Songklanakarin J. Sci. Technol.
47 (5), 314–319, Sep. – Oct. 2025

http://www.sjst.psu.ac.th

Original Article

Orthogonal generalized reverse derivations in semiprime Γ-semirings

V. S. V. Krishna Murty[1*], C. Jaya Subba Reddy[2], and Sk. Haseena[2]

[1] *Government Degree College, Razole, Andhra Pradesh, 533242 India*

[2] *Department of Mathematics, S.V. University, Tirupati, Andhra Pradesh, 517502 India*

Received: 24 September 2024; Revised: 3 April 2025; Accepted: 6 June 2025

Abstract

Let M be a semiprime Γ-semiring. In this paper, we introduce the notion of orthogonal generalized reverse derivations in a semiprime Γ-semiring. Some characterizations of semiprime Γ-semirings are obtained by means of orthogonal generalized reverse derivations. We also establish the conditions for two reverse derivations in Γ-semiring to be orthogonal.

Keywords: Γ-semiring, semiprime Γ-semiring, reverse derivation, generalized reverse derivation, orthogonal generalized reverse derivations

1. Introduction

The concept of rings with derivations, which integrates the aspects of analysis, algebraic geometry and algebra, has a long history. Although the study of derivations in rings began early, it gained significant attention only after Posner's (1957) impactful results on derivations in prime rings. The concepts of derivation and reverse derivation have been widely studied in rings, semirings and other algebraic systems because of their role in characterizing algebraic properties. In particular, the conditions for the orthogonality of two reverse derivations have been studied in semiprime, prime and other algebraic structures. As the theory extends, researchers have turned their view to investigate reverse derivations and their orthogonality in Γ-semirings. In this paper, we introduce the concept of reverse derivations and establish the conditions of orthogonality in the context of Γ-semirings.

Vandiver (1934) introduced the notion of semirings, while Nobusawa (1964) later expanded on this theory by proposing the concept of a Γ-ring to generalize rings. Sen (1981) introduced Γ-semigroups, which Rao (1995, 1997) further developed into Γ-semirings. Herstein (1957) was the first to introduce reverse derivation. Later, subsequent studies (Aboubakr & Gonzalez, 2015; Bresar & Vukman,1989; Samman & Alyamani, 2007) expanded this topic. Javed, Aslam, and Hussain (2013) focused on the theory of derivations in prime Γ-semiring while Suganthameena and Chandramouleeswaran (2014) introduced orthogonal derivations in semirings. Venkateswarlu, Rao, & Narayana (2018, 2019) established conditions for orthogonal derivations and reverse derivations in semiprime Γ-semirings. Majeed and Hamil (2020) explored orthogonal generalized derivations in semiprime Γ-semirings while Kim (2022a, 2022b) studied orthogonal reverse derivations in semirings and Γ-semirings. Meanwhile, some significant results on non-associative rings, fuzzy ideals, ordered h-regular semirings and fuzzy bi-ideal in LA rings have been established (Anjum *et al*. 2020; Kausar, 2019; Kausar, Alesemi, Salahuddin, & Munir, 2020; Kausar, Islam, Amjad, & Waqar 2019; Kausar, Islam, Javaid, Amjad, & Ijaz, 2019; Kausar *et al*. 2020; Kausar & Waqar, 2019a, 2019b).

Recently, some results on the orthogonality of generalized symmetric reverse bi- (σ,τ) -derivations in semiprime rings and generalized reverse (σ,τ)-derivations in semiprime Γ-rings, Γ-near rings and Γ-semirings have been established (Jaya Subba Reddy, & Krishna Murty, 2024; Krishna Murty, Chennakesavulu, & Jaya Subba Reddy, 2024; Krishna Murty, & Jaya Subba Reddy, 2024a, 2024b; Krishna Murty, Jaya Subba Reddy, & Chennakesavulu, 2024; Krishna Murty, Jaya Subba Reddy, & Haseena, 2024; Krishna Murty, Jaya Subba Reddy, & Sukanya, 2024).

*Corresponding author
Email address: krishnamurty.vadrevu@gmail.com

This study introduces orthogonal generalized reverse derivations on semiprime Γ-semirings. Our objective is to prove some results on generalized reverse derivations and establish some conditions for the orthogonality of two generalized reverse derivations in semiprime Γ-semirings by extending the results established by (Majeed & Hamil, 2020).

A set M is called a semiring when it has two associative binary operations, addition (+) and multiplication (·), if the following conditions are met:

(i) The addition operation is commutative.

(ii) The multiplication operation distributes over addition from both the left side and the right side.

(iii) An element $0 \in M$ exists such that $a + 0 = a$ and $a.0 = 0.a = 0$, for every a ∈M.

If $(M, +)$ and $(\Gamma, +)$ are two abelian semigroups, with identity elements 0 and θ of M and Γ respectively, and there exists a mapping of $M \times \Gamma \times M \to M$ satisfying the following properties for $s, t, u \in M$ and $\alpha, \beta \in \Gamma$:

1. $(s + t)\alpha u = s\alpha u + t\alpha u$
2. $s(\alpha + \beta)t = s\alpha t + s\beta t$,
3. $s\alpha(t + u) = s\alpha t + s\alpha u$,
4. $(s\alpha t)\beta u = s\alpha(t\beta u)$,
5. $s\alpha 0 = 0\alpha s = 0$ and $s\theta t = 0$, then M is termed a Γ-semiring.

Let M be a Γ-semiring. M is said to be a prime if $s\Gamma M\Gamma t = \{0\}$ implies $s = 0$ or $t = 0$, for $s, t \in M$. M is said to be a semiprime if $s\Gamma M\Gamma s = \{0\}$ implies $s = 0$, for all $s \in M$. A Γ-semiring M is said to be 2-torsion free if $2s = 0$ implies $s = 0$, for all $s \in M$. An additive mapping $d_1: M \to M$ is called a derivation if $d_1(s\alpha t) = d_1(s)\alpha t + s\alpha d_1(t)$, for all s, t ∈ M. An additive mapping $d_1: M \to M$ is called a reverse derivation if $d_1(s\alpha t) = d_1(t)\alpha s + t\alpha d_1(s)$, for all s, t ∈ M. An additive mapping $D_1: M \to M$ is called a generalized derivation if $D_1(s\alpha t) = D_1(s)\alpha t + s\alpha d_1(t)$, for all s, t ∈ M, $\alpha \in \Gamma$, where d_1 is an associated derivation. An additive mapping $D_1: M \to M$ is called a generalized reverse derivation if $D_1(s\alpha t) = D_1(t)\alpha s + t\alpha d_1(s)$, for all s, t ∈ M , $\alpha \in \Gamma$ where d_1 is an associated reverse derivation. Two additive mappings d_1 and d_2 of M into M are said to be orthogonal if $d_1(s)\Gamma M\Gamma d_2(t) = \{0\} = d_2(s)\Gamma M\Gamma d_1(t)$; for all $s, t \in M$.

2. Preliminaries

Lemma 1: [Lemma 3.1, Venkateswarlu, Rao, & Narayana, 2018] Let s and t be two elements of a 2-torsion free semiprime Γ-semiring M. Then the following statements are equivalent:

(i)$s\Gamma x\Gamma t = 0$, (ii) $t\Gamma x\Gamma s = 0$, (iii) $s\Gamma x\Gamma t + t\Gamma x\Gamma s = 0$, for every $x \in M$,

If one of these conditions is fulfilled then $s\Gamma t = t\Gamma s = 0$.

Lemma 2: [Lemma 3.2, Venkateswarlu, Rao, & Narayana, 2018] Let M be a 2-torsion free semiprime Γ-semiring. If two additive mappings d_1 and d_2 of M into itself satisfy if for all $s \in M$, $d_1(s)\Gamma M\Gamma d_2(s) = \{0\}$, then $d_1(s)\Gamma M\Gamma d_2(t) = \{0\}$, for all $s, t \in M$.

Lemma 3: [Theorem 3.2, Kim, 2022b]: Let M be a 2-torsion free semiprime Γ-semiring and d_1,d_2 be reverse derivations, then d_1,d_2 are orthogonal if and only if $d_1 d_2 = 0$.

3. Main Results

Theorem 1: Let M be a 2-torsion free semiprime Γ-semiring and let D_1,D_2 be two generalized reverse derivations on M associated with the reverse derivations d_1,d_2 of M into itself respectively. If D_1 and D_2 are orthogonal on M then the following relations hold:

(a) (i)$D_1(s)\Gamma D_2(t) = D_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$.

(ii)$D_1(s)\Gamma D_2(t) + D_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$.

(b) d_1 and D_2 are orthogonal; $d_1(s)\Gamma D_2(t) = D_2(s)\Gamma d_1(t) = \{0\}$, for all $s, t \in M$.

(c) d_2 and D_1 are orthogonal; $d_2(s)\Gamma D_1(t) = D_1(s)\Gamma d_2(t) = \{0\}$, for all $s, t \in M$.

(d) d_1 and d_2 are orthogonal.

Proof.

Proving (a)

It is given that D_1 and D_2 are orthogonal generalized reverse derivations on M. Then by the definition of orthogonality,

we have $D_1(s)\Gamma M\Gamma D_2(t) = D_2(s)\Gamma M\Gamma D_1(t) = \{0\}$, for all $s, t \in M$. (3.1)

By Lemma 1, we can write $D_1(s)\Gamma D_2(t) = \{0\} = D_2(s)\Gamma D_1(t)$, for all $s, t \in M$.
Hence, the result is evident.

Proving (b)

From the condition (i) of (a), we can have $D_1(s)\Gamma D_2(t) = \{0\}$, for all $s, t \in M$.

Then $D_1(s)\Gamma D_2(t) = \{0\}$. (3.2)

Replacing s by $u\alpha s$, $u \in M, \alpha \in \Gamma$ in the equation (3.2) and using (3.1), we get

$s\alpha d_1(u)\beta D_2(t) = 0$, for all $s, t, u \in M, \alpha, \beta \in \Gamma$. (3.3)

Premultiplying (3.3) by $d_1(u)\beta D_2(t)\alpha$, we get $d_1(u)\beta D_2(t)\alpha s\alpha d_1(u)\beta D_2(t) = 0$.
Since M is semiprime, we obtain

$d_1(u)\Gamma D_2(t) = \{0\}$, for all $u, t \in M$. (3.4)

Replacing u by $s\alpha u, s \in M, \alpha \in \Gamma$ in (3.4) and using it again, we obtain $d_1(u)\alpha s\Gamma D_2(t) = 0$ which can be taken as

$d_1(u)\Gamma M\Gamma D_2(t) = \{0\}$. (3.5)

Using Lemma 1, we get $d_1(u)\Gamma D_2(t) = 0$, for all $u, t \in M$. (3.6)

From the result (i) of condition (a), we can have

$D_2(s)\Gamma D_1(t) = \{0\}$, for $u, t \in M$. (3.7)

Replacing t by $t\beta u$, $u \in M, \beta \in \Gamma$ in (3.7) and using it, we get $D_2(s)\Gamma u\alpha d_1(t) = \{0\}$.

$D_2(s)\Gamma M\Gamma d_1(t) = \{0\}$. (3.8)

Using Lemma 1, we get $D_2(s)\Gamma d_1(t) = \{0\}$, for all $s, t \in M$. (3.9)

From (3.5) and (3.8), we can conclude that d_1 and D_2 are orthogonal.

Proving (c)

By proceeding in the similar manner as in (b), we can easily prove the result (c).

Proving (d)

From the condition (i) of (a), we have $D_1(s)\Gamma D_2(t) = 0$, for all $s, t \in M$. Replacing s by $u\alpha s$ and t by $v\beta t$, $u, v \in M, \alpha, \beta \in \Gamma$ in the above equation, we get $D_1(s)\alpha u\Gamma D_2(t)\beta v + s\alpha d_1(u)\Gamma D_2(t)\beta v + D_1(s)\alpha u\Gamma t\beta d_2(v) + s\alpha d_1(u)\Gamma t\beta d_2(v) = 0$. Using (3.1) and the conditions (b) and (c) of the hypothesis

we get $s\alpha d_1(u)\Gamma t\beta d_2(v) = 0$, for all $s, t, u, v \in M, \alpha, \beta \in \Gamma$. (3.10)

Premultiplying the equation (3.10) by $d_1(u)\Gamma t\beta d_2(v)\alpha$ and using the semiprimeness property of M,

we get $d_1(u)\Gamma M\Gamma d_2(v) = \{0\}$. (3.11)

Also, from the condition (i) of (a), we have $D_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$. Replacing s by $u\alpha s$ and t by $v\beta t$, where $u, v \in M, \alpha, \beta \in \Gamma$ in the above equation, we get $D_2(s)\alpha u\Gamma D_1(t)\beta v + D_2(s)\alpha u\Gamma t\beta d_1(v) + s\alpha d_2(u)\Gamma D_1(t)\beta v + s\alpha d_2(u)\Gamma t\beta d_1(v) = \{0\}$. Using (3.1) and the conditions (b) and (c) of the hypothesis, we get $s\alpha d_2(u)\Gamma t\beta d_1(v) = \{0\}$, for all s, t, $u, v \in M, \alpha, \beta \in \Gamma$. Premultiplying the above equation by $d_2(u)\Gamma t\beta d_1(v)\alpha$ and using the semiprimeness of M, we get $d_2(u)\Gamma t\beta d_1(v) = \{0\}$.

which can be written as $d_2(u)\Gamma M\Gamma d_1(v) = \{0\}$, for all $u, v \in M$. (3.12)

From (3.11), (3.12), we can conclude that d_1 and d_2 are orthogonal.

Theorem 2: Suppose M is a 2-torsion-free semiprime Γ-semiring. Let D_1 be a generalized reverse derivation on M associated with a reverse derivation of M. If $D_1(s)\Gamma D_1(t) = \{0\}$, for any elements s and t in M, then it must be that D_1 and d_1 are both zero.

Proof: By hypothesis, $D_1(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$. (3.13)

Replacing t by $u\alpha t$, $u \in M$ in (3.13) and using (3.13), we get $D_1(s)\Gamma t\alpha d_1(u) = \{0\}$, for all $u, s \in M, \alpha \in \Gamma$.
we can write $D_1(s)\Gamma M\Gamma d_1(u) = \{0\}$.

Using Lemma 1, we get $D_1(s)\Gamma d_1(u) = \{0\} = d_1(u)\Gamma D_1(s)$, for $u, s \in M$. (3.14)

Replacing s by $u\alpha s$, $u \in M$ in (3.14) and using the same, we get
$d_1(u)\Gamma s\alpha d_1(u) = \{0\}$, for all $s \in M, \alpha \in \Gamma$ and so $d_1(u)\Gamma M\Gamma d_1(u) = \{0\}$.
Using Lemma 1, we get $d_1(u)\Gamma d_1(u) = \{0\}$.
Since M is semiprime, we can have $d_1(u) = 0$, for all $u \in M$.
Again replacing s by $s\alpha u$, $u \in M, \alpha \in \Gamma$ in (3.13) and using (3.14), we get $D_1(u)\alpha s\Gamma D_1(t) = \{0\}$.
we can write $D_1(u)\Gamma M\Gamma D_1(t) = \{0\}$, for all $u, t \in M$.
Replacing u by t in the above equation, we get $D_1(t)\Gamma M\Gamma D_1(t) = \{0\}$, for all $t \in M$.
By the semiprimeness of M, we get $D_1(t) = 0$.
Hence, we get $D_1 = d_1 = 0$

Theorem 3: Consider M as a 2-torsion-free semiprime Γ-semiring, and let D_1, D_2 be two generalized reverse derivations on M associated with reverse derivations d_1, d_2. If the conditions (i) $D_1(s)\Gamma D_2(t) + D_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$.
(ii) $d_1(s)\Gamma D_2(t) + d_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$, are true,
then D_1 and D_2 are orthogonal on M.

Proof: By the condition (i) of the hypothesis, we have

$D_1(s)\Gamma D_2(t) + D_2(s)\Gamma D_1(t) = \{0\}$, for all $s, t \in M$. (3.15)

Replacing s by $u\alpha s$, $u \in M, \alpha \in \Gamma$ in (3.15), we get
$D_1(s)\alpha u\Gamma D_2(t) + D_2(s)\alpha u\Gamma D_1(t) + s\alpha d_1(u)\Gamma D_2(t) + s\alpha d_2(u)\Gamma D_1(t) = \{0\}$.

$D_1(s)\alpha u\Gamma D_2(t) + D_2(s)\alpha u\Gamma D_1(t) = \{0\}$. (Using condition (ii)) (3.16)

Replacing t by s in (3.16), we get $D_1(s)\alpha u\Gamma D_2(s) + D_2(s)\alpha u\Gamma D_1(s) = \{0\}$.
Using Lemma 1, we obtain $D_1(s)\Gamma M\Gamma D_2(s) = \{0\} = D_2(s)\Gamma M\Gamma D_1(s)$.
Using Lemma 2, we get $D_1(s)\Gamma M\Gamma D_2(t) = \{0\} = D_2(s)\Gamma M\Gamma D_1(t)$, for all $s, t \in M$.
Therefore, D_1 and D_2 are orthogonal.

Theorem 4: Suppose M is a 2-torsion-free semiprime Γ-semiring. Let D_1, D_2 be two generalized reverse derivations on M associated with reverse derivations d_1, d_2. If $D_1(s)\Gamma D_2(t)$ and $d_1(s)\Gamma D_2(t)$ are both zero for all $s, t \in M$, then D_1, D_2 are orthogonal. for all $s, t \in M$.

Proof: Given that $D_1(s)\Gamma D_2(t) = \{0\}$, for all$s, t \in M$. (3.17)

$d_1(s)\Gamma D_2(t) = \{0\}$, for all $s, t \in M$. (3.18)

Replacing s by $u\alpha s, u \in M, \alpha \in \Gamma$ in (3.17) and using (3.18), we get $D_1(s)\alpha u\Gamma D_2(t) = \{0\}$,

$D_1(s)\Gamma M\Gamma D_2(t) = \{0\}$, (3.19)

$D_2(t)\Gamma M\Gamma D_1(s) = \{0\}$. (By Lemma 1) (3.20)

Replacing t by s in (3.20), we get, $D_2(s)\Gamma M\Gamma D_1(s) = \{0\}$.

$D_2(s)\Gamma M\Gamma D_1(t) = \{0\}$, for all $s, t \in M$. (Using Lemma 2) (3.21)

From (3.19) and (3.21), we can conclude that D_1, D_2 are orthogonal.

Theorem 5: Suppose M is a 2-torsion-free semiprime Γ-semiring. Let D_1, D_2 be two generalized reverse derivations on M associated with reverse derivations d_1, d_2. If $D_1(s)\Gamma D_2(t) = \{0\}$, for all $s, t \in M$ and $d_1 D_2 = d_1 d_2 = 0$, then D_1, D_2 are orthogonal.

Proof: By hypothesis, we have $d_1 d_2 = 0$.
By Lemma 3, we have that d_1 and d_2 are orthogonal.

Hence $d_1(s)\Gamma M\Gamma d_2(t) = \{0\} = d_2(s)\Gamma M\Gamma d_1(t)$, for all $s, t \in M$. (3.22)

By using Lemma1, we can write

$d_1(s)\Gamma d_2(t) = d_2(s)\Gamma d_1(t) = \{0\}$, for all $s, t \in M$. (3.23)

It is also given that $d_1 D_2 = 0$. (by hypothesis)
$d_1 D_2(s\alpha t) = 0$, for all $s, t \in M, \alpha \in \Gamma$.
$d_1(s)\alpha D_2(t) + d_2(s)\alpha d_1(t) + s\alpha d_1 D_2(t) + d_1 d_2(s)\alpha t = 0$
Since $d_1 D_2 = d_1 d_2 = 0$ and using (3.23), we get

$d_1(s)\alpha D_2(t) = 0$, for all $s, t \in M$. (3.24)

$d_1(s)\Gamma D_2(t) = \{0\}$, for all $s, t \in M$.
Also, by the hypothesis, we have $D_1(s)\Gamma D_2(t) = \{0\}$, for all $s, t \in M$.
Thus, we have $D_1(s)\Gamma D_2(t) = \{0\} = d_1(s)\Gamma D_2(t)$, for all $s, t \in M$.
By Theorem 4, we can conclude that D_1 and D_2 are orthogonal.

4. Future Research

We can investigate how orthogonal generalized reverse derivations can classify semiprime Γ-semirings more comprehensively and explore applications in related algebraic structures such as prime Γ-semirings or near-rings to extend the theoretical framework. We can study the impact of orthogonal generalized reverse derivations on the lattice of ideals or submodules within Γ-semirings and analyze how these derivations influence specific algebraic properties such as commutativity, associativity or idempotency in Γ-semirings. We can extend the concept of orthogonality to other types of derivations such as symmetric or skew derivations and examine their roles in semiprime Γ-semirings. We can compare and contrast generalized reverse derivations with classical derivations in terms of their effects on the structure of the semiring. We can investigate whether orthogonal generalized reverse derivations have interpretations in terms of algebraic geometry or module representations of Γ-semirings. We can analyze the dynamic behaviour of sequences of orthogonal generalized reverse derivations and their limits within semiprime Γ-semirings and explore operator-theoretic interpretations of these derivations, potentially leading to applications in functional analysis. We can develop computational tools or algorithms to identify orthogonal generalized reverse derivations in Γ-semirings and investigate connections with combinatorial structures, such as graphs or networks, where these derivations could play a role. We can extend the study to Γ-semirings that are not semiprime, exploring whether weaker conditions still allow for meaningful definitions, results and analyze whether orthogonality in generalized reverse derivations holds any relevance or utility in non-semiprime settings. We can study the applications of these mathematical concepts in fields like cryptography, coding theory, or theoretical computer science, where semirings play a role.

References

Aboubakr, A., & Gonzalez, S. (2015). Generalized reverse derivations on semiprime rings. *Siberian Mathematical Journal, 56*(2), 199–205.

Anjum, R., Ullah, S., Chu, Y. M., Munir, M., Kausar, N., & Kadry, S. (2020). Characterizations of ordered h-regular semirings by ordered h-ideals. *American Institute of Mathematical Sciences, 2020*.

Bresar, M., & Vukman, J. (1989). On some additive mappings in rings with involution. *Aequationes Mathematicae, 38*, 178-185.

Herstein, I. N. (1957). Jordan derivations of prime rings. *Proceedings of the American Mathematical Society, 8*, 1104-1110.

Javed, M. A., Aslam, A., & Hussain, M. (2013). On derivations of prime Γ-semirings. *South Asian Bulletin of Mathematics, 37*, 859-865.

Jaya Subba Reddy, C., & Krishna Murty, V. S. V. (2024). Orthogonal symmetric reverse bi-(σ, τ)-derivations in semiprime rings. *Tuijin Jishu/Journal of Propulsion Technology, 45*(1), 5133-5138.

Kausar, N. (2019). Direct product of finite intuitionistic anti-fuzzy normal subrings over non associative rings. *European Journal of Pure and Applied Mathematics, 12*, 622-648.

Kausar, N., Alesemi, M., Salahuddin, & Munir, M. (2020). Study on LA-rings by their intuitionistic fuzzy ideals. *Mathematica Montisnigri, 47*, 22-42.

Kausar, N., Islam, B., Amjad, S., & Waqar, M. (2019). Intuitionistic fuzzy ideals with thresholds (α, β) in LA-rings. *European Journal of Pure and Applied Mathematics, 12*(3), 906-943.

Kausar, N., Islam, B., Javaid, M., Amjad, S., & Ijaz, U. (2019). Characterizations of non-associative rings by the properties of their fuzzy ideals. *Journal of Taibah University for Science, 13*, 820-833.

Kausar, N., Munir, M., Islam, B., Alesemi, M., Salahuddin, & Gulzar, M. (2020). Fuzzy bi-ideal in LA-rings. *Italian Journal of Pure and Applied Mathematics, 44*, 745-763.

Kausar, N., & Waqar, M. (2019). Direct product of finite fuzzy normal subrings over non associative rings. *International Journal of Analysis and Applications, 17*(5), 752-770.

Kausar, N., & Waqar, M. (2019). Characterizations of non-associative rings by their intuitionistic fuzzy bi-ideals. *European Journal of Pure and Applied Mathematics, 12*(1), 226-250.

Kim, K. (2022). On orthogonal reverse derivations of semiprime semirings. *Korean Journal of Mathematics, 30*(2), 341–349.

Kim, K. H. (2022). On orthogonal reverse derivations of semiprime Γ-semirings. *Journal of the Chungcheong Mathematical Society, 35*(2), 115-124.

Krishna Murty, V.S.V.,Chennakesavulu, K., & Jaya Subba Reddy, C. (2024). Orthogonal generalized symmetric reverse bi-(σ,τ)-derivations in semiprime rings. *Communications on Applied Nonlinear Analysis, 31*(3), 156-168.

Krishna Murty, V. S. V., & Jaya Subba Reddy, C. (2024). Orthogonality of generalized reverse (σ,τ)-derivations in semiprime Γ-rings. *Journal of Nonlinear Analysis and Optimization, 15*(4), 20-28.

Krishna Murty, V. S. V., & Jaya Subba Reddy, C. (2024). Orthogonal generalized symmetric reverse biderivations in semiprime rings. *Journal of Applied and Pure Mathematics, 6*(3-4), 155-165.

Krishna Murty, V. S. V., Jaya Subba Reddy, C., & Chennakesavulu, K. (2024). Orthogonal (σ, τ)-derivations in semiprime Γ-semirings. *International*

Journal of Mathematics Trends and Technology, 70(7), 13-17.

Krishna Murty, V. S. V., Jaya Subba Reddy, C., & Haseena, Sk. (2024). Orthogonal generalized (σ, τ)-derivations on an ideal of semiprime Γ-rings. *JP Journal of Algebra and Number Theory, 23*(6), 481-503.

Krishna Murty, V. S. V., Jaya Subba Reddy, C., & Sukanya, J. S. (2024). Orthogonal generalized (σ, τ)-derivations in semiprime Γ-near rings. *Advances in Mathematics: A Scientific Journal, 13*(3), 311-321.

Majeed, A. H., & Hamil, S. A. (2020). Orthogonal generalized derivations on Γ-semiring. *Journal of Physics: Conference Series, 1530*(1), 1-6.

Nobusawa, N. (1964). On a generalization of the ring theory. *Osaka Journal of Mathematics, 1*(1), 81-89.

Posner, E. C. (1957). Derivations in prime rings. *Proceedings of the American Mathematical Society, 8*, 1093–1100.

Rao, M. M. K. (1995). Γ-semirings-I. *Southeast Asian Bulletin of Mathematics, 19*(1), 49-54.

Rao, M. M. K. (1997). Γ-semirings-II. *Southeast Asian Bulletin of Mathematics, 21*, 281-287.

Samman, M., & Alaymani, N. (2007). Derivations and reverse derivations in semiprime rings. *International Mathematical Forum, 2*(39), 1895-1902.

Sen, M. K. (1981). On Γ-Semigroups. *Proceedings of International Conference on Algebra and its Applications* (*New Delhi, 1981*) (pp. 301-308). *Lecture Notes in Pure and Applied Mathematics,* New York, NY: Decker Publications.

Suganthameena, N., & Chandramouleeswaran, M. (2014). Orthogonal derivations on semirings. *International Journal of Contemporary Mathematical Sciences, 9*(13), 645-651.

Vandiver, H. S. (1934). Note on a simple type of algebra in which the cancellation law of addition does not hold. *Bulletin of the American Mathematical Society, 40*(12), 914-921.

Venkateswarlu, B., Rao, M. M. K., & Narayana, Y. A. (2018). Orthogonal derivations on Γ-semirings. *Bulletin of the International Mathematical Virtual Institute, 8*, 543-552.

Venkateswarlu, B., Rao, M. M. K., & Narayana, Y. A. (2019). Orthogonal reverse derivations on semiprime Γ-semirings. *Mathematical Sciences and Applications e-Notes, 7*(1), 71-77.