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Original Article

Residual-based MEWMA control charts in the presence of multicollinearity

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Abstract

Statistical Process Control has been performing a critical role in attaining quality assurance from historic times to the modern era. Examining and governing the process variables involves rigorous stages and several control charts. The multivariate process is considered for a more comprehensive understanding of handling multiple correlated variables of the process. The study here focuses on the unique creation and deployment of residual-based Multivariate Exponentially Weighted Moving Average control charts in the presence of multicollinearity, specially constructed and evaluated for Phase I and Phase II. The chart offers a reliable framework for understanding shifts in multivariate processes across time from minor to moderate changes in process parameters. Agro-Economy data of Indian States for the years 2019 and 2020 are utilized in an application example. The proposed residual-based MEWMA control charts detect out-of-control circumstances with few false alarms and this is critical for rapid interventions, resulting in optimal crop management and production.

Keywords: control charts, multivariate exponentially weighted moving average, phase I, phase II, quality control

1. Introduction

Industries frequently need to monitor many quality traits or factors at the same time to increase production and turnover. Incorporating theories and procedures of Statistical Process Control (SPC) with multivariate processes via the Multivariate Exponentially Weighted Moving Average (MEWMA) control charts has revolutionized the area of Statistical Quality Control (SQC). MEWMA charts help to evaluate the overall quality of a process by considering many factors simultaneously. In recent years, measuring and tracking several process variables at the same time has been monitored by MEWMA charts, making them appropriate for identifying changes across multiple dimensions. MEWMA charts provide improved management of correlation and sensitivity when compared to single-variate charts, expanding numerous sectors with operational excellence in multivariate processes. For the past three decades, MEWMA charts have gained favorable interest in detecting deviations and shifts in the process characteristics.

Lowry discussed the concept of MEWMA with two illustrative applications and charts, providing insights about the detection of out-of-control signals (Lowry, Woodall, Champ, & Rigdon, 1992). This was followed by the regression adjustments process in MEWMA charts (Hawkins, 1993). Wierda explored the advances in multivariate SPC such as CUSUM charts, multivariate charts, and capacity indices, emphasizing theoretical gaps and the importance of identifying factors that cause out-of-control signals (Wierda, 1994) and, during the same period Lowry published a detailed survey of multivariate control charts contributing to the spread of these ideas for multivariate processes (Lowry & Montgomery, 1995). MEWMA charts for detecting smaller shifts were further demonstrated by (Prabhu & Runger, 1997) whereas Stoumbos devised robustness against non–normality in the same chart

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(Stoumbos, & Sullivan, 2002). Yeh and his co-researchers proved the effectiveness of the combined MEWMA and EWMA–V charts regarding the process mean and variability (Yeh, Lin, Zhou, & Venkataramani, 2003). Slightly altering the EWMA of each observation and its transposition, Yeh and his team further proposed MEWMA when the variables are highly correlated (Yeh, Huwang, & Wu, 2005). Extending this version, Zhang *et al*. identified a rapid detection of mean, variance, or both shifts in terms of correct detection percentages using the multiple testing procedure (Zhang & Chang, 2008). Park and Jun (2015) facilitated a new MEWMA for every mean shift to improve the identification of tiny alterations in multivariate systems. In the presence of a gamma model for performance of the MEWMA charts, Flury and Quaglino (2018) obtained control limits with known parameters. Adaptive MEWMA charts were used by Haq and Khoo (2019) for monitoring process mean in a normally distributed procedure. By identifying a robust MEWMA with dispersion when the covariance matrix decreases, their research focuses on monitoring process variability using individual observations. However, the suggested chart also insisted on monitoring aggregated data by including the sample covariance matrix in the EWMA statistic (Ajadi, Zwetsloot, & Tsui, 2021). Nidsunkid and Yeesa (2023) analyzed the MEWMA chart when the normality assumption is violated, and this assumption has a significant impact on multivariate control charts intended for quick detection.

Based on our literature survey, addressing the residual-based MEWMA control charts in the presence of multicollinearity provides a potential for future study, notably in creating and assessing strategies for dealing with multicollinearity within residual-based MEWMA frameworks. Traditional MEWMA charts might overlook some minor deviations; and yet, significant shifts with respect to process variations and averages, in the presence of multicollinearity, will be a challenge for the monitoring. The motivation to propose the chart is to improve the ability of early detection and be consistent in tracking multivariate processes, and the proposed chart minimizes the impact of multicollinearity by using residuals of biased estimators resulting in higher quality, so that the two phases in process control are further strengthened. This technique helps in early detection, producing a practical solution to intricate situations.

This article proposes unique residual-based Multivariate Exponentially Weighted Moving Average charts (MEWMA) in the presence of multicollinearity. The adaptability of MEWMA from sensitive to minor shifts helps to enhance measuring and variation evaluations from conventional techniques, making it a robust chart with the hybrid approach in the context of multicollinearity. The performance of the residual-based MEWMA charts is measured using the Average Run Length (AvgRL) and further justified with an application in the field of Agronomy of Indian states and union territories for two successive years.

The article is laid out as follows: Section 2 describes the model methodology, followed by Section 3 briefly outlining the proposed design of the residual-based MEWMA charts in the presence of multicollinearity. An application is illustrated in Section 4 to facilitate understanding the proposed charts. Lastly, the inference is highlighted in Section 5.

2. Model Methodology

A multiple linear regression model in the general form is denoted as:

$$X_i = \gamma_0 + \gamma_1 R_{i1} + \gamma_2 R_{i2} + \cdots + \gamma_q R_{iq} + \epsilon_i, i = 1,2,\ldots,m \quad (1)$$

$where, E[\epsilon_i] = 0, Var[\epsilon_i] = \sigma^2\ i.e., \epsilon_i \sim N(0,\sigma^2).$

X_i refers to the response variable of size $(m \times 1)$, R of size $(m \times q)$ has q regressors and, γ is the unknown regression coefficient with order $(q \times 1)$. The variables are standardized to estimate the $(q\times1)$ vector γ of regression coefficients. The multiple linear regression model in equation (1) can be written in matrix form as:

$$X = R\gamma + \epsilon \quad (2)$$

$$i.e., \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} 1 & r_{11} & r_{12} & \cdots & r_{1q} \\ 1 & r_{21} & r_{22} & & r_{2q} \\ & \vdots & & \ddots & \vdots \\ 1 & r_{m1} & r_{m2} & \cdots & r_{mq} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_q \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix}$$

$R'R$ is the correlation matrix of size $q \times q$ and $R'X$ is the vector of the correlation coefficients. Assumptions are crucial during the analysis of regression; in case the assumptions are violated the model becomes unfit and unreliable in estimating the parameters. These assumptions may include non-linearity, autocorrelation, homoscedasticity, normality, and multicollinearity. There always exists a correlation between the response and the explanatory variables in a regression model. The study presented here examines the case when the multiple independent variables in a regression model are correlated leading to multicollinearity. A small to high degree of multicollinearity can cause significant issues. To address this, various techniques are employed. One such technique that is often used to detect multicollinearity is the Variance Inflation Factor (VIF). $VIF = \frac{1}{Tolerance} = \frac{1}{1-p_i^2}$, p_i^2 is the unadjusted coefficient of determination for predicting the i[th] independent variable on the remaining ones. During the absence of multicollinearity, the ordinary least square estimator will be applied to estimate the parameters; but since the interest of the study is concerning multicollinearity, we further proceed with the estimators that help us to overcome the issue of multicollinearity. To address this issue, several estimators such as Principal component, ridge regression, (r-k) class estimator, etc. are used to estimate the coefficients of the regression model, which stabilize and improve regression model dependability. The functional form of these estimators depends on the shrinkage parameter K which was further estimated by various researchers to enhance the performance of the regression model. Based on a few inspections and extensive reviews it was found that the estimator proposed by Azar and Genc (2017) for the shrinkage parameter gives the minimum expected mean squared error and performs well for strong multicollinearity. and it is utilized in this study. It is given by $K_{A\&G} = maximum\sqrt{\frac{\hat{\sigma}^2}{\hat{\gamma}^2}}$, where $\hat{\gamma} = \lambda^{-1} R'X$. Further, this $K_{A\&G}$ estimator is used to estimate the regression coefficients using the ridge and (r-k) class estimators. The ridge estimator $(\hat{\gamma}_r)$ proposed by Hoerl and Kennard (1970), and (r-k) class estimator ($\widehat{\gamma}_{rk}$) was proposed by Baye and Parker (1984) combining PCR and ridge estimator, are given by:

$$\widehat{\gamma}_r=(R^{'}R + K_{A\&G}\ I_q)^{-1}\ R^{'}X, \quad (3)$$

$$\widehat{\gamma}_{rk}=P_c(P_c^{'}R^{'}RP_c + K_{A\&G}I_q)^{-1}\ P_c^{'}R^{'}X \quad (4)$$

where, $P_c = P_1, P_2, \ldots, P_q$ is an orthogonal matrix with principal components as columns, I_q denotes the identity matrix and $P_c^{'}R^{'}RP_c = \xi_c = \text{diag}(\lambda_1, \lambda_2, .., \lambda_c)$, $\lambda_1, \lambda_2, .., \lambda_c$ are the eigenvalues of matrix $R^{'}R$. Further, the regression models are estimated using ridge and (r-k) class estimators and the residual-based MEWMA charts have been proposed in this article.

3. Proposed Residual -Based MEWMA Charts

3.1 Layout for the proposed residual-based MEWMA charts

To enhance the flow of the study and to visualize the process of residual-based MEWMA charts in two phases, a layout is constructed and is represented in Figure 1.

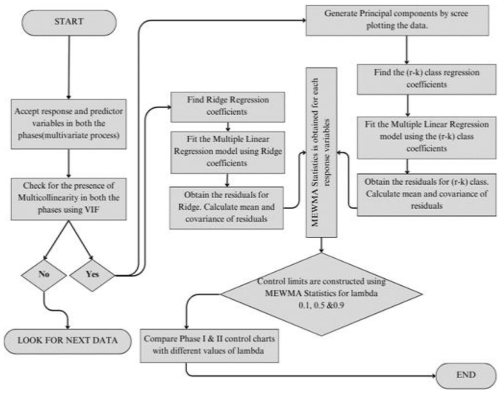


Figure 1. Residual-based MEWMA control charts layout

3.2 Proposed residual-based MEWMA charts

A statistical method for examining the association between several independent variables and one dependent variable is the multiple linear regression model and it takes into account two or more predictors, in contrast to basic linear regression.

The multivariate regression is given by,

$$X_i = \gamma R_i + \epsilon_i \quad (5)$$

with process mean vector μ_r and variance covariance matrix Σ of correlated variables i.e., $R_i \sim N_q(\mu, \Sigma)$, and $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1q} \\ \vdots & & \ddots & \vdots \\ \sigma_{q1} & \sigma_{q2} & \cdots & \sigma_{qq} \end{pmatrix}$. The idea here is to propose residual-based MEWMA control charts using ridge and (r-k) class estimators that account for multicollinearity to monitor process variability. The MEWMA charts of Lowry (1992) are an extension of the EWMA charts (Robert 1959) for detecting smaller shifts, based on the EWMA statistic given as,

$$Z_i = \xi R_i + (1-\xi)Z_{i-1}, \mathit{for}\ i = 1,2, \ldots, m \quad (6)$$

where, $Z_1 = 0$, and $0 < \xi \le 1$. To summarize, $\xi = \lambda_1, \lambda_2 \ldots, \lambda_q$ are the individual smoothing constants for each of the q variables. These constants regulate the amount of smoothing applied to each variable. A low λ number (near 0) indicates that previous observations are weighted higher, lowering the chart's sensitivity to current changes. A larger λ value (near 1) emphasizes current data, making the graphic more responsive to process changes. In a multivariate situation, the vector of EWMA's is as follows:

$$Z_i = \wedge R_i + (1-\wedge)Z_{i-1}, \mathit{for}\ i = 1,2, \ldots, m \quad (7)$$

where, $Z_1 = 0$, and $\wedge = \xi_1, \xi_2, \ldots, \xi_q$,($0 < \xi_j \le 1), j = 1,2, .., q$. The MEWMA control charts signal is obtained as:

$$MU_i^2 = Z_i^{'}\Sigma_{zi}^{'}Z_i\ \mathit{and}\ SD(\ MU_i^2) = \Sigma_{z_i} = \frac{\xi}{(2-\xi)}\Sigma \quad (8)$$

where, Σ_{z_i} is the variance-covariance matrix of Z_i (Mahmoud *et al*. 2010).

To propose the M-EWMA charts instead of considering the out-of-signal $UCL = H > 0$ (always generated using simulation), the current study uses residuals MU_i^2 calculated using the ridge and r-k class estimators. Hence, the study presents an adaptive structure of EWMA limits to frame the M-EWMA control charts, to detect shifts via the Multivariate Exponentially Weighted Moving Average (MEWMA) control charts. The beginning value $Z_i = Z_1 = 0$ is set to state that the process is in control and to initialize the control chart for the monitoring process. With the univariate EWMA framework, though the control limits' form is consistent, the monitoring statistic is multivariate and residual-based to maintain the recursive nature and memory features. The limits offer a tractable, non-simulation-based technique for identifying process behaviour abnormalities in linked variables. By employing estimator-specific residuals, the adaption expands the use of EWMA limits to multivariate, multicollinearity contexts while maintaining their well-known interpretability. Without the computational strain of simulation to ascertain the crucial value H, this architecture enables practical implementation. Now, the proposed Upper Control Limit (UCL), Center Line (CL) and Lower Control Limit (LCL) for residual-based MEWMA statistics in the presence of multicollinearity are:

$$LCL = \mu_{err} - L\sigma_{err}\sqrt{\frac{\wedge}{(2-\wedge)}[1-(1-\wedge)^{2i}]} \quad (9)$$

$$CL = \mu_{err} \quad (10)$$

$$UCL = \mu_{err} + L\sigma_{err}\sqrt{\frac{\wedge}{(2-\wedge)}[1-(1-\wedge)^{2i}]} \quad (11)$$

Here, $\mu_{err} = mean(MU_i^2)$, $\sigma_{err} = SD(MU_i^2)$ and L represents the various sigma limits $i.e., 1\sigma, 2\sigma\ or\ 3\sigma$ respectively.

Further, the performance of the proposed control chart is evaluated with the Average Run Length (AvgRL) which measures the number of observations to signal an out-of-control process i.e.,

$$AvgRL = \frac{1}{Raise\ of\ false\ alarm} \quad (12)$$

To make the performance more applicable, AvgRL is transformed to

$$AvgRL = \frac{No\ of\ runs\ above\ UCL\ or\ below\ LCL}{Total\ no\ of\ observations} \quad (13)$$

By proposing the MEWMA charts and checking their performances using AvgRL, the study presents unique MEWMA control charts to monitor the process variability in the presence of multicollinearity using regression models.

4. Application of the Proposed Residual-Based MEWMA Control Charts with Multicollinearity

To demonstrate the proposed residual-based MEWMA charts with multicollinearity and to monitor the quality control process, the Indian Agro-Economy data of Indian States/Union territories are analyzed. These data are taken from the RBI website (www.rbi.org.in). The per hectare consumption of fertilizer (CFR) such as Nitrogen(N), Phosphorus(P), and Potassium(K) (N+P+K) is assumed to be the response variable. The explanatory variables are taken as the production of food grains (PFG), Area of Food grains grown (AFG), gross irrigated area (GIA), net irrigated area (NIA) and, cropping intensity (CI) for two subsequent years 2019 and 2020 as Phase I & II data respectively. Since all the States/Union territories do not produce all kinds of crops, crops that are common for all states are taken into consideration, so we have a count of 27 States/Union territories under consideration. The entire study is completed in two phases, Phase I for 2019 data and Phase II for 2020 data respectively. Multicollinearity is inspected for both phases, and in the presence of multicollinearity, ridge and (r-k) class estimators are fitted to construct the proposed residual-based MEWMA charts for monitoring despite multicollinearity. The research also extends prior work by using 3 different ∧ values, and performance is evaluated using the Average run length for two different ∧ values to monitor the moderate and high sensitivity of consumption of fertilizers.

4.1 Phase I and II: Residual-based MEWMA charts with ridge and (r-k) class estimators

From equation (1) we obtain the regression equation for the year 2019 data set.

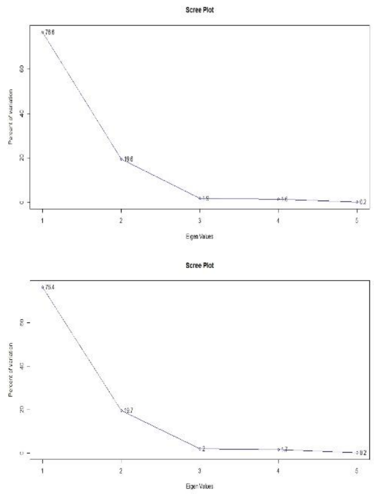
i.e., $$CFR_{(2019\,\&\,2020)} = \hat{\gamma}_1\ PFG + \hat{\gamma}_2\ AFG + \hat{\gamma}_3\ GIA + \hat{\gamma}_4\ NIA + \hat{\gamma}_5\ CI$$

After standardization of the variables. Multicollinearity is tested for the dataset using VIF. VIF for phase I is 14.53, 9.32, 49.49, 35.99, 1.09, and for phase II it is 13.30, 10.35, 53.32, 43.49, 1.10, respectively. These values indicate the presence of multicollinearity, since 4 regressors' are above 5 in both phases. By scree plotting the eigenvalues shown in Figures 2 (a) and (b) against the percentage of variation, we analyze the principal components, and the first 3 components of phases I & II contribute 98 % of the variation; hence P_c for both phases retain only the first 3 principal components.

Using equations 3 and 4, we obtain $\hat{\gamma}_r$ and $\hat{\gamma}_{rk}$ coefficients and fit equation (1) to obtain the ridge and (r-k) class estimator regression models. Table 1 presents the $\hat{\gamma}_r$ and $\hat{\gamma}_{rk}$ coefficients for both phases.

After fitting the multiple regression model for the multivariate process, we obtain the MEWMA statistic using equation (7). μ_{err} and σ_{err} are calculated to construct the proposed control limits equations (9), (10), and (11) that are used for three different values of ∧ , for ridge and (r-k) class estimators in phase I & II. Tables 2 and 3 present the Phase I MEWMA statistics along with the control limits for 3 different ∧ values, followed by residual-based MEWMA control charts for Phase I using ridge and (r-k) class estimators, and these are visualized in Figure 3. Similarly, the Phase II MEWMA statistics along with the control limits for 3 different ∧ values are constructed, followed by residual-based MEWMA control charts for Phase II using ridge and (r-k) class estimators that are visualized in Figure 4.

After obtaining the control limits, control charts are plotted for MEWMA for the initial phase and monitoring phase with smoothing parameters $\wedge = 0.1, 0.5,$ and 0.9 respectively. For both the Phases MEWMA statistics along with the control limits for 3 different ∧ values are constructed and are visualized in Figures 3 and 4 respectively. The chart supports the policy makers in making appropriate decisions for the geographic evaluation of per hectare consumption of fertilizer among states/union territories, better in comparison to tracking fluctuations over time in a conventional MEWMA.



(a)

(b)

Figure 2. (a) Scree plot for initial phase (b) scree plot for monitoring phase

Table 1. $\hat{\gamma}_r$ and $\hat{\gamma}_{rk}$ coefficients for phases I & II

Coefficient	$\hat{\gamma}_r$		$\hat{\gamma}_{rk}$	
	Phase I	Phase II	Phase I	Phase II
$\hat{\gamma}_1$	0.889	0.803	0.631	0.233
$\hat{\gamma}_2$	-0.449	-0.461	-0.761	0.195
$\hat{\gamma}_3$	0.664	0.602	0.395	-0.051
$\hat{\gamma}_4$	-0.792	-0.660	0.036	-0.099
$\hat{\gamma}_5$	0.233	0.173	0.239	0.270

Table 2. Residual-based MEWMA statistics and their control limits using ridge estimator in phase I for various ʌ values

State/Union territory	ʌ= 0.1				ʌ= 0.5				ʌ= 0.9			
	MEWMA	UCL	CL	LCL	MEWMA	UCL	CL	LCL	MEWMA	UCL	CL	LCL
Andhra Pradesh	0.2	0.7	0.7	0.6	0.9	1.9	0.9	0.0	1.7	4.2	1.0	-2.3
Assam	0.2	0.8	0.7	0.6	0.8	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Bihar	0.5	0.8	0.7	0.5	2.1	2.1	0.9	-0.2	3.1	4.2	1.0	-2.3
Chhattisgarh	0.5	0.8	0.7	0.5	1.1	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
NCT of Delhi	0.5	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.0	4.2	1.0	-2.3
Goa	0.5	0.8	0.7	0.5	0.7	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Gujarat	0.5	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Haryana	0.5	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
Himachal Pradesh	0.5	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Jammu & Kashmir	0.6	0.8	0.7	0.5	0.9	2.1	0.9	-0.2	1.2	4.2	1.0	-2.3
Jharkhand	0.6	0.8	0.7	0.5	0.9	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Karnataka	0.6	0.8	0.7	0.5	0.7	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
Kerala	0.6	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.9	4.2	1.0	-2.3
Madhya Pradesh	0.6	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.4	4.2	1.0	-2.3
Maharashtra	0.6	0.8	0.7	0.5	0.4	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Manipur	0.5	0.8	0.7	0.5	0.3	2.1	0.9	-0.2	0.2	4.2	1.0	-2.3
Mizoram	0.6	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	1.1	4.2	1.0	-2.3
Odisha	0.6	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Puducherry	1.2	0.8	0.7	0.5	3.5	2.1	0.9	-0.2	5.9	4.2	1.0	-2.3
Punjab	1.1	0.8	0.7	0.5	1.8	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Rajasthan	1.0	0.8	0.7	0.5	1.0	2.1	0.9	-0.2	0.2	4.2	1.0	-2.3
Tamil Nadu	0.9	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.6	4.2	1.0	-2.3
Telangana	0.9	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Tripura	1.1	0.8	0.7	0.5	1.6	2.1	0.9	-0.2	2.2	4.2	1.0	-2.3
Uttar Pradesh	1.0	0.8	0.7	0.5	1.3	2.1	0.9	-0.2	1.1	4.2	1.0	-2.3
Uttarakhand	1.0	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.4	4.2	1.0	-2.3
West Bengal	0.9	0.8	0.7	0.5	0.4	2.1	0.9	-0.2	0.1	4.2	1.0	-2.3

Table 3. Residual-based MEWMA statistics and their control limits using (r-k) class

State/Union territory	ʌ= 0.1				ʌ= 0.5				ʌ= 0.9			
	MEWMA	UCL	CL	LCL	MEWMA	UCL	CL	LCL	MEWMA	UCL	CL	LCL
Andhra Pradesh	0.2	0.7	0.7	0.6	0.9	1.9	0.9	0.0	1.7	4.2	1.0	-2.3
Assam	0.2	0.8	0.7	0.6	0.8	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Bihar	0.5	0.8	0.7	0.5	2.1	2.1	0.9	-0.2	3.1	4.2	1.0	-2.3
Chhattisgarh	0.5	0.8	0.7	0.5	1.1	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
NCT of Delhi	0.5	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.0	4.2	1.0	-2.3
Goa	0.5	0.8	0.7	0.5	0.7	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Gujarat	0.5	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Haryana	0.5	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
Himachal Pradesh	0.5	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Jammu & Kashmir	0.6	0.8	0.7	0.5	0.9	2.1	0.9	-0.2	1.2	4.2	1.0	-2.3
Jharkhand	0.6	0.8	0.7	0.5	0.9	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Karnataka	0.6	0.8	0.7	0.5	0.7	2.1	0.9	-0.2	0.5	4.2	1.0	-2.3
Kerala	0.6	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.9	4.2	1.0	-2.3
Madhya Pradesh	0.6	0.8	0.7	0.5	0.6	2.1	0.9	-0.2	0.4	4.2	1.0	-2.3
Maharashtra	0.6	0.8	0.7	0.5	0.4	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Manipur	0.5	0.8	0.7	0.5	0.3	2.1	0.9	-0.2	0.2	4.2	1.0	-2.3
Mizoram	0.6	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	1.1	4.2	1.0	-2.3
Odisha	0.6	0.8	0.7	0.5	0.5	2.1	0.9	-0.2	0.3	4.2	1.0	-2.3
Puducherry	1.2	0.8	0.7	0.5	3.5	2.1	0.9	-0.2	5.9	4.2	1.0	-2.3
Punjab	1.1	0.8	0.7	0.5	1.8	2.1	0.9	-0.2	0.7	4.2	1.0	-2.3
Rajasthan	1.0	0.8	0.7	0.5	1.0	2.1	0.9	-0.2	0.2	4.2	1.0	-2.3
Tamil Nadu	0.9	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.6	4.2	1.0	-2.3
Telangana	0.9	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.8	4.2	1.0	-2.3
Tripura	1.1	0.8	0.7	0.5	1.6	2.1	0.9	-0.2	2.2	4.2	1.0	-2.3
Uttar Pradesh	1.0	0.8	0.7	0.5	1.3	2.1	0.9	-0.2	1.1	4.2	1.0	-2.3
Uttarakhand	1.0	0.8	0.7	0.5	0.8	2.1	0.9	-0.2	0.4	4.2	1.0	-2.3
West Bengal	0.9	0.8	0.7	0.5	0.4	2.1	0.9	-0.2	0.1	4.2	1.0	-2.3

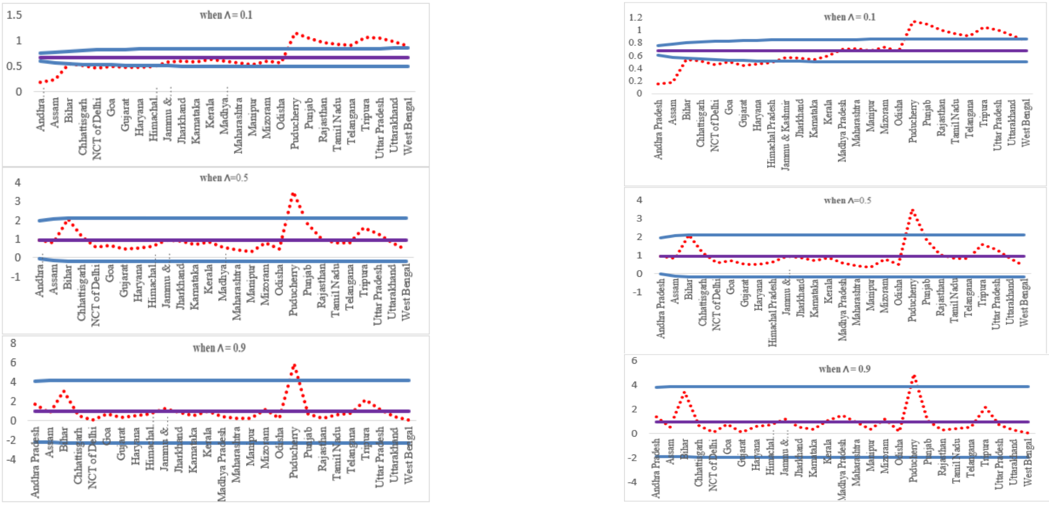


Figure 3. Residual-based MEWMA control chart for phase I using ridge and (r-k) class estimators

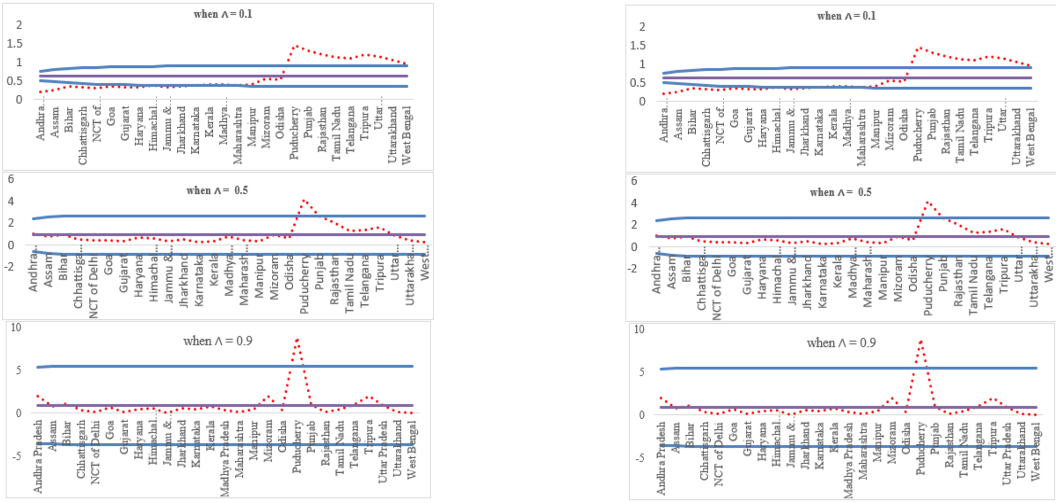


Figure 4. Residual-based MEWMA control chart for phase II using ridge and (r-k) class estimators

Here, X-axis denotes States /Union Territories (not tracking process over time but instead comparing multiple entities at a single point or aggregate level) & the Y-axis denotes the MEWMA statistic.

The average run length metric in equation 10 is used in the chart performance evaluations. AvgRL is demonstrated in Table 4 and a comparative study is done to check the residual-based MEWMA chart's consistency and adaptability. Here we consider the AvgRL values when $\wedge = 0.5$ *and* $\wedge = 0.9$, since $\wedge = 0.1$ gives more weightage to older data.

5. Inference

5.1 Phase I and II breakthroughs

AvgRL when $\wedge = 0.9$ is lower than with $\wedge = 0.5$, indicating that the proposed residual-based MEWMA charts detect process shift more quickly in both of the estimators similarly when $\wedge = 0.5$, AvgRL is comparatively lower indicating that the proposed residual-based MEWMA charts detect process shift in both the estimators with the average value. Consistent performance is expected for both phases due to similar values.

Table 4. Average run length for phases I and II

AvgRL	Ridge estimator		(r-k) class Estimator	
	$\wedge = 0.5$	$\wedge = 0.9$	$\wedge = 0.5$	$\wedge = 0.9$
Phase I	0.074	0.037	0.074	0.037
Phase II	0.037	0.037	0.037	0.037

5.2 Phase I and II monitoring

Λ values are not significantly affecting the residual-based MEWMA charts. Phase I monitoring of Puducherry indicates significant variability or a process shift, interpreting deviations and fluctuations in the process of the state. Identifying the out-of-control signal in the per-hectare consumption of fertilizers has increased variability in the form of shifts of mean or variance factors. A closer examination and corrective measures might solve the issue. Since in both phases, Puducherry has exceeded the UCL, a clear factor has been out of control and needs a quick remedy for the union territory. Sustainable practices, resource allocation, and policy formulation have to be looked upon seriously concerning agricultural practices. Areas for future study could be approached by including several alternative estimators.

By closely monitoring stakeholders, the proposed charts can improve resource management and enhance productivity, and decrease the use of fertilizers while maintaining quality standards. The findings show that the residual-based MEWMA control charts are successful in detecting process shifts even when multicollinearity exists. Our solution incorporates a two-phase architecture, allowing for the initial phase of estimation and the succeeding phase of monitoring, which improves the control charts' sensitivity and accuracy. More research and corrective measures are required to address these discrepancies and restore the border boundaries. The findings are supported by the following web source published on 2 April 2021, in THE HINDU businessline;https://www.thehindubusinessline.com/economy/agri-usiness/puducherry-telangana-punjab-top-in-fertilizer-consumption/article34224656.ece).

The proposed residual-based MEWMA chart can be extended to high-dimensional and nonlinear situations. To do away with simulation-based thresholds, efforts might also be focused on obtaining analytical expressions for ARL. The goal of these initiatives is to increase MEWMA charts' durability and usefulness in contemporary, data-rich settings.

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