ORIGINAL ARTICLE

Efficient algorithm and computing tool for shading calculation

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Abstract

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The window is always part of a building envelope. It earns its respect in creating architectural elegance of a building. Despite a major advantage of daylight utilization, a window would inevitably allow heat from solar radiation to penetrate into a building. Hence, a window design must be performed under a careful consideration in order to achieve an energy-conscious design for which the daylight utilization and heat gain are optimized. This paper presents the validation of the vectorial formulation of shading calculation by comparing the computational results with experimental ones, overhang, fin, and eggcrate. A computational algorithm and interactive computer software for computing the shadow were developed. The software was designed in order to be user-friendly and capable of presenting profiles of the shadow graphically and computing corresponding shaded areas for a given window system. It was found that software simulation results were in excellent agreement with experimental results. The average percentage of error is approximately 0.25%, 0.52%, and 0.21% for overhang, fin, and eggcrate, respectively.

Key words : shading devices, shading calculation, window, energy conservation

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กระจกเป็นส่วนหนึ่งของกรอบอาคารที่ช่วยเพิ่มความสวยงามให้กับอาคาร แม้ว่ากระจกจะช่วยให้สามารถนำแสงธรรมชาติมาใช้ประโยชน์ แต่ขณะเดียวกันความร้อนจากรังสีแสงอาทิตย์สามารถผ่านเข้าสู่อาคารได้มากขึ้น ดังนั้นในการออกแบบระบบหน้าต่างจึงต้องคำนึงถึงประสิทธิภาพสูงสุดทั้งในด้านการนำแสงธรรมชาติมาใช้งานและความร้อนที่ผ่านกระจกโดยการใช้อุปกรณ์บังแดด เพื่อลดปริมาณรังสีตรงจากดวงอาทิตย์ที่ตกกระทบบนหน้าต่าง บทความได้นำเสนอวิธีการคำนวณเงาที่ให้ความแม่นยำสูงโดยอาศัยการคำนวณแบบเวกเตอร์ พร้อมทั้งได้เสนอผลการคำนวณที่ได้จากโปรแกรมคอมพิวเตอร์ที่พัฒนาขึ้น โปรแกรมคำนวณเงาดังกล่าวมีลักษณะที่ง่ายต่อการใช้งานสามารถคำนวณและแสดงผลลักษณะเงาแบบกราฟิกได้กับอุปกรณ์บังแดดหลากหลายรูปร่าง และสามารถคำนวณพื้นที่เงาที่เกิดขึ้น ผลการคำนวณเปรียบเทียบลักษณะเงาและพื้นที่เงาของอุปกรณ์บังแดดให้ผลสอดคล้องกับการทดลองกับระบบหน้าต่างที่มีอุปกรณ์บังแดดในแนวราบ แนวดิ่ง และแบบผสม โดยมีค่าผิดพลาดเฉลี่ยประมาณ 0.25% 0.52% และ 0.21% ตามลำดับ

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The window is always part of a building envelope. It earns its respect in creating architectural elegance of a building. Despite a major advantage of daylight utilization, a window would inevitably allow heat to penetrate into a building. Hence, a window design must be performed under a careful consideration in order to achieve an energy-conscious design for which the daylight utilization and heat gain are optimized. In addition to a proper selection of glazing materials, shading devices play a major role in enhancing the performance of a window. The main objective of having shading devices installed on a window is to prevent a window from being exposed to the direct solar radiation. As a result, a large amount of heat gain is diminished and an illumination level is not in excess of the desirable level where glare and visual discomfort are eliminated. Hence, a properly designed shading device would be directly beneficial to the performance of a window.

In order to evaluate the performance of a shading device, the ability to compute the amount of shaded and unshaded area due to the shadow cast on a window by the sunray is required. Several methods of shading computation have been established. For example, the shadow-angle protractor is typically used among architects to graphically determine a shadow profile of a shading device and any object adjacent to a building (Kreith *et al.*, 1978). Jones (Jones, 1980) employed the concept of a shading plane defined by the leading edge of the overhang and the bottom of the window to estimate the shaded area on a window. This method was also used for computing the amount of solar radiation on a vertical plane (Yanda *et al.*, 1983). The aforementioned methods have several disadvantages especially when graphical realization and the amount of shaded and unshaded areas needed to be determined. This drawback has been overcome by the development of some computer tools, for example 'Shading Mask' (Setiadarma, 1996) that employed sun path diagrams as a basis for determining shadow cast on a vertical plane. 'AWNShade' is another interesting tool that calculates the unshaded fraction of a rectangular window for any given solar position coordinates relative to the window, where the window is shaded by an awning with or without side walls or an overhang of arbitrary dimensions above the window (McCluney, 1990). However, those tools lack flexibility in simulating windows and shading devices possessing high geometrical complexity. To this end, the effective shading computation thus requires a method that could handle all possible types and geometries of both shading devices and windows.

The most effective method, that employs the vector algebra, was proposed by Budin and Budin (Budin *et al*., 1982). Chirarattananon *et al.* (2000) further presented an algorithmic procedure for the computation of shading of radiation on a surface by an object based on Budin and Budin's method. This vectorial formulation has advantages over the others in its ability to be conveniently implemented in a computer program that allows the shading calculation be made for any type of a window system at any specific time and place. This would be useful for simulating and, hence, determining the performance of a window.

This paper presents the validation of the vectorial formulation of shading calculation by comparing the computational results with experimental ones. A computer program developed on Matlab in conjunction with Autocad was developed to easily configure a window system and determine shaded points cast on a window. Consequently, the shaded and unshaded area can be found from an efficient algorithm.

Theoretical background

The shading calculation is based on vectorial formulation of the geometrical relationships of the sunís position, a shading point, and an inclined plane. Detailed derivation of the formulation can be found in Budin *et al.* (1982) and Chirarattananon *et al.* (2000). The following summarizes the whole concept and essential relationships from which the computational scheme is developed.

In Figure 1, let's consider an arbitrarily inclined plane having a tilt angle of β and a plane azimuth of γ from the south (west positive). At particular moment, the sun is situated at the solar altitude of α and solar azimuth of γ_s. The values of α and γ_s depend on time, date and latitude of the plane being considered. A straightforward methodology for determining α and γ_s can be found in Duffie and Beckman (1980). The x-coordinates system, that is a rectangular system having the unit vectors $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ in the respective direction of zenith, east and north, is introduced to define vectors involved in the calculation. As such, the position of the sun and the plane can be represented by the unit vectors $\hat{v}_s$ and $\hat{v}_y$, respectively, written as follows:

$$\hat{v}_s = \begin{bmatrix} \sin\alpha \\ -\cos\alpha\sin\gamma_s \\ -\cos\alpha\cos\gamma_s \end{bmatrix} \tag{1}$$

$$\hat{v}_y = \begin{bmatrix} \sin\beta \\ -\cos\beta\sin\gamma \\ -\cos\beta\cos\gamma \end{bmatrix} \tag{2}$$

Suppose there is a point P, represented by the vector $\bar{y}_P$, in front of the inclined plane shown in Figure 1, the shadow is then cast by the sunray onto the plane on which the position can be represented by the vector $\bar{y}_s$. By simply employing a vector algebra, $\bar{y}_s$ can be found from

$$\bar{y}_s = \bar{y}_P + \lambda\hat{v}_s \tag{3}$$

where λ is arbitrary and yet to be determined. By performing dot operation on Eq.(3) with $\hat{v}_y$ and exerting the fact that $\bar{y}_s$ is perpendicular to $\hat{v}_y$, the value of λ can immediately be written as

$$\lambda = -\frac{\bar{y}_p \cdot \hat{v}_y}{\hat{v}_s \cdot \hat{v}_y} = -\frac{h}{\cos\theta} \tag{4}$$

where $\bar{y}_p \cdot \hat{v}_y = h$ which is the height of the point P above the plane and $\hat{v}_s \cdot \hat{v}_y = \cos\theta$, that is the cosine of the incident angle θ. Finally, the position of the shadow of the point P cast by the sunray can be found from

$$\bar{y}_s = \bar{y}_p - \frac{h}{\cos\theta} \cdot \hat{v}_s \tag{5}$$

It is worth mentioning that writing $\bar{y}_p$ and $\hat{v}_y$ or even allocating $\bar{y}_s$ with respect to the x-coordinates system might be cumbersome. It would be more convenient if the rectangular y-coordinates system, as also shown in Figure 1, is introduced by designating the unit vectors $\vec{y}_1$ coincident with the normal to the plane, $\vec{y}_2$ in the direction along the line where the inclined plane intersects with the horizontal plane and $\vec{y}_3$ in the direction of the cross product between $\vec{y}_1$ and $\vec{y}_2$, that is exactly on the inclined plan itself. With respect to the y-coordinates system, $\vec{y}_p$ and $\hat{v}_y$ will immediately be easily determined. For example, the vector $\vec{y}_p$ of the point P at the corner of an overhang as shown in Figure 2 can be written as $[0.8,0,1.5]^T$. Clearly, the unit vector $\hat{v}_y$ will always be $[1,0,0]^T$ in the y-coordinates. In maintaining the consistency of the coordinates system, $\hat{v}_s$ must as well be in the y-coordinates. The transformation of any vectors in the x to y coordinates can be performed through the use of

$$[Y] = \left[A_{xy}\right][X]^T \tag{6}$$

where $\left[A_{xy}\right]$ is a transformation matrix written as Eq.(7)

$$A_{xy} = \begin{bmatrix} \cos\beta & \sin\gamma\sin\beta & -\cos\gamma\sin\beta \\ 0 & \cos\gamma & \sin\gamma \\ \sin\beta & -\cos\beta\sin\gamma & \cos\gamma\cos\beta \end{bmatrix}. \tag{7}$$

Once all vectors have been represented with respect to the y-coordinates, Eq.(5) will then be used for computing the position of the shade on the inclined plane under two following important conditions: (i) the sun must be up, i.e. *sunrise time < solar time under consideration < sunset time*, and (ii) the sun must be facing the plane, i.e. the angle between $\hat{v}_s$ and $\hat{v}_y$ must be positive, $\hat{v}_s \cdot \hat{v}_y = \cos\theta > 0$. Otherwise, the plane is completely shaded.

Eq.(5) is essentially a key formula that computes a shadow of a point cast by sun ray. More importantly, it can be extended for determining a pattern of a shadow of any types of shading devices. For simple shading devices such as an overhang, a fin and an egg-crate type as shown in Figure 3, the pattern of the shadow can be found from interconnecting the position of the shadow corresponding to all corner points of the shading

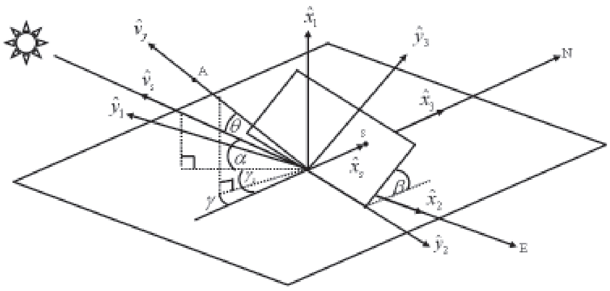


Figure 1. Vector representation for the sun, a shading point and shadow of the point.

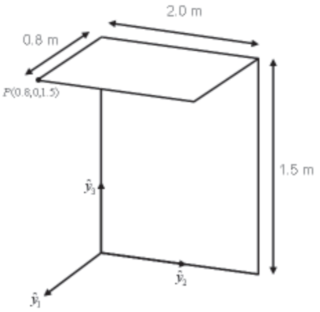


Figure 2. Coordinates of the point P.

device. For example of an overhang shown in Figure 4, the points $P_1^{'}$, $P_2^{'}$, $P_3^{'}$ and $P_4^{'}$ are the shadows of the points P_1, P_2, P_3 and P_4, respectively. The pattern of the shade can then be determined by interconnecting $P_1^{'}$ and $P_2^{'}$, $P_2^{'}$ and $P_3^{'}$, and $P_3^{'}$ and $P_4^{'}$.

However, the method described above could only be effectively applied(for shading devices having such simple and regular geometry. For irregular shaped shading devices, more points along the boundary of the shading device must be traced as shown in Figure 5. The method of point tracing offers its generality not only in determining the shade pattern but also in easily implementing the computational algorithm to compute the amount of shaded area. This can be best explained by considering the shading device shown in Figure 6. By starting from the first three shaded points $P_1^{'}(0, y_{21}, y_{31}), P_2^{'}(0, y_{22}, y_{32})$ and $P_3^{'}(0, y_{23}, y_{33})$, a triangular of the area A_1 having its vertices at $P_1^{'}$, $P_2^{'}$ and $P_3^{'}$ is formed. The area A_1 can be simply computed from

$$A_1 = \frac{1}{2}\left[\begin{vmatrix} y_{21} & y_{31} \\ y_{22} & y_{32} \end{vmatrix} + \begin{vmatrix} y_{22} & y_{32} \\ y_{23} & y_{33} \end{vmatrix} + \begin{vmatrix} y_{23} & y_{33} \\ y_{21} & y_{31} \end{vmatrix}\right]$$

$$\frac{1}{2}[y_{21}y_{32} + y_{22}y_{33} + y_{23}y_{31} - y_{22}y_{31} - y_{23}y_{32} - y_{21}y_{32}] \quad (8)$$

Further moving along the boundary of the shading device, there exist $P_4^{'}$, $P_5^{'}$, $P_6^{'}$, …, $P_n^{'}$ that corres-

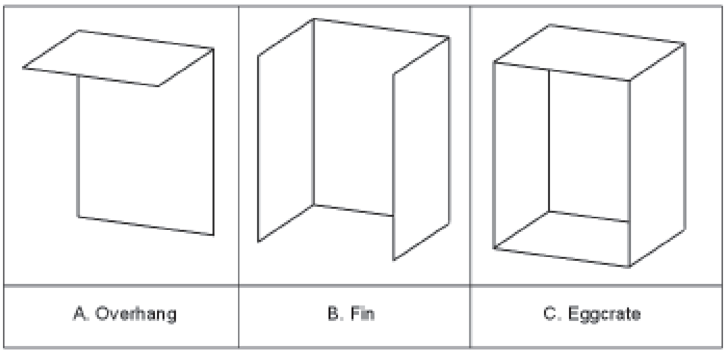


Figure 3. Typical shading device: overhang, fin and eggcrate.

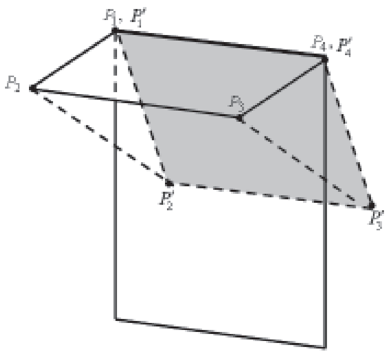


Figure 4. Shadow pattern obtained from interconnecting the position of the shadow corresponding to all corner points of the shading device.

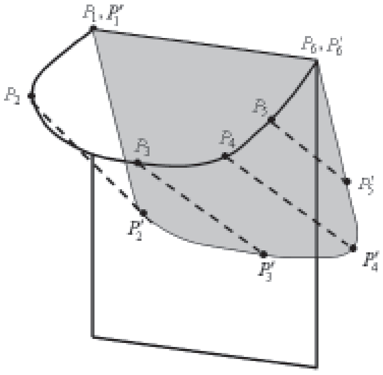


Figure 5 Shadow pattern of an irregular-shape shading device.

pondingly produce additional areas A_2, A_3, A_4,..., A_k. Consequently, the total shaded area A_s can be found from

$$A_s = \sum_{j=1}^{k} A_j \qquad (9)$$

where A_j is the incremental area that can be computed from the coordinates of all three vertices and using the Eq. (7).

This section is concluded by stating that the methodology for shading computation is explained. It is worth mentioning again that this paper shall present the development of efficient computational algorithm based on the methodology described above. Its generality and validity will be investigated by comparing the experimental results and the simulation ones presenting in the following sections.

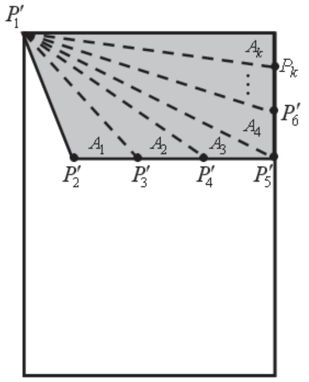


Figure 6. A series of contiguous triangles created by shadow point vertices.

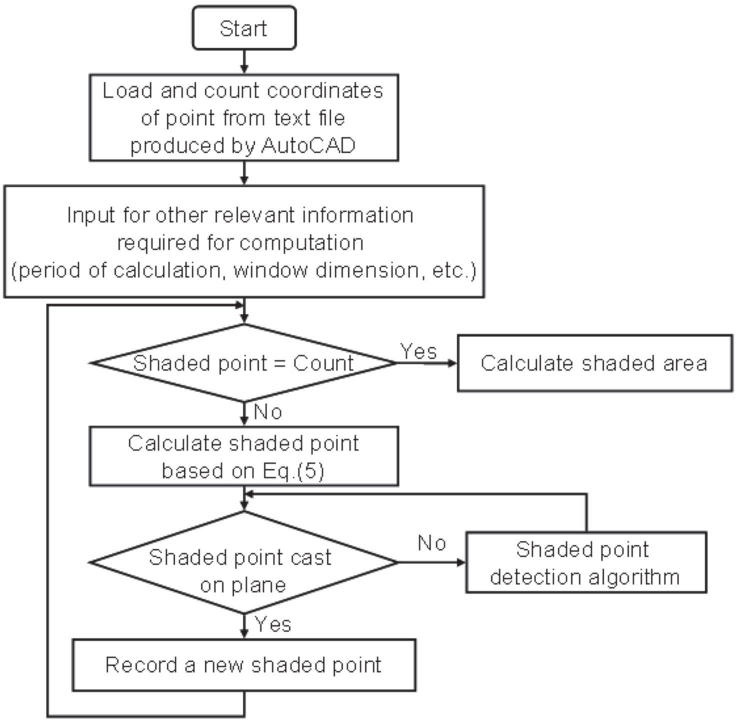
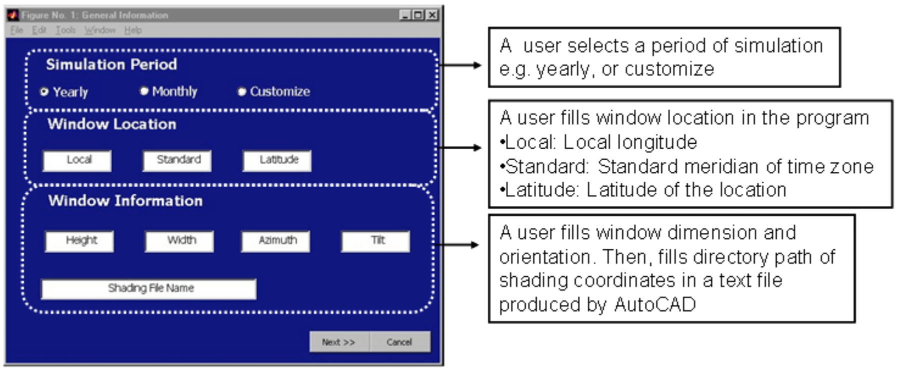

Start
Load and count coordinates of point from text file produced by AutoCAD
Input for other relevant information required for computation (period of calculation, window dimension, etc.)
Shaded point = Count
Yes
Calculate shaded area
No
Calculate shaded point based on Eq.(5)
Shaded point cast on plane
No
Shaded point detection algorithm
Yes
Record a new shaded point

Figure 7. Flow chart of shading calculation program.

Development of Shading Computational Tool

Based on the methodology described in the previous section, shading computational tool was developed using MATLAB, which is widely used in engineering computer programming. The calculation flow-chart is shown in Figure 7. The front-end of the developed tool is illustrated in Figure 8. In addition, AutoCAD - software used for computer aided design was additionally used for graphically creating a plane and shading device geometry. Moreover, it can automatically produce a text file that contains the information about the coordinates of a plane and shade points. For example, the window in Figure 2 can be drawn in AutoCAD front-end as shown in Figure 9 and then

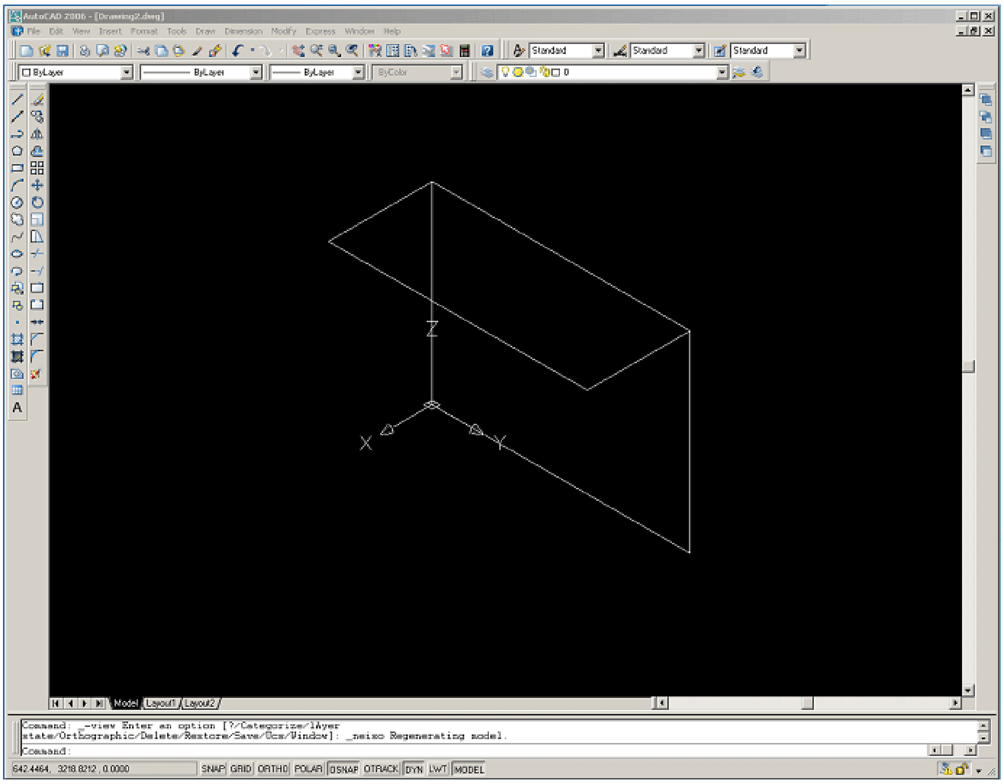


A user selects a period of simulation e.g. yearly, or customize

A user fills window location in the program
- Local: Local longitude
- Standard: Standard meridian of time zone
- Latitude: Latitude of the location

A user fills window dimension and orientation. Then, fills directory path of shading coordinates in a text file produced by AutoCAD

Figure 8. Matlab front-end of shading calculation program.



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Command: _-view Enter an option [?/Categorize/lAyer
state/Orthographic/Delete/Restore/Save/Ucs/Window]: _neiso Regenerating model.
Command:
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Figure 9. A window system created on AutoCAD.

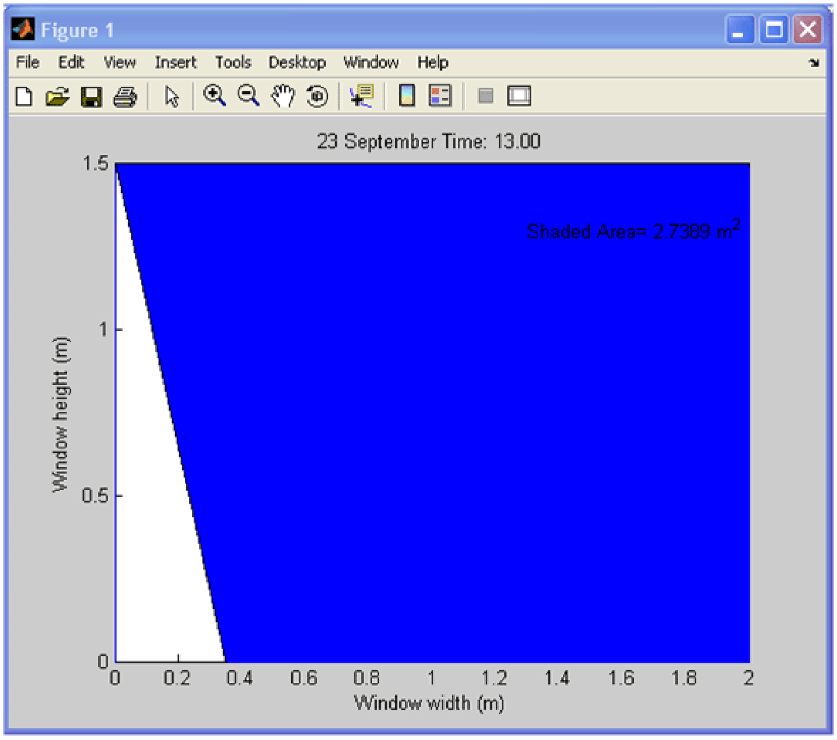


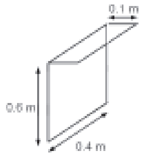
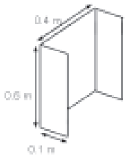
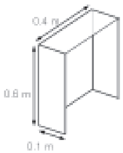
Figure 10. Example of simulated pattern and amount of a shaded area of a 1.5×2.0 window on 23 September at 13.00 hrs.

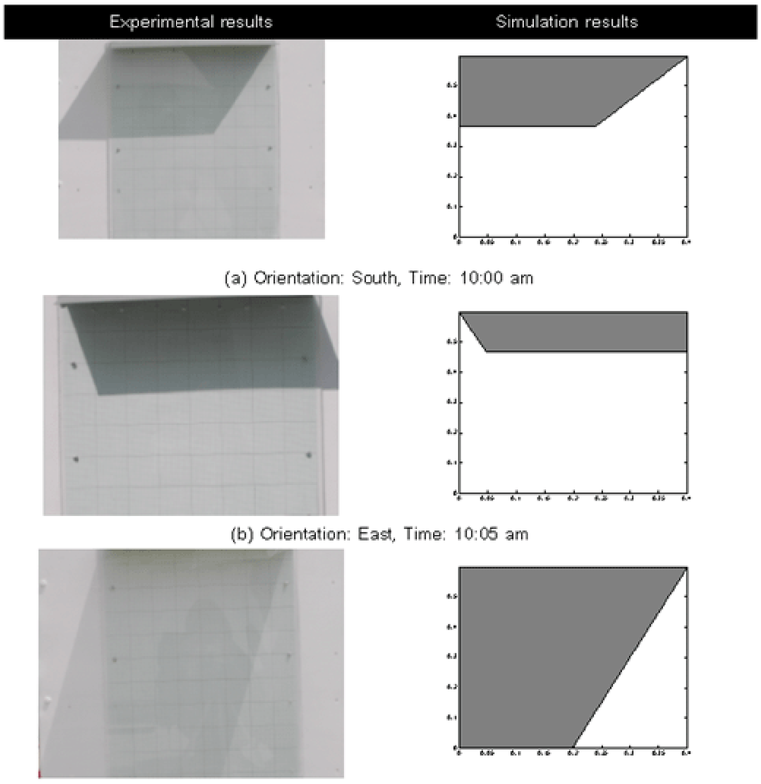
the coordinates of all relevant points required for shading computation were translated into a text file. Once the computing module was executed, these coordinates were then called to compute the corresponding positions of the shade based on Eq.5. Simultaneously, the shaded area was also computed based on the method of point tracing, in which Eq (8) and (9) are essentially employed. Figure 10 shows an example of the results including a graphical view of the shade pattern and the value of the shaded area.

Results Validation

The validation of shading calculation algorithm was performed by comparing experimental results with simulation ones. Experiments were carried out by observing and positioning shade cast on a 0.6m x 0.4m - plane that was equipped with a shading device and oriented in various directions. Three typical shading devices, namely overhang , fin and eggcrate, were taken for observation in the experiments. The experiments were conducted on the rooftop of School of Energy and Materials, King Mongkut's University of Technology Thonburi, Bangkok, Thailand. Geometry of the shading devices, dates of experiment, and orientations of the plane are presented in Table 1. Simultaneously, a series of simulations was performed for all cases. The pattern of the shade cast on a plane and the amount of shaded area can be obtained directly from the simulations. Examples of comparisons between experimental and simulated shading patterns for an overhang on March 10, 2004 and for a fin on March 13, 2004 are shown in Figure 11 and 12, respectively. Furthermore, the amount of area obtained from simulations are investigated and compared with experimental results as shown in Figure 13, 14 and 15 for the case of overhang, fin, and eggcrate, respectively. The results clearly show excellent agreement in which the average percentage of error

Table 1. Geometry of shading configurations, dates of experiment and orientations of the plane.

Types of shading devices	Geometrical descriptions	Dates of experiments	Orientations
Overhang		10 March 2004 13 March 2004 13 August 2005 15 August 2005	North East South West
Fin		10 March 2004 13 March 2004 13 August 2005 15 August 2005	North East South West
Eggcrate		4 March 2004 10 March 2004 13 August 2005 15 August 2005	North East South West

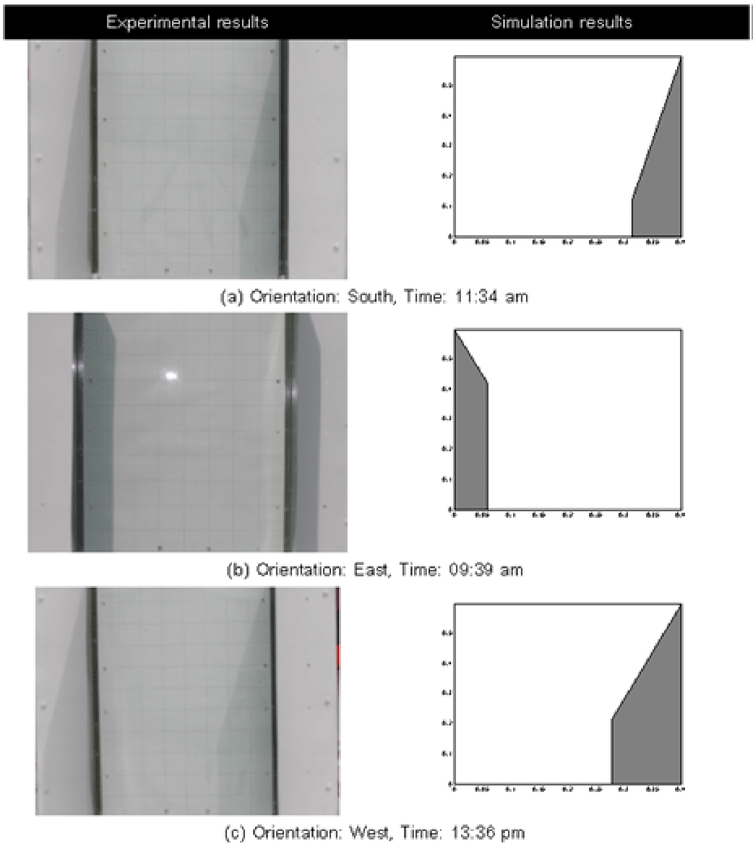


(a) Orientation: South, Time: 10:00 am

(b) Orientation: East, Time: 10:05 am

(c) Orientation: West, Time: 13:07 pm

Figure 11. Comparisons of shading patterns cast on a 0.6m x 0.4m plane equipped with a 0.1 m overhang at different orientations and time on 10 March 2004.



(a) Orientation: South, Time: 11:34 am

(b) Orientation: East, Time: 09:39 am

(c) Orientation: West, Time: 13:36 pm

Figure 12. Comparisons of shading patterns cast on a 0.6m x 0.4m - plane equipped with a pair of 0.15 m - fin at different orientation and time on 13 March 2004.

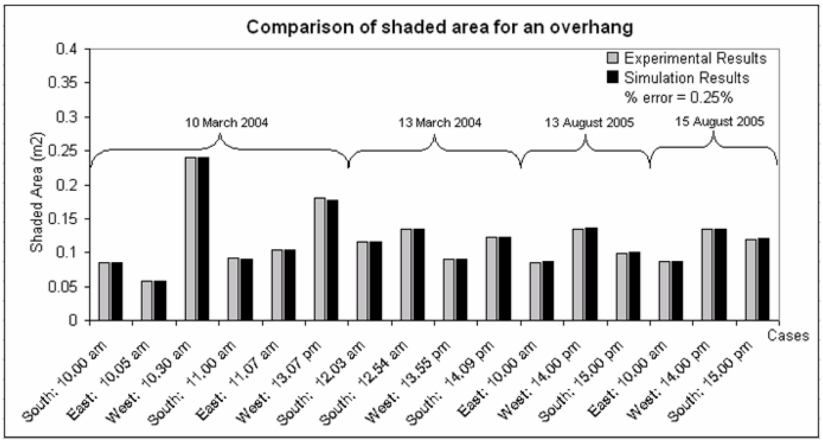


Figure 13. Comparison of the amount of shaded area obtained from experiment and simulation for the case of overhang on four different days: March 10, 2004, March 13, 2004, August 13, 2005 and August 15, 2005.

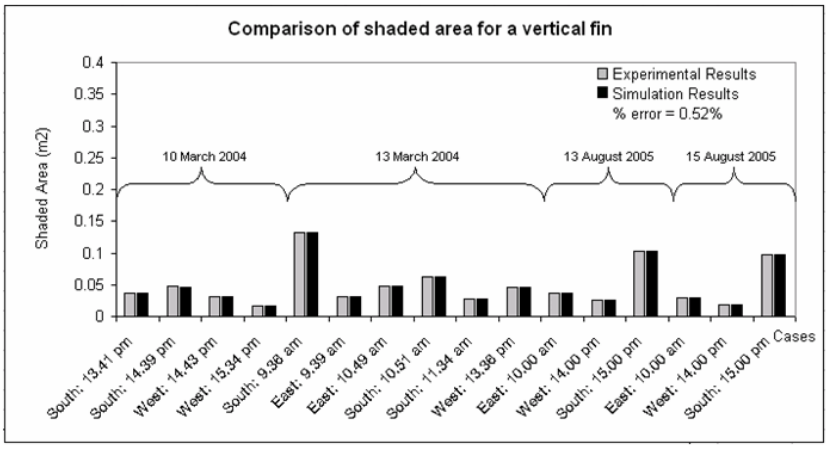


Figure 14. Comparison of the amount of shaded area obtained from experiment and simulation for the case of fin on four different days: March 10, 2004, March 13, 2004, August 13, 2005 and August 15, 2005.

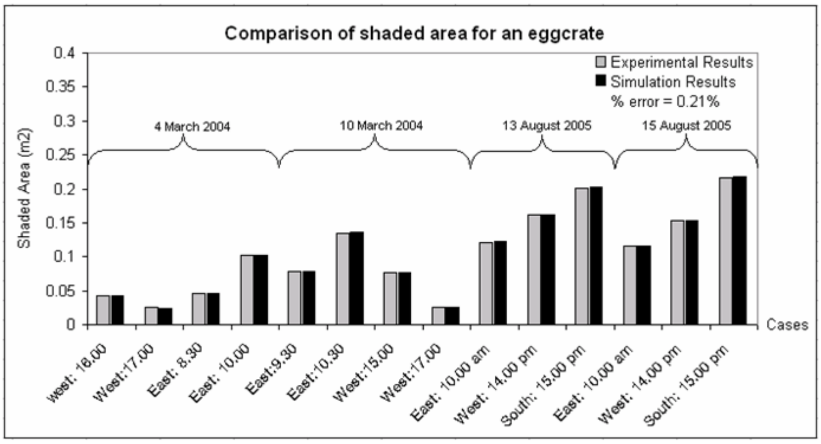


Figure 15. Comparison of the amount of shaded area obtained from experiment and simulation for the case of eggcrate on four different days: March 4, 2004, March 10, 2004, August 13, 2005 and August 15, 2005.

is approximately 0.25%, 0.52%, and 0.21% for the respective cases.

Conclusion

A computational algorithm and interactive computer software for computing the shadow were developed. The software was designed in order to be user-friendly and capable of presenting profiles of the shadow graphically and computing corresponding shaded areas for a given window system. The algorithm was experimentally tested to examine its validity and generality. It was found that software simulation results were in excellent agreement with experimental results. Comparisons showed that maximum errors between the two are less than 1%.

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