

In this segment, simulation has been made for one-dimensional, the scheme is tested according to the conditions mentioned in section (2.3). So, it is a good choice compared to others because of its simplicity, and ease of implementation, making accessible for a wide range of problems. Its computational efficiency arises from the direct calculation of each time step from previous values, eliminating the need for complex matrix inversions. It also, provides a clear understanding of how solutions progress over time, and is particularly effective for problems with well-defined boundaries and conditions. Additionally, its flexibility allows for adaptation to various geometries and boundary conditions, while the independence of calculations at each grid point facilitates straightforward parallelization of computation, and avoids degeneracy during the computational process.

To determine the effects of physical determinant parameters emerging on the velocity profile (w) and volumetric rate of flow of the blood (Q), considerations have been taken to the values of parameters and constants shown in the Table 1, and graphs are displayed in Figures 2 to 9 and discussed.

At the specific location of an arterial segment $z = 0.5$ and at time instant $t = 1$, Figure 2 illustrates the influence of stenotic height (e) on velocity profile (w), in light of presumption that blood geometrical vessel exhibits axial symmetry, the velocity of the blood arrives at the highest value around the middle of an artery, that is, the velocity diminishes with stenotic height, whereas from the highest value at the

Table 1. Parameter and constant values

Parameter / constant	Value(s)	Source(s)
Stenotic height (e)	0.1 - 0.3	(Sankar, Goh, & Ismail, 2010)
Velocity of a moving particle (wp)	0.0 - 0.2	Assumed
Particle concentration (R)	0.2 - 0.6	(Priyadharshini & Ponalagusamy, 2019)
Negative pressure gradient (A)	1.0 - 3.0	Assumed
Dynamic viscosity (μ)	`0.0305	(Sankar, Goh, & Ismail, 2010)

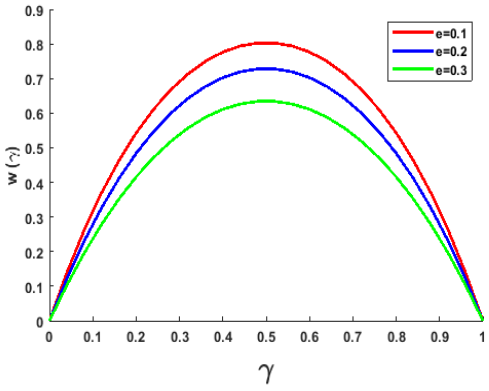

Figure 2. The influence of stenotic height (e) on the velocity profile (w)

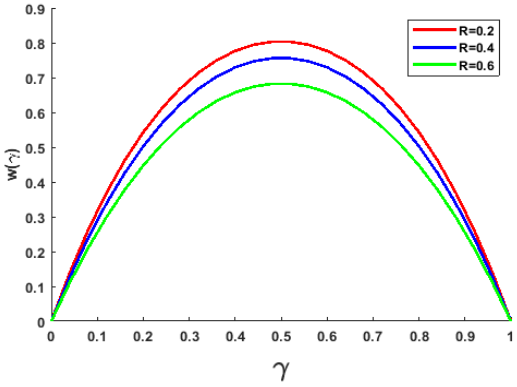

Figure 3. The influence of particle concentration (R) on the velocity profile (w)

middle of an artery, it goes down as arterial distance increases. The increase in stenotic height prevents the blood from flowing easily in the constricted region as the result causes an increase in the stress at the artery walls.

Figure 3 shows the effect of particle concentration (R) on velocity profile (w). When the particle concentration (R) is introduced to the system, the velocity of the blood shows some changes, it tends to decrease. Decrease in velocity may indicate that the presence of particles is increasing resistance to flow within the narrowed region. This can lead to higher shear stress on the arterial walls, potentially exacerbating vascular damage. Also, it can impair oxygen and nutrient delivery to tissues downstream of the stenosis, increasing the risk of insufficient blood flow, and related health issues.

Figure 4 depicts the effects of blood pressure gradient (A) on velocity profile (w). The velocity of the blood increases as the negative pressure gradient of the blood increases, since the flow normally takes place from high pressure to low pressure which results in the speed of pumping up the blood into the muscles in which it is supplying more oxygen, and facilitates the efficient transport of nutrients and hormones throughout the body parts, which is essential for overall health and recovery processes. Therefore, the increase of negative pressure gradient enhances velocity of blood in the positive manner.

Figure 5 illustrates the influence of a moving particle (wp) on the velocity profile (w), it is normally during the flow, blood moves much faster than a moving particle, this is due to collision of drag and other forces of retardation such as inertial forces and viscous forces. However, the velocity of a moving particle increases due to collision between forces. When the velocity of a moving particle is zero, it implies that the flow of blood remains less disrupted due to lack of the particle interaction, but when moving particle gets involved or there is

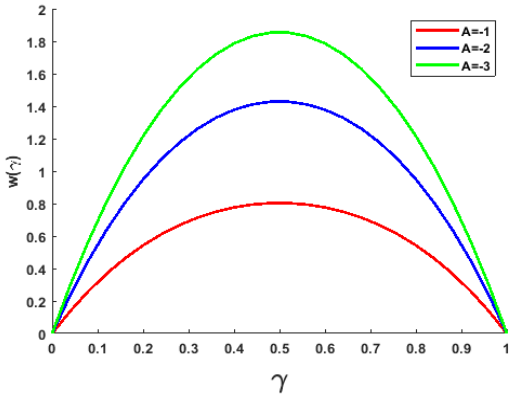

Figure 4. The influence of pressure gradient (A) on the velocity profile (w)

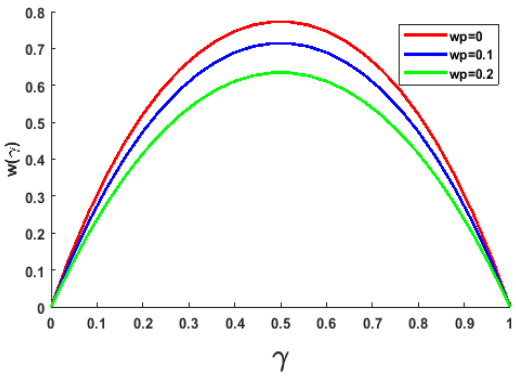

Figure 5. The influence of a moving particle (wp) on the velocity profile (w)

no interaction in the movement (i.e. presence in motion) the graph illustrates that the velocity of the blood increases because of the additional velocity in the flow system. This suggests that interactions within the blood flow can enhance particle movement, potentially affecting their delivery to target areas, such as tissues.

Figure 6 and 7 show the volumetric flow rate (Q) of the blood against distance (γ) of an artery plots with increasing three different values of stenotic height (e), particle velocity (wp) and pressure gradient (A). In both figures, we observe that at the beginning of the down part of an artery wall, when stenotic height and particle velocity increases accordingly, the volumetric flow rate shows no variation or there is very slight variation, but when it goes up, it experiences a clear variation, meaning that increasing the stenotic height and particle velocity result in decreasing the volumetric flow rate of the blood in the artery for a given set of parameters. These results can lead to significant health consequences, including tissue damage due to insufficient oxygen and nutrient delivery, which may cause pain and organ dysfunction. These agree well with those of (Gujral & Singh, 2020, Uddin *et al*., 2019; Sankar, Goh, & Ismail, 2010) in relation to stenotic height.

Figure 8, shows that under the effect of pressure gradient (A), the flow rate (Q) increases as pressure gradient increases, reaching the highest value at the mid of an artery but it slightly converges at the upper part of the wall of an artery. Thus an increase in flow rate due to a higher pressure gradient can enhance circulation and tissue perfusion, it also poses risks that require careful regulation to maintain vascular and cardiovascular health.

In Figure 9, when the particle concentration (R) increases, the blood flow rate (Q) decreases. This phenomenon is primarily due to the increased viscosity of the blood, which occurs when there are higher levels of particle concentration, and blood encounters greater resistance as it flows through narrowed arteries, impeding efficient flow. Consequently, more effort must be exerted to pump viscous blood, lowering the overall flow rate. Additionally, increased particle concentration can lead to turbulence, particularly in stenotic regions, which diminishes flow efficiency. This reduction in blood flow not only compromises the delivery of oxygen and nutrients to downstream tissues, but also highlights the clinical significance of managing conditions like stenosis. In such cases, both elevated particle concentration and arterial narrowing can independently affect blood flow, necessitating targeted interventions to restore adequate perfusion. Overall, the interplay between particle concentration and blood flow in stenosed arteries underscores the importance of maintaining optimal blood characteristics for cardiovascular health.

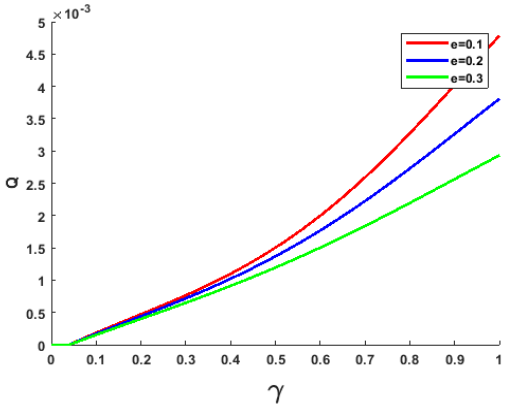

Figure 6. The influence of stenotic height (e) on the flow rate (Q)

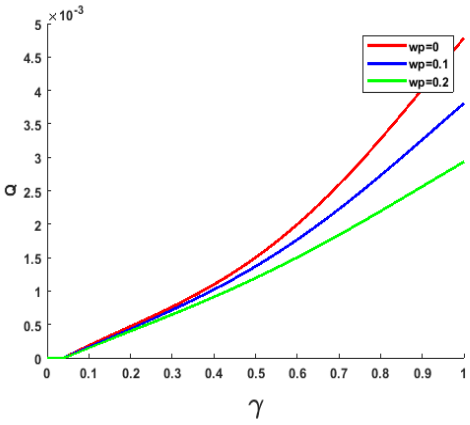

Figure 7. The influence of a moving particle (wp) on the flow rate (Q)

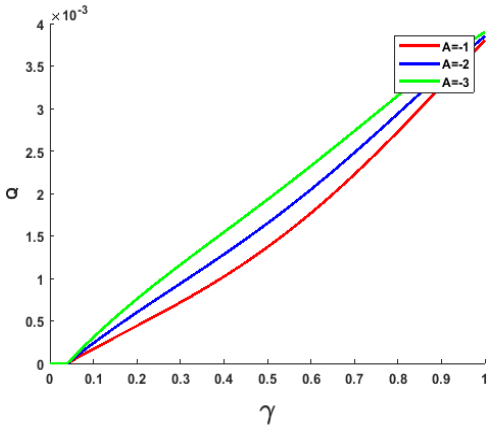

Figure 8. The influence of pressure gradient (A) on the flow rate (Q)

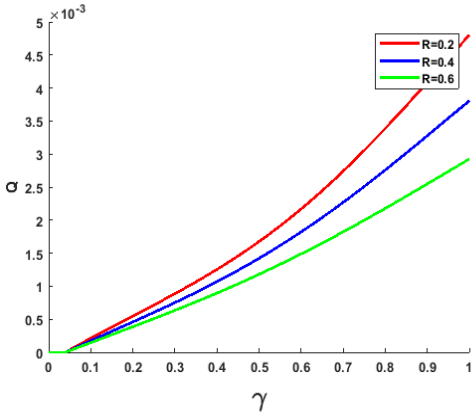

Figure 9. The influence of particle concentration (R) on the flow rate (Q)

4. Conclusions

A mathematical model to ascertain the effects of physical determinants in an unsteady blood flow in a stenosed artery have been developed, analyzed and solved numerically using the explicit finite difference method. The findings for this study indicate that increases in stenotic height, particle concentration, a moving particle in flow of blood, diminish the velocity of the blood, but increase the pressure gradient. These results of the diminishing velocity of the blood are not good for the health of human being because they can affect the supply of oxygen in the body parts especially in the brain and they may cause cardiovascular diseases. Thus, it is significant to recall that the normal and abnormal flow of blood can be affected by these determinants. Also, increases in stenotic height, particle velocity, and the particle concentration decrease the flow rate of blood, whereas they increase the negative pressure gradient. These results are of significant interest to health practitioner for comprehend the specific flow behaviors that are being depicted and keeping the blood flow rate at the wanted level during treatment of cardiovascular diseases. In other words, these findings are technically important for treating diseases associated with blood flow.

Nomenclature

a : Radius of an artery in the absent of stenosis (mm)
A : Pressure gradient (Pa)
D : Stokes' constant
e : stenotic height (mm)
h : radius of affected part of an artery (dimensionless)
$H(\bar{z})$: radius of geometry part of an artery (mm)
Q : volumetric flow rate (mm^3/s)
r : radial coordinate
R : particle concentration
t : time (s)
u, v : velocities of blood and a moving particle (m/s)
w, wp : velocities of blood and a moving particle (dimensionless)
$w(\gamma)$: velocity profile (dimensionless)
z : axial length of stenosis (dimensionless)
z_0 : axial length of stenosis part (mm)
$\bar{z}$: axial length of an artery (mm)
γ : coordinate transformation parameter (dimensionless)
δ : stenotic height parameter (mm)

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