Resultant geometric variation of a fixtured workpiece Part I: a simulation

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Abstract

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When a workpiece is fixtured for a machining or inspection operation, the accuracy of an operation is mainly determined by the efficiency of the fixturing method. Variability in manufactured workpiece is hardly inevitable. When such variability is found at contact areas between the workpiece and the fixture, errors in location are expected. The errors will affect quality of features to be produced. This paper developed an algorithm to determine variant final locations of a displaced workpiece given normally distributed errors at contact points. Resultant geometric variation of workpiece location reveals interesting information which is beneficial in tolerance planning.

บทคัดย่อ

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รูปแบบความไม่แน่นอนของชิ้นงานที่ถูกจับยึดด้วยฟิกซ์เจอร์

ส่วนที่ 1 แบบจำลองทางคณิตศาสตร์

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ปกติแล้วในกระบวนการผลิต ชิ้นงานจะถูกจับยึดบนเครื่องจักรด้วยตัวจับชิ้นงานหรือฟิกซ์เจอร์ (Fixture) ฟิกซ์เจอร์มีความสำคัญเป็นอย่างมากในการควบคุมคุณภาพของผลิตภัณฑ์ ผลิตภัณฑ์ที่ผ่านกระบวนการผลิตจะมีความเที่ยงตรงแม่นยำถูกต้องตามลักษณะที่กำหนดไว้เพียงใดขึ้นอยู่กับประสิทธิภาพในการจับยึดชิ้นงานของฟิกซ์เจอร์ อย่างไรก็ตาม ในกระบวนการผลิตมักจะพบว่ามีความไม่แน่นอนเกิดขึ้นเสมอและหลีกเลี่ยงได้ยาก ซึ่งอาจเกิดจากความผิดพลาดของผู้ปฏิบัติงาน การเสื่อมสภาพของเครื่องจักร อุปกรณ์ต่าง ๆ เป็นต้น ทำให้ผลิตชิ้นงานที่มีขนาดและรูปร่างที่ผิดเพี้ยนไปจากที่กำหนดไว้ เมื่อนำชิ้นงานนั้นมาจับยึด ความผิดพลาดที่จุดสัมผัสระหว่างฟิกซ์เจอร์กับชิ้นงานจะถูกส่งผ่านไปยังฟีทเจอร์ (Feature) ที่จะทำการผลิต งานวิจัยนี้มีจุดประสงค์เพื่อที่จะพัฒนากระบวนวิธีที่ใช้ในการคำนวณหาความไม่แน่นอนของตำแหน่งชิ้นงานที่ถูกจับยึด ภายใต้สมมติฐานที่ว่าชิ้นงานมีความผิดพลาด ณ จุดสัมผัสโดยมีลักษณะการกระจายเป็นแบบปกติ ผลจากการทดสอบแบบจำลองทำให้ทราบว่า ความไม่แน่นอนของฟีทเจอร์ที่จะทำการผลิตมีลักษณะเป็นรูปแบบที่น่าสนใจ และจากข้อมูลนี้สามารถนำไปใช้ในการกำหนดระยะพิกัดเผื่อที่มีประสิทธิภาพในกระบวนการผลิตต่อไป

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Workpiece displacement because of locating or fixturing errors is a threat in the early stage of precision machining. It has been discussed widely either in the workpiece deviation determination or fixture component placement optimization. Researchers working in fixturing area have developed algorithms providing complete constraint of a workpiece (King and Hutter, 1993; Trappey and Matrubhutam, 1993; Brost and Goldberg, 1996; Dai, Nee *et al*., 1997; Jeng and Gill, 1997; De Meter, 1998; Wu, Rong *et al*., 1998), determining sufficient support, performance of the fixture, and accessibility (Pham and Lazaro, 1990; Fuh, Chang *et al*., 1993; De Meter, 1994; De Meter, 1994; De Meter, 1995; Ong and Nee, 1998). Rong and Bai (1996) analyzed a dependent relationship of operational dimensions to estimate machining errors in terms of linear and angular dimensions of a workpiece. Cai *et al*. (1997) proposed a method to conduct robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture set-up errors. A method for modeling and analyzing the impact of a locator

tolerance scheme on geometric errors of machined features was developed by Choudhuri and De Meter (1999). There were also algorithms to predict a deviation of a prismatic workpiece located by 3-2-1 fixturing method (Salisbury and Peters, 1998) and a cylindrical workpiece in a v-block (Sangnui and Peters, 2001). The results from Salisbury and Peters (1998) and Sangnui and Peters (2001) indicated that the largest deviation did not necessarily come from the largest errors at contact points. Djurdjanovic and Ni (2003) developed procedures for determining the influence of errors in fixtures, locating datum features and measurement datum features on dimensional errors in machining. These studies were conducted when a static case is assumed. Although variability existing in a production line is playing an obstructive role in gaining an efficiently precise control over manufacturing operations, only a few researchers employed a variational model to evaluate fixture performance. A model in the absence of workpiece variability consideration would not be applicable and functional in most cases. Bhat and De Meter

(2000) carried out a simulation analysis to evaluate performance of three different datum-establishment methods: 3-2-1, sequential least-squares (SQLS) and simultaneous least-squares (SMLS). The authors assumed variant errors of datum features. However, only the magnitude of relative positional workpiece displacement was presented. Variation in workpiece position was yet to be analyzed. A probabilistic fixturing model was also developed by Sangnui (2002) and geometric variation of a feature was determined. Because the analytical model was established according to a sequential locating scheme of 3-2-1 fixturing method, calculation was somewhat tedious.

In this paper, an approach to assess prismatic workpiece displacement resulting from misalignment in locating is introduced. When errors of a datum feature are assumed probabilistic, the resultant workpiece geometric variation reveals a significant systematic pattern. Unlike Sangnui (2002), a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. By implementing the method developed in this study, a designer will be able to determine variation of subsequent processes as affected by fixturing errors.

Methodology

1. Fixturing Model

In a manufacturing process, a fixture is used to locate and restrain a workpiece from movements during machining or inspection. A reference to locate the workpiece at its nominal position and orientation is called a datum, which can be established from fixture or machine components as shown in Figure 1. A realistic part of the work-piece used to map with a datum is called 'a datum feature'. When the workpiece is fixtured for an operation, drilling a hole for example, exact mapping between a datum and a datum feature is desirable. However, in case that a workpiece surface is used as a datum feature, any surface deviations from a perfect topography would lead to workpiece displacement, and therefore inaccuracy of final products.

The impact of erratic fixturing on a feature accuracy is illustrated in Figure 2. A perfect workpiece is shown located at its theoretical location as opposed to an actual workpiece. It is assumed that the machine coordinate system, XYZ, is positioned coincidently with that of the nominal workpiece. The workpiece is to be drilled at a location defined by $\bar{L}$ with respect to the machine coordinate system. When the workpiece is displaced from its theoretical position, the hole will be moved to $\bar{L}'$ with respect to the workpiece, XYZ'. Thus, discrepancy in location of the hole occurs.

If surface variation exists at contact points between a workpiece and a fixture, deviation of the workpiece from its nominal location is expected. A typical 3-2-1 fixturing method is composed of six locators forming three mutually perpendicular datum planes. The primary datum plane as shown in Figure 3 is constructed from the first three locators (no. 1, 2, 3). Perpendicular to the primary datum plane, the secondary datum plane is established from the contacts of the locator no. 4 and 5. Finally, the tertiary datum plane is the plane perpendicular to the preceding ones in which the last contact point (no. 6) lies. Note that the workpiece surfaces in contact with the datum planes are called workpiece datum features. Given a distribution of surface errors at the contact locations,

Figure 1. The mapping between a datum plane and a workpiece datum feature.
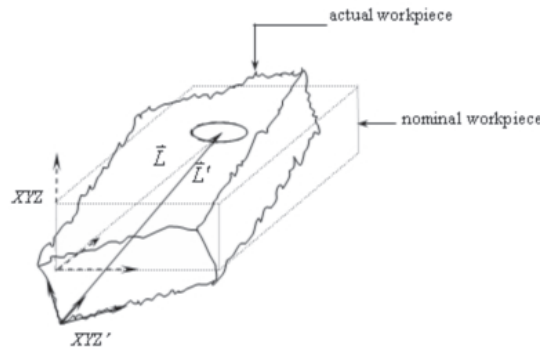
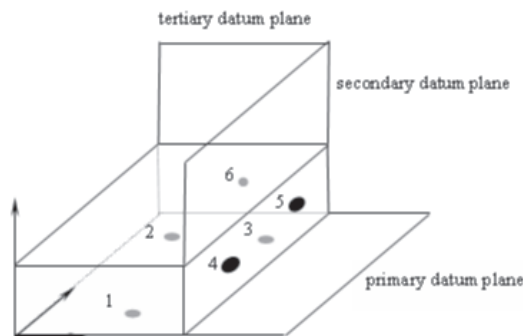**Figure 2. The workpiece and feature displacement.**

Figure 3. Datum planes in 3-2-1 fixturing method.

statistics allows us to determine variability of final position of the fixtured workpiece.

2. Assumptions

The following assumptions regarding the fixture and the workpiece were used.

1. A workpiece is considered a rigid body. Deformation of the workpiece during transformation is not allowed.

2. The workpiece surface errors are measured perpendicularly to the perfect form of datum features at the contact points. The determination of the surface error sign is shown in Figure 4.

3. The errors are assumed to be normally distributed with mean μ and variance σ^2, or $N(\mu, \sigma^2)$.

3. Analysis

When an imperfect workpiece is fixtured for an operation, variation at the contact point between the workpiece and the fixture would be imparted to a feature to be produced. The impact of such errors on the feature can be analyzed through workpiece deviation from its nominal location. The deviation is explained by means of geometric transformation: translation and rotation. Workpiece location and orientation after located in a fixture will be mathematically analyzed. Workpiece location in the fixture can be represented by an arbitrary point, which could be a location of a feature of interest. Workpiece orientation in this paper is defined by vectors normal to the primary and secondary datum features. A normal vector to the primary datum feature $\vec{n}_p$ is derived from

$$\vec{n}_p = \vec{P}_{31} \times \vec{P}_{21} \qquad (1)$$

where $\qquad \vec{P}_{21} = \vec{P}_2 - \vec{P}_1$

$$\vec{P}_{31} = \vec{P}_3 - \vec{P}_1$$

Once the orientation of the primary datum feature is computed, a vector normal to the second-

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ary datum feature, $\vec{n}_s$, can be obtained from Eq.(2). According to the right-hand rule, $\vec{n}_s$ must stay perpendicular to the vector normal to the primary datum plane, $\vec{n}_p$. Therefore,

$$\vec{n}_s = \vec{P}_{54} \times \vec{n}_p \tag{2}$$

where $\quad \vec{P}_{54} = \vec{P}_5 - \vec{P}_4$

When the errors at the contact point $\vec{P}_i$ are considered in the equations, Eq. (1) and (2) become

$$\vec{n}_{pe} = \vec{P}_{31e} \times \vec{P}_{21e} \tag{3}$$

$$\vec{n}_{se} = \vec{P}_{54e} \times \vec{n}_{pe} \tag{4}$$

where $\quad \vec{P}_{21e} = \vec{P}_{2e} - \vec{P}_{1e}$

$$\vec{P}_{31e} = \vec{P}_{3e} - \vec{P}_{1e}$$

$$\vec{P}_{54e} = \vec{P}_{5e} - \vec{P}_{4e}$$

Because of errors at the contact areas, the workpiece will be disoriented and dislocated from nominal location and orientation to allow complete contacts with the six locators. To guarantee such contacts, the following constraints must be satisfied.

$$\vec{n}_{pe} - R_1(\alpha,\beta,\gamma,dx,dy,dz)\vec{n}_p = 0 \tag{5}$$

$$\vec{n}_{se} - R_1(\alpha,\beta,\gamma,dx,dy,dz)\vec{n}_s = 0 \tag{6}$$

$$\vec{n}_{pe} \cdot \vec{P}_{1e} - D_p = 0 \tag{7}$$

$$\vec{n}_{se} \cdot \vec{P}_{4e} - D_s = 0 \tag{8}$$

$$\vec{n}_{te} \cdot \vec{P}_{6e} - D_t = 0 \tag{9}$$

where R_1 is a homogeneous transformation matrix represented by

$$R_1 = \begin{bmatrix} Cos(\alpha)Cos(\beta) & -Sin(\alpha)Cos(\beta) & Sin(\beta) & dx \\ Sin(\alpha)Cos(\gamma)+Cos(\alpha)Sin(\beta)Sin(\gamma) & Cos(\alpha)Cos(\gamma)-Sin(\alpha)Sin(\beta)Sin(\gamma) & -Cos(\beta)Sin(\gamma) & dy \\ Sin(\alpha)Sin(\gamma)+Cos(\alpha)Sin(\beta)Cos(\gamma) & Cos(\alpha)Sin(\gamma)+Sin(\alpha)Sin(\beta)Cos(\gamma) & Cos(\beta)Cos(\gamma) & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system of the above equations is solved by breaking down the procedure into 2 steps. The first step is to find an appropriate rotation which matches $\vec{n}_p$ with $\vec{n}_{pe}$ and $\vec{n}_s$ with $\vec{n}_{se}$ by using the *Newton-Raphson* method. A set of objective functions is in the following equations.

$$
\begin{aligned}
f_1 &: \vec{n}_{xp} - \vec{n}_{xpe} \\
f_2 &: \vec{n}_{yp} - \vec{n}_{ype} \\
f_3 &: \vec{n}_{zp} - \vec{n}_{zpe} \\
f_4 &: \vec{n}_{xs} - \vec{n}_{xse} \\
f_5 &: \vec{n}_{ys} - \vec{n}_{yse} \\
f_6 &: \vec{n}_{zs} - \vec{n}_{zse}
\end{aligned}
\tag{10}
$$

where $\quad \vec{n}_{ie} = R_2(\alpha,\beta,\gamma)\vec{n}_i$

$$R_2 = \begin{bmatrix} Cos(\alpha)Cos(\beta) & -Sin(\alpha)Cos(\beta) & Sin(\beta) \\ Sin(\alpha)Cos(\gamma)+Cos(\alpha)Sin(\beta)Sin(\gamma) & Cos(\alpha)Cos(\gamma)-Sin(\alpha)Sin(\beta)Sin(\gamma) & -Cos(\beta)Sin(\gamma) \\ Sin(\alpha)Sin(\gamma)+Cos(\alpha)Sin(\beta)Cos(\gamma) & Cos(\alpha)Sin(\gamma)+Sin(\alpha)Sin(\beta)Cos(\gamma) & Cos(\beta)Cos(\gamma) \end{bmatrix}$$

$$\tag{11}$$

Following the *Newton-Raphson* procedure, the algorithm will be executed repeatedly until it finds a solution (α, β, γ) which makes Eq.(10) true. Once such a rotation is found, it will be used to
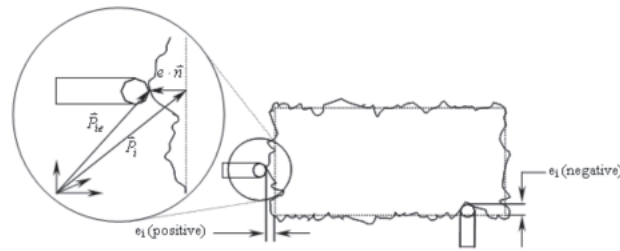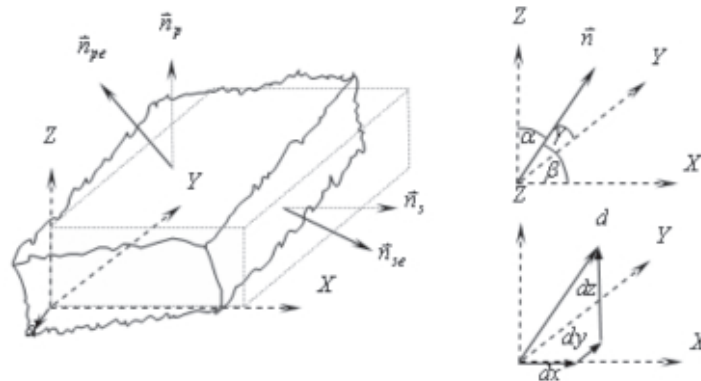
Figure 4. Error determination.

Figure 5. Workpiece location affected by variability at workpiece surface.

transform the workpiece to its final orientation. The second step is to calculate a translation vector needed to bring the workpiece to its final location while maintaining the orientation obtained from the first step (Figure 6). The vector is derived from the difference in the distance from the origin, D, of the nominal and actual datum features along corresponding normal vectors, $\vec{n}_{pe}, \vec{n}_{se}$ and $\vec{n}_{te}$. In summary, a translation vector bringing the workpiece to make contact with all locators will be calculated from Eq. (12).

$$\vec{T} = \vec{n}_{pe} \cdot \Delta D_p + \vec{n}_{se} \cdot \Delta D_s + \vec{n}_{te} \cdot \Delta D_t \qquad (12)$$

$$\Delta D_p = D_{pe} - D_p$$
$$\Delta D_s = D_{se} - D_s \qquad (13)$$
$$\Delta D_t = D_{te} - D_t$$

The final location of the workpiece is then

$$\vec{L}' = R_2(\alpha, \beta, \gamma)\vec{L} + \vec{T} \qquad (14)$$

where $\vec{L}'$ is a point of interest.

$\vec{L}$ is the point after transformation.

Note that the algorithm simultaneously solves for the transformation satisfying the objective functions (Eq. 5-9). It may sound contradictive to the practical use of a 3-2-1 fixture, in which the workpiece must be sequentially located starting from the primary, secondary and tertiary datum planes. It is also realized that different orders of locating the planes will be associated with different contact locations. However, it is considered in this paper that the sequential process is not necessary since only variability of the workpiece location is estimated, not its exact arrangement.

Results and Discussion

Once the algorithm to characterize the workpiece behavior was developed, a Monte Carlo simulation was conducted to assess the variability of workpiece location by using the following method.
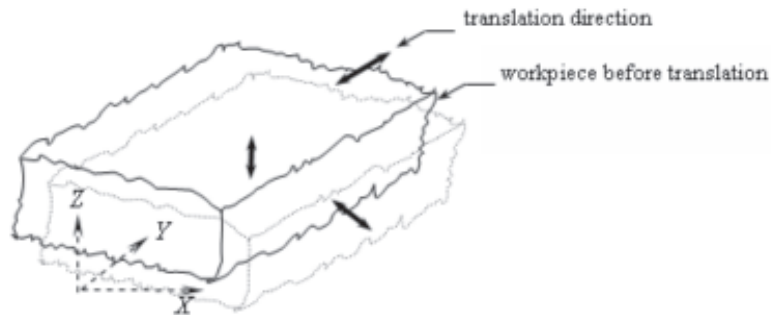
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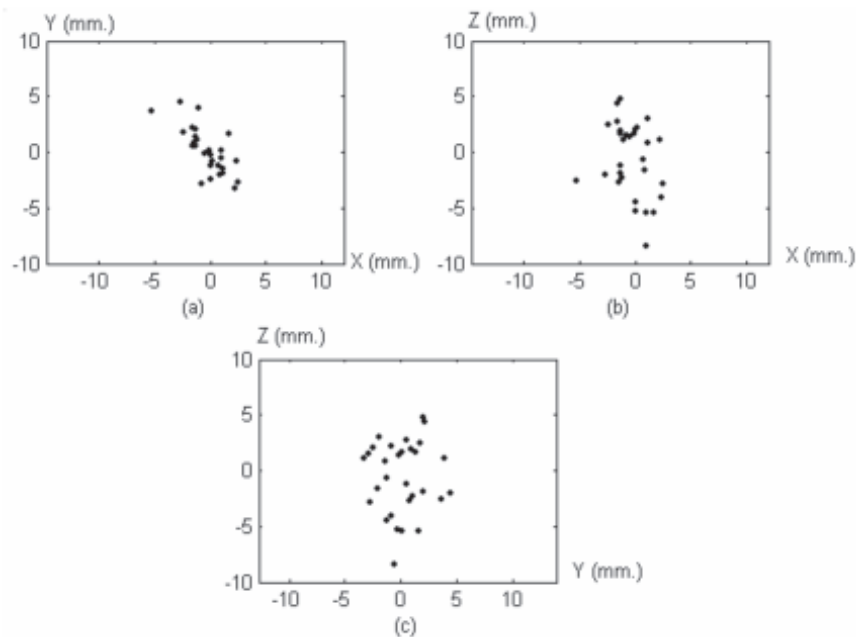**Figure 6. Workpiece translation.**

Figure 7. Variability in workpiece location when n=30.

1. A set of normal or Gaussian distributed random numbers represented the errors at each contact point was generated.

2. The error samples were used in the algorithm and the results were collected.

3. A multivariate statistical process was used to evaluate the variability of the sample distribution.

Examples of fixturing constraints are shown in Table 1. Table 2 contains the results which are the distributions of hole location under the constraints.

From the scatter plots (Figure 7 and 8), it can be seen that when the number of samples is greater (i.e. 300) the pattern of distribution is clearly shown. The elliptical-shaped point cloud preliminary indicates a normal or Gaussian distribution pattern. In 3-dimensional system, however, it does not guarantee a global multivariate normal relationship among the three axes.

To evaluate the distribution of multivariate data, a normality test is performed by constructing a chi-square plot (see detail in (Johnson and Wichern, 1998)). If the data are drawn from a
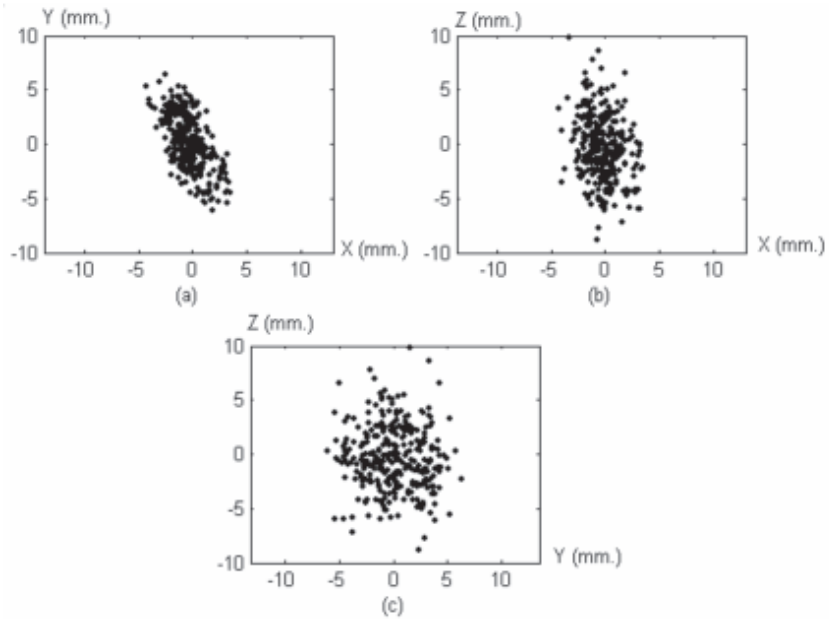
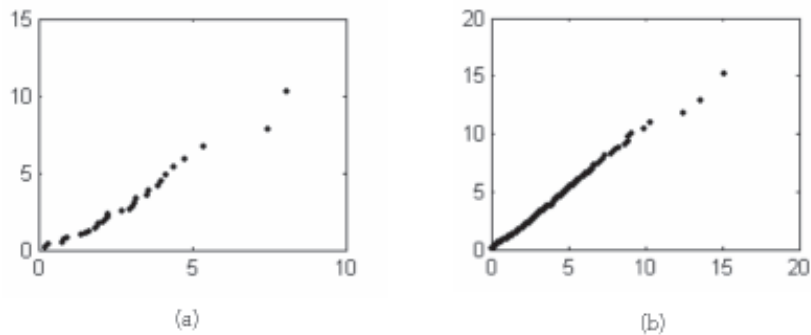**Figure 8. Variability in workpiece location when n=300.**

Figure 9. Normality plot (a) when a sample size is 30 (b) when a sample size is 300.

multivariate normal distribution, the plot should be similar to a straight line through the origin with slope equal to one. Any systematic curves indicate lack of normality and points far from the line suggest outlying observations. The data from the simulation plotted in Figure 9 is reasonably assumed a multivariate normal distribution. These statistical results are beneficial in tolerance analysis and will be discussed later in future work.

Relationships among the components of the locations of interest in Table 2 are in well agreement with the scatter plot. It can be seen that strong correlations between a component 'x' and 'y' of

workpiece location is notable, while the other pairs ('y' and 'z', 'x' and 'z') do not demonstrate any significant evidence of correlations. The distinct correlation between 'x' and 'y' suggests extra care if a circular positional tolerance is in use. How much a proportion in the elliptical area should be included to establish the circular tolerance is decisive and needs further study.

Conclusion

This study proposes an analysis of the variability in location of the workpiece as affected

Table 1. Coordinates and distributions of errors at each locator.

Plane	Locators (coordinates)	Errors Distribution				
		1	2	3	4	5
Primary Plane	1 (30 30 0)	N(0.0, 4.00)	N(-1.0, 1.21)	N(0.0, 3.61)	N(1.5, 1.44)	N(0.0, 4.00)
	2 (120 30 0)	N(0.0, 2.25)	N(2.0, 3.24)	N(2.0, 2.89)	N(2.3, 1.69)	N(-1.0, 2.25)
	3 (75 120 0)	N(0.0, 1.44)	N(0.0, 1.69)	N(-1.2, 1.44)	N(0.0, 1.44)	N(-1.5, 1.44)
Secondary Plane	4 (150 30 15)	N(0.0, 1.69)	N(-1.2, 2.56)	N(0.0, 0.81)	N(-1.5, 3.61)	N(0.0, 1.69)
	5 (150 120 15)	N(0.0, 3.61)	N(2.0 ,2.25)	N(2.0, 4.41)	N(-1.0, 4.00)	N(2.3, 3.61)
Tertiary Plane	6 (75 150 15)	N(0.0, 2.25)	N(0.0 ,1.44)	N(-1.0, 2.25)	N(0.0, 3.24)	N(1.5, 2.25)

Table 2. Results obtained from the constraints shown in Table 1.

Error Set	μ			σ_x^2	σ_y^2	σ_z^2	σ_{xy}	σ_{xz}	σ_{yz}
1	0.0040	-0.0285	0.2120	4.6756	6.5716	10.1982	**-3.2095**	-1.1812	-1.2336
2	-1.6142	2.4725	-1.8671	5.5041	5.1305	2.9554	**-3.9664**	-0.1632	-0.1694
3	-0.4418	0.4104	0.0755	2.1175	6.1266	8.2082	**-2.3401**	-0.3508	-0.9357
4	-1.6176	0.0155	1.9616	7.3417	10.0237	4.7778	**-5.6818**	-0.7146	-0.5029
5	-0.8809	3.2380	0.5825	3.0245	5.8589	9.8903	**-2.5032**	-1.1441	-0.8828

by variant errors at contact areas. The errors are assumed normally distributed. Unlike previous studies, a step-wise analysis is considered unnecessary and the mathematical formulation is simplified. The repetitive *Newton-Raphson* is employed to obtain a transformation which matches a theoretical with an actual position and orientation. Multivariate statistical techniques were used to analyze resultant variability. The information gained is beneficial to manufacturers as it reveals how surface variability becomes influential on location variation of the fixtured workpiece. It is recommended that such surface variability should be accounted in tolerancing establishment. Tighter tolerance ensures a functional assembly; however, usually associated with higher cost. It is the responsibility of a designer to find ways that would benefit the production the most. Implementing the concept proposed in this work would help the designer impose the tolerances more efficiently, and consequently reduce manufacturing cost and improve product quality.

Nomenclature

$\vec{n}_p, \vec{n}_s, \vec{n}_t$	Nominal vectors normal to primary, secondary and tertiary datum features, respectively
$\vec{n}_{pe}, \vec{n}_{se}, \vec{n}_{te}$	Actual vectors normal to primary, secondary, and tertiary datum features, respectively
$\bar{P}_i$	Nominal contact points when $i = 1,2,..6$
$\bar{P}_{ie}$	Actual contact points when $i = 1,2,..6$
D_p, D_s, D_t	Nominal distances from the origin to primary, secondary, and tertiary datum features, respectively
D_{pe}, D_{se}, D_{te}	Actual distances from the origin to primary, secondary, and tertiary datum features, respectively

| α, β, γ | Rotational angles about *Z*, *Y*, and *X* axes, respectively |
| *dx,dy,dz* | Translational distances along *X*, *Y*, and *Z* axes, respectively |

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