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Original Article

Intuitionistic fuzzy α* closed sets

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Abstract

On the basis of α fuzzy closed sets and intuitionistic fuzzy sets, the conception of intuitionistic fuzzy α* closed sets is introduced in intuitionistic fuzzy topological spaces. Here, we have deliberated the relations between the newly introduced set with other intuitionistic fuzzy closed sets and looked into their properties. In addition, we have obtained a few interesting propositions.

Keywords: intuitionistic fuzzy topology, intuitionistic fuzzy sets, intuitionistic fuzzy closed sets, intuitionistic fuzzy α* closed sets

1. Introduction

Fuzzy sets, sets with a smooth boundary, were brought in by Zadeh (1965) as an extension of the classical notion of a set. The fuzzy notion has been applied in all branches of mathematics. Chang (1968) introduced and developed fuzzy topology. After that, the concept of intuitionistic fuzzy sets was introduced by Atanassov (1986), as an extension of fuzzy sets, which has both membership and non-membership degree. In the last two decades, various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. Coker (1997) founded the conceptualization of intuitionistic fuzzy topological space. In topological space, Hatir, Noiri, and Yuksel (1996) introduced α* sets and spoke of their properties. Here, we have found a set in intuitionistic fuzzy topological spaces, as an extension of α* set, named intuitionistic fuzzy α* closed sets. We have examined several properties of this set type and acquired some propositions in suitable instances. The significance of a set can be identified if it is applied to real-life problems and an appropriate solution is found. Hence, the proposed set can be used in various applications like pattern recognition, decision-making problems, and image processing. In particular, the set itself can be used in decision-making problems, whereas continuity and connectedness can be used in image processing.

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2. Preliminaries

Definition 2.1 (Atanassov, 1986) An intuitionistic fuzzy set (IFS) T is

$$\mathrm{T} = \{\langle k, \mu_{\mathrm{T}}(k), \nu_{\mathrm{T}}(k)\rangle : k \in \mathrm{K}\},$$

where μ_{T} and ν_{T} are functions and $\mu_{\mathrm{T}} : \mathrm{K} \to [0,1]$ and $\nu_{\mathrm{T}} : \mathrm{K} \to [0,1]$ denote the degree of membership ($\mu_{\mathrm{T}}(k)$) and the degree of non-membership ($\nu_{\mathrm{T}}(k)$) of each element $k \in \mathrm{K}$ in the set T, respectively, and $0 \le \mu_{\mathrm{T}}(k) + \nu_{\mathrm{T}}(k) \le 1$ for each $k \in \mathrm{K}$. In K, an intuitionistic fuzzy set T is represented as $\mathrm{T} = \langle k, \mu_{\mathrm{T}}, \nu_{\mathrm{T}}\rangle$ instead of $\mathrm{T} = \{\langle k, \mu_{\mathrm{T}}(k), \nu_{\mathrm{T}}(k)\rangle : k \in \mathrm{K}\}$.

Definition 2.2 (Atanassov, 1986) Let T and V be two IFSs in K where $\mathrm{T} = \langle k, \mu_{\mathrm{T}}, \nu_{\mathrm{T}}\rangle$ and $\mathrm{V} = \langle k, \mu_{\mathrm{V}}, \nu_{\mathrm{V}}\rangle$, then

(i) $\mathrm{T} \subseteq \mathrm{V} \Leftrightarrow \mu_{\mathrm{T}} \le \mu_{\mathrm{V}}$ and $\nu_{\mathrm{T}} \ge \nu_{\mathrm{V}}$
(ii) $\mathrm{T} = \mathrm{V} \Leftrightarrow \mathrm{T} \subseteq \mathrm{V}$ and $\mathrm{T} \supseteq \mathrm{V}$
(iii) $\mathrm{T}^c = \langle k, \nu_{\mathrm{T}}, \mu_{\mathrm{T}}\rangle$
(iv) $\mathrm{T} \cup \mathrm{V} = \langle k, \mu_{\mathrm{T}} \cup \mu_{\mathrm{V}}, \nu_{\mathrm{T}} \cap \nu_{\mathrm{V}}\rangle$
(v) $\mathrm{T} \cap \mathrm{V} = \langle k, \mu_{\mathrm{T}} \cap \mu_{\mathrm{V}}, \nu_{\mathrm{T}} \cup \nu_{\mathrm{V}}\rangle$

The whole set K and the empty set are respectively the IFSs $1_{\sim} = \langle k,1,0\rangle$ and $0_{\sim} = \langle k,0,1\rangle$.

Definition 2.3 (Coker, 1997) An ***intuitionistic fuzzy topology*** (IFT) on K is a family ζ of IFSs in K fulfilling these conditions:

- $0_{\sim}, 1_{\sim} \in \zeta$,
- $H_1 \cap H_2 \in \zeta$ for any $H_1, H_2 \in \zeta$,
- $\cup H_i \in \zeta$ for any family $\{H_i : i \in J\} \in \zeta$

where (K,ζ) is an *intuitionistic fuzzy topological space* (IFTS) and all IFS in ζ is an *intuitionistic fuzzy open set* (IFOS) in K. The complement (T^c) of T in (K,ζ) is termed as an *intuitionistic fuzzy closed set* (IFCS) in (K,ζ).

Definition 2.4 (Thakur & Chaturvedi, 2006) The necessary and sufficient condition for two IFSs T and V to be *not q-coincident* ($T_{\bar{q}}V$) is $T \subseteq V^c$.

Definition 2.5 (Santhi & Jayanthi, 2012) In an IFTS (K,ζ), an IFS T is known to be an *intuitionistic fuzzy Q-set* (IFQ-set) if cl(int(T)) = int(cl(T)).

Definition 2.6 (Coker, 1997) If T = $\langle k, \mu_T, \nu_T \rangle$ is an IFS in an IFTS (K,ζ), then the *IF closure and IF interior* are denoted by
cl(T) = ⋂{ Y / Y is an IFCS in K and T ⊆ Y},
int(T) = ∪{ P / P is an IFOS in K and P ⊆ T}.
(i) cl(T^c) = $(\text{int}(T))^c$ (ii) int(T^c) = $(\text{cl}(T))^c$

Definition 2.7 (Thakur & Dhavaseelan, 2015) An IFS T = $\langle k, \mu_T, \nu_T \rangle$ in an IFTS (K,ζ) is an *IF nowhere dense set* if there exists no IFOS U such that U ⊆ cl(T) so that int(cl(T)) = $0_\sim$.

Result 2.8 (Thakur & Dhavaseelan (2015) Let T = $\langle k, \mu_T, \nu_T \rangle$ be an IFS in an IFTS (K,ζ). If T is an *IF nowhere dense set* in K, then int(T) = $0_\sim$.

Definition 2.9 An IFS T = $\langle k, \mu_T, \nu_T \rangle$ in an IFTS (K,ζ) is an
(i) IFαCS (Jeon, Jun, & Park, 2005) if cl(int(cl(T))) ⊆ T
(ii) IFRCS (Jeon *et al*., 2005) if cl(int(T)) = T
(iii) IFPCS (Jeon *et al*., 2005) if cl(int(T)) ⊆ T
(iv) IFβCS (Jeon *et al*., 2005) if int(cl(int(T))) ⊆ T
(v) IFSCS (Jeon *et al*., 2005) if int(cl(T)) ⊆ T
(vi) IFγCS (Hanafy, 2009) if cl(int(T)) ∩ int(cl(T)) ⊆ T
(vii) IFSPCS (Jun & Song, 2005) if there exists an IFPCS V such that int(V) ⊆ T ⊆ V

Definition 2.10 (Coker, 1997) The IFSs T and V in an IFTS (K,ζ) satisfy the following claims:
- int(T) ⊆ T ⊆ cl(T)
- T ⊆ V ⇒ cl(T) ⊆ cl(V) and int(T) ⊆ int(V)
- int(int(T)) = int(T) and cl(cl(T)) = cl(T)
- int(T ∩ V) = int(T) ∩ int(V)
- cl(T ∪ V) = cl(T) ∪ cl(V)
- int($1_\sim$) = $1_\sim$ and cl($0_\sim$) = $0_\sim$

Result 2.11 (Annalakshmi & Chandramouleeswaran, 2015) The IFSs T and V in an IFTS (K,ζ) satisfy the conditions:
(i) int(T ∪ V) ⊇ int(T) ∪ int(V)
(ii) cl(T ∩ V) ⊆ cl(T) ∩ cl(V)

3. Intuitionistic Fuzzy α* Closed Sets

We have innovated intuitionistic fuzzy α* closed set (IFα*CS) and acquired the inter-connection amidst IFα*CS and already existing intuitionistic fuzzy closed sets in IFTS.

Definition 3.1 An IFS T of an IFTS (K,ζ) is said to be an *intuitionistic fuzzy α* closed set* (IFα*CS) if cl(int(cl(T))) = cl(T) whenever T ⊆ L and L is an IFOS in (K,ζ).

Example 3.2 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K, where $H_1 = \langle k, (0.5_e, 0.4_f), (0.5_e, 0.6_f) \rangle$, $H_2 = \langle k, (0.8_e, 0.6_f), (0.2_e, 0.4_f) \rangle$ then (K,ζ) is an IFTS. The IFS $T = \langle k, (0.4_e, 0.3_f), (0.6_e, 0.7_f) \rangle$ in K is an IFα*CS.

Proposition 3.3 In (K,ζ), every IFRCS is an IFα*CS while the converse is untrue.

Proof: Given: T is an IFRCS in (K,ζ). IFRCS ⇒ IFCS.
Consider, cl(int(cl(T))) = cl(int(T)) = T = cl(T). Hence, cl(int(c(T))) = cl(T) and T is an IFα*CS in (K,ζ).

Example 3.4 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.5_e, 0.4_f), (0.5_e, 0.6_f) \rangle$, $H_2 = \langle k, (0.8_e, 0.5_f), (0.2_e, 0.5_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.5_e, 0.3_f), (0.5_e, 0.5_f) \rangle$ in K is an IFα*CS but not an IFRCS in K.

Remark 3.5 Every IFCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.6 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.2_e, 0.3_f), (0.8_e, 0.7_f) \rangle$, $H_2 = \langle k, (0.3_e, 0.4_f), (0.7_e, 0.6_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.3_e, 0.2_f), (0.7_e, 0.7_f) \rangle$ in K is an IFα*CS but not an IFCS in K.

Example 3.7 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.4_e, 0.5_f), (0.6_e, 0.5_f) \rangle$, $H_2 = \langle k, (0.5_e, 0.6_f), (0.5_e, 0.4_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.5_e, 0.4_f), (0.5_e, 0.6_f) \rangle$ in K is an IFCS but not an IFα*CS in K.

Remark 3.8 Every IFSCS in an IFTS (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.9 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.3_e, 0.6_f), (0.7_e, 0.4_f) \rangle$, $H_2 = \langle k, (0.5_e, 0.9_f), (0.5_e, 0.1_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.2_e, 0.5_f), (0.8_e, 0.4_f) \rangle$ in K is an IFα*CS but not an IFSCS in K.

Example 3.10 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.8_e, 0.4_f), (0.2_e, 0.6_f) \rangle$, $H_2 = \langle k, (0.6_e, 0.3_f), (0.4_e, 0.7_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.2_e, 0.4_f), (0.8_e, 0.6_f) \rangle$ in K is an IFSCS but not an IFα*CS in K.

Remark 3.11 Every IFγCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.12 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.8_e, 0.4_f), (0.2_e, 0.6_f) \rangle$, $H_2 = \langle k, (0.6_e, 0.3_f), (0.4_e, 0.7_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.7_e, 0.3_f), (0.3_e, 0.7_f) \rangle$ in K is an IFα*CS but not an IFγCS in K.

Example 3.13 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.3_e, 0.6_f), (0.7_e, 0.4_f) \rangle$, $H_2 = \langle k, (0.5_e, 0.9_f), (0.5_e, 0.1_f) \rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.5_e, 0.4_f), (0.5_e, 0.6_f) \rangle$ in K is an IFγCS but not an IFα*CS in K.

Remark 3.14 Every IFPCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.15 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.8_e, 0.3_f), (0.2_e, 0.7_f)⟩ in K is an IFα*CS but not an IFPCS in K.

Example 3.16 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.1_e, 0.3_f), (0.6_e, 0.7_f)⟩ in K is an IFPCS but not an IFα*CS in K.

Remark 3.17 Every IFαCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.18 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.5_e, 0.7_f), (0.5_e, 0.3_f)⟩, H_2 = ⟨k, (0.3_e, 0.6_f), (0.7_e, 0.4_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.4_e, 0.5_f), (0.5_e, 0.4_f)⟩ in K is an IFα*CS but not an IFαCS in K.

Example 3.19 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.1_e, 0.3_f), (0.6_e, 0.7_f)⟩ in K is an IFαCS but not an IFα*CS in K.

Remark 3.20 Every IFβCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.21 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.8_e, 0.3_f), (0.2_e, 0.7_f)⟩ in K is an IFα*CS but not an IFβCS in K.

Example 3.22 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.2_e, 0.4_f), (0.8_e, 0.6_f)⟩ in K is an IFβCS but not an IFα*CS.

Remark 3.23 Every IFSPCS in (K,ζ) is independent of every IFα*CS in (K,ζ).

Example 3.24 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.8_e, 0.4_f), (0.2_e, 0.6_f)⟩, H_2 = ⟨k, (0.6_e, 0.3_f), (0.4_e, 0.7_f)⟩, then (K,ζ) is an IFTS. The IFSs T = ⟨k, (0.7_e, 0.3_f), (0.3_e, 0.7_f)⟩ and V = ⟨k, (0.8_e, 0.3_f), (0.2_e, 0.7_f)⟩ in K are IFα*CSs but T is not an IFSPCS in K as V is not an IFPCS.

Example 3.25 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.3_e, 0.6_f), (0.7_e, 0.4_f)⟩, H_2 = ⟨k, (0.5_e, 0.9_f), (0.5_e, 0.1_f)⟩, then (K,ζ) is an IFTS. The IFS T = ⟨k, (0.4_e, 0.4_f), (0.6_e, 0.6_f)⟩ is an IFSPCS as V = ⟨k, (0.5_e, 0.4_f), (0.5_e, 0.6_f)⟩ in K is an IFPCS. But T and V are not IFα*CSs in K.

The interconnection between various intuitionistic fuzzy closed sets with newly found IFα*CS is provided in Figure 1.

Proposition 3.26 Let T and V be any two IFα*CSs in an IFTS (K,ζ), then T ∪ V is also an IFα*CS in (K,ζ).

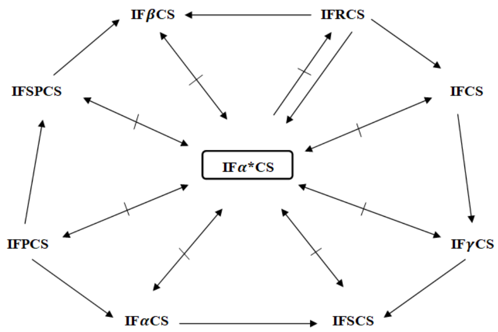

Figure 1. Interconnections

Proof: Given: T and V are IFα*CSs in (K,ζ)
Consider, cl(int(cl(T ∪ V))) ⊆ cl(cl(T ∪ V)) = cl(T ∪ V)
Therefore,
cl(int(cl(T ∪ V))) ⊆ cl(T ∪ V) (1)

Now, cl(int(cl(T ∪ V))) = cl(int(cl(T) ∪ cl(V))) ⊇ cl(int(cl(T)) ∪ int(cl(V))) = cl(int(cl(T))) ∪ cl(int(cl(V))) = cl(T) ∪ cl(V) = cl(T ∪ V)

Therefore,
cl(int(cl(T ∪ V))) ⊇ cl(T ∪ V) (2)

From (1) and (2), cl(int(cl(T ∪ V))) = cl(T ∪ V) which implies T ∪ V is an IFα*CS in (K,ζ).

Remark 3.27 In an IFTS (K,ζ), the intersection of any two IFα*CSs is not required to be an IFα*CS.

Example 3.28 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where H_1 = ⟨k, (0.6_e, 0.8_f), (0.4_e, 0.2_f)⟩, H_2 = ⟨k, (0.5_e, 0.5_f), (0.4_e, 0.4_f)⟩, then (K,ζ) is an IFTS. The IFSs T = ⟨k, (0.5_e, 0.4_f), (0.4_e, 0.5_f)⟩ and V = ⟨k, (0.4_e, 0.6_f), (0.5_e, 0.2_f)⟩ are IFα*CSs in (K,ζ). The intersection of T and V, T ∩ V = ⟨k, (0.4_e, 0.4_f), (0.5_e, 0.5_f)⟩ is not an IFα*CS in (K,ζ).

Proposition 3.29 If T is both an IFCS and an IFα*CS in an IFTS (K,ζ), then T is an IFRCS in (K,ζ).

Proof: Given: T is an IFCS and IFα*CS in (K,ζ). Consider, cl(int(T)) = cl(int(cl(T))) = cl(T) = T. Therefore, T is an IFRCS in (K,ζ).

Proposition 3.30 For an IFS T in an IFTS (K,ζ), the followings are selfsame:

(i) T ⊆ cl(int(cl(T)))
(ii) T is an IFα*CS

Proof:
(i) ⇒ (ii) Let T ⊆ cl(int(cl(T))). Then, cl(T) ⊆ cl(cl(int(cl(T)))) = cl(int(cl(T))). Therefore, cl(T) ⊆ cl(int(cl(T))). Consider, cl(int(cl(T))) ⊆ cl(cl(T)) = cl(T). Therefore, cl(int(cl(T))) ⊆ cl(T). Hence, cl(int(cl(L))) = cl(L) and T is an IFα*CS in (K,ζ).
(ii) ⇒ (i) Assume T be an IFα*CS in (K,ζ).
Consider, cl(int(cl(T))) = cl(T) ⊇ T.

Proposition 3.31 In an IFTS (K,ζ), every IF clopen set is an IFα*CS but not necessarily vice versa.

Proof: Given: T is an IF clopen set in (K,ζ),
Consider, cl(int(cl(T))) = cl(int(T)) = cl(T).
Hence, cl(int(cl(T))) = cl(T) and T is an IFα*CS in (K,ζ).

Example 3.32 Let K = {e,f}, ζ = {$0_\sim$, H_1, H_2, $1_\sim$} be an IFT on K where $H_1 = \langle k, (0.5_e, 0.7_f), (0.5_e, 0.3_f)\rangle$, $H_2 = \langle k, (0.3_e, 0.6_f), (0.7_e, 0.4_f)\rangle$, then (K,$\zeta$) is an IFTS. The IFS $T = \langle k, (0.2_e, 0.6_f), (0.8_e, 0.4_f)\rangle$ in K is an IFα*CS but not an IF clopen set in K.

Proposition 3.33 An IFS T is an IFαCS in an IFTS (K,ζ) if T is an IFCS and IFα*CS in (K,ζ).

Proof: Given: T is an IFCS and IFα*CS in (K,ζ),
Consider, cl(int(cl(T))) ⊆ cl(T) = T. Hence, T ⊇ cl(int(cl(T))) and T is an IFαCS in (K,ζ).

Proposition 3.34 For an IF clopen set T in (K,ζ), the following are equivalent:
(i) T is an IFRCS,
(ii) T is an IFα*CS and an IF Q-set.

Proof: Assume T be an IF clopen set,
(i) ⇒ (ii) Assume T be an IFRCS. Since, T is an IF clopen set, by proposition 3.31, T is an IFα*CS in (K,ζ).
Consider, int(cl(T)) = int(T) = T = cl(int(T)).
Therefore, cl(int(T)) = int(cl(T)) and T is an IF Q-set in (K,ζ).
(ii) ⇒ (i) Let T be an IFα*CS and IF Q-set in (K,ζ)
Consider, cl(int(cl(T))) = cl(T)
cl(cl(int(T))) = T
cl(int(T)) = T
Hence, T is an IFRCS in (K,ζ).

Proposition 3.35 In an IFTS (K,ζ), an IFCS T is an IFα*CS if $T_{\bar{q}}N \Rightarrow [\mathrm{cl}(\mathrm{int}(\mathrm{cl}(T))) = \mathrm{cl}(T)]_{\bar{q}}N$ for all IFCS N of K.

Proof: Assume N be an IFCS. Let $T_{\bar{q}}N$, then T ⊆ N^c.
Consider, cl(int(cl(T))) = cl(T) = T ⊆ N^c.
Therefore, $[\mathrm{cl}(\mathrm{int}(\mathrm{cl}(T))) = \mathrm{cl}(T)]_{\bar{q}}N$.

Proposition 3.36 In an IFTS (K,ζ), an IFS T is an IFγCS if T is both IFCS and IFα*CS.

Proof: Let T be an IFCS and IFα*CS,
Consider, cl(int(T)) ∩ int(cl(T)) = cl(int(cl(T))) ∩ int(T) = cl(T) ∩ int(T) = int(T) ⊆ T.
Hence, cl(int(T)) ∩ int(cl(T)) ⊆ T and T is an IFγCS in (K,ζ).

Proposition 3.37 If an IFCS T is both nowhere dense and IFα*CS in (K,ζ), then T is IFγCS in K.

Proof: Let T be an IFCS, nowhere dense and IFα*CS in K
Consider, cl(int(T)) ∩ int(cl(T)) = cl(int(cl(T))) ∩ int(cl(T)) = cl(L) ∩ $0_\sim$ = $0_\sim$ ⊆ T. Hence, T is an IFγCS in (K,ζ).

Proposition 3.38: If an IFCS T is both IFβCS and IFα*CS in (K,ζ), then T is an IFSCS in K.

Proof: Let T be an IFCS, IFβCS and IFα*CS in K.
Then, int(cl(int(T))) ⊆ T
t(cl(int(cl(T)))) ⊆ T
int(cl(T)) ⊆ T
Hence, T is an IFSCS in (K,ζ).

Proposition 3.39: If an IFS T is both IF Q-set and IFα*CS in (K,ζ), then T is an IFSOS in K.

Proof: Let T be an IF Q-set and IFα*CS
Then, cl(int(cl(T))) = cl(T)
cl(cl(int(T))) = cl(T)
cl(int(T)) = cl(T) ⊇ T
Hence, T is an IFSOS in (K,ζ).

Proposition 3.40: The following claims are equivalent for an IFS T of an IFTS (K,ζ).
(i) T is both IFαCS and IFα*CS
(ii) T is an IFRCS in (K,ζ).

Proof:
(i) ⇒ (ii) Let T be an IFαCS and IFα*CS in K ,
cl(int(cl(T))) ⊆ T (1)

cl(int(cl(T))) = cl(T) (2)

From (1) & (2),
cl(T) ⊆ T (3)

W.K.T., T ⊆ cl(T) (4)

From (3) & (4) T = cl(T)
Consider, cl(int(T)) = cl(int(cl(T))) = cl(T) = T. Hence, cl(int(T)) = T. Therefore, T is an IFRCS in (K,ζ).
(ii) ⇒ (i) Let T be an IFRCS. IFRCS ⇒ IFCS.
By Proposition 3.3, T is an IFα*CS in (K,ζ).
Consider, cl(int(cl(T))) = cl(T) = T ⊆ T.
Hence, T is an IFαCS in (K,ζ).

4. Conclusions

In the field of general topology, the theory of α* closed sets is essential. Since its inception in general topology, weak and strong forms of α closed sets have been introduced in fuzzy topology, and in intuitionistic fuzzy topology. In this article, we defined intuitionistic fuzzy α* closed sets in intuitionistic fuzzy topological spaces. The proposed set's dependence on and independence from other existing sets were investigated, with several related propositions. Numerous examples are provided to demonstrate the results. It is possible to deduce new kinds of continuity, a new decomposition of continuity, and a new separation axiom using intuitionistic fuzzy α* closed sets. This set may be used in the fields of decision-making, pattern recognition, and image processing.

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