
INVITED ARTICLE

Analytical model of interaction of tide and river flow

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Abstract

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Hydrodynamic characteristics of a river resulting from interaction of tide and river flow are important since problems regarding flood, salinity intrusion, water quality and sedimentation are ubiquitous. The lower reach of the river strongly influenced by tides from the sea, when interacting with river flows, results in a complicated pattern which is simplified to its interaction with four main constituents of tides obtained from harmonic analysis. An analytical model is developed in this study for simulating the hydrodynamic processes in estuarine waters, with the emphasis being given to the interaction between tides and river flows. The perturbation method is used to derive the analytical solution, in which the estuarine flow is separated into steady and unsteady components. Thus the analytical solutions derived consist of two distinct parts; one represents the influence of river flows and the other represents the influence of tides. The application of the model to a case study, the Chao Phraya river, which requires a time series of discharges and loadings at the river mouth to model water quality in the Gulf of Thailand, shows that the model can beautifully and completely simulate the hydrodynamic features of tide and river flow interaction especially in the rainy season when the river discharge is high. Data of tidal discharges are scarce because of high cost of measurement especially in the lower reach of the river strongly influenced by tides from the sea. From this study of relation between tidal discharges and tides, the analytical model can compute tidal discharges from tides correctly. The results of tides and tidal flow can subsequently be used to calculate eddy viscosity and dispersion coefficient for describing salinity and water quality profiles.

Key words : analytical techniques, estuaries, interaction, river flow, tides

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บทคัดย่อ

สุกัธ วงศ์เสมโนใจ¹ และ ไฟเรงน์ พัตรอนันท์เวช²
แบบจำลองทางทฤษฎีของปฏิกิริยาระหว่างคลื่นน้ำขึ้นน้ำลงกับอัตราการไหลของน้ำจืด
จากแม่น้ำ

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สภาพคลาสตร์ของแม่น้ำจากปฏิกิริยาระหว่างน้ำขึ้นน้ำลงกับอัตราการไหลของน้ำจืดจากแม่น้ำ มีความสำคัญ เพราะปัญหาน้ำท่วม การรุกล้ำของน้ำเค็ม คุณภาพน้ำและการตกตะกอนมีอยู่ทั่วไปทุกสถานที่ บริเวณปากแม่น้ำได้รับอิทธิพลของน้ำขึ้นน้ำลงจากทะเลเมื่อทำปฏิกิริยากับน้ำจืดจากแม่น้ำจะมีรูปแบบที่ยุ่งยาก เนื่องจากคลื่นน้ำขึ้น น้ำลงมีหลายตัว ซึ่งสามารถปรับให้ง่ายโดยการแทนรูปแบบที่ยุ่งยากด้วยตัวแทนที่สำคัญเพียง 4 ตัว ซึ่งได้จากการวิเคราะห์โดยวิธีสาร์มอนิก (Harmonic Analysis)

แบบจำลองทางทฤษฎีได้พัฒนาขึ้นจากการศึกษาเพื่อจำลองกระบวนการทางชลศาสตร์ในพื้นที่น้ำกร่อยโดยเน้นปฏิกิริยาระหว่างน้ำขึ้นน้ำลงกับอัตราการไหลของน้ำจืดจากแม่น้ำโดยใช้วิธี การปรับเปลี่ยน (Perturbation) ในการทำคำตบอนโดยแบ่งอัตราการไหลของน้ำในพื้นที่น้ำกร่อยเป็นสองส่วน คือ ส่วนของน้ำจืดจากแม่น้ำซึ่งไม่เปลี่ยนแปลงกับเวลาและส่วนของน้ำขึ้นน้ำลงซึ่งเปลี่ยนแปลงกับเวลา ดังนั้นคำตบอนที่ได้จากการจำลองทางทฤษฎีประกอบด้วยสองส่วนคือ อิทธิพลของน้ำจืดจากแม่น้ำและอิทธิพลของน้ำขึ้นน้ำลง เมื่อนำผลจากแบบจำลองมาใช้กับแม่น้ำเจ้าพระยาซึ่งต้องการใช้อัตราการไหลของแม่น้ำที่ปักแม่น้ำในการจำลองคุณภาพของน้ำในอ่าวไทยปรากฏว่าได้ผลถูกต้องเป็นอย่างดี ในการจำลองสภาพทางชลศาสตร์ของปฏิกิริยาระหว่างน้ำขึ้นน้ำลงกับอัตราการไหลของน้ำจืดจากแม่น้ำโดยเฉพาะอย่างยิ่งในฤดูฝน ซึ่งอัตราการไหลของน้ำจืดจากแม่น้ำมีค่าสูง

ข้อมูลอัตราการไหลของน้ำไม่ค่อยมี เพราะต้องใช้ค่าใช้จ่ายสูงในการตรวจวัด โดยเฉพาะอย่างยิ่งบริเวณปากแม่น้ำที่ถูกอิทธิพลของน้ำขึ้นน้ำลง จากการศึกษาความสัมพันธ์ระหว่างอัตราการไหลของน้ำกับระดับน้ำขึ้นน้ำลง บริเวณปากแม่น้ำในการศึกษานี้ สามารถคำนวณอัตราการไหลของน้ำจากข้อมูลระดับน้ำขึ้นน้ำลงได้อย่างถูกต้อง ผลที่ได้จากการจำลองทางทฤษฎีสามารถนำไปใช้คำนวณสัมประสิทธิ์ความปั่นปวนของมวลน้ำ หรือสัมประสิทธิ์การผสมผสานของน้ำเค็มเพื่อใช้ในการทำการแพร่กระจายของน้ำเค็มและคุณภาพน้ำ

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While studies on the simplified features of tides in canals and gulfs are plentiful, studies on the interaction of tide and river flow are scarce. Tides generated by astronomical forces can be properly represented by harmonic analysis of recorded tides. In rivers directly adjacent to an ocean, the variation of water surface along the rivers relies on tides at the estuary and the fresh-water flows from upstream. Logs of water levels along rivers show that an increase in the discharges damps tides along the rivers and reduces the celerity of the tides resulting in a raising of the mean water level along the rivers. When a high tide

is superimposed on a high discharge, the risk of flooding in the risk area is high. Accordingly, it is essential to investigate and evaluate quantitatively the interaction of tide and river flow to better understand the phenomenon and be able to formulate the suitable approach for its solution. Acquired information on water surface variation is very useful for flood protection and mitigation as well as navigation, while that on flow characteristics is very useful for analyses of salinity intrusion, water quality, sedimentation, etc.

Ippen and Harleman (1966) reported results of tests in a tidal flume at the Waterways Ex-

periment Station, Vicksburg, U.S.A., on the interaction of tide with river flow. They also investigated analytically the characteristics of damped tides with linear frictional force. Leblond (1978) examined the flow regime relevant to tidal propagation in shallow rivers using the dimensionless equations derived from the dimensional hydrodynamic governing equations. He considered 1-D tidal propagation with narrow rectilinear channel of uniform depth and width, and took into account downstream freshwater discharge as well. After reexamination of the momentum balance in shallow rivers with scaling suitable for the Saint Lawrence and the Fraser rivers in the study, he assumed the frictional forces exceeded acceleration over most of the tidal cycle. Consequently, the tidal propagation in shallow rivers was envisaged as a diffusion phenomenon rather than as a wave propagation phenomenon. The long time lags associated with low waters, unexplainable in terms of a simple wave propagation model, were then accounted for by the simple diffusive model.

Godin (1985) analyzed the tidal records of three rivers in Canada to determine the influences of freshwater discharge on the time of travel and on the range of the tides. He found a clear relationship between the tide and the discharge: friction and freshwater flow were shown to be intrinsically linked, because friction was perceived only when currents were flowing. Based on 1-D hydrodynamic equations, he considered the current to be made up of a steady component created by freshwater discharge and a time-dependent component contributed by tide. The solutions were expressed in functional forms and the important parameters describing the interaction of tide and river flow were identified. The analytical solution, however, was not derived because of the complicated governing equations. Poor correlation was obtained from the analysis since the recorded water levels used were not decomposed into various constituents of tides through harmonic analysis. Vongvisessomjai and Rojanakamthorn (1989) applied the perturbation method to solve analytically the interaction of tide and river flow taking into account the nonlinear convective inertia and

bottom friction. Nevertheless, the application of the analytical solution to natural rivers was still incomplete. Moreover, emphasis was made only on water level fluctuation data.

In the present study, the analytical model simulating water surface fluctuation and tidal flow is developed based on the analytical results from Vongvisessomjai and Rojanakamthorn (1989) and theory of harmonic analysis. The manifestation of the usefulness of the analytical model, which successfully simulates hydrodynamic characteristics of tide and river flow interaction, can be seen in the case study part of this paper. Emphasis is made in this study on prediction of tides and tidal flow in variations of both spatial and temporal scales.

Theoretical considerations

1. Governing equations

The unsteady one-dimensional equation of continuity and the equation of motion are used to describe the interaction of tide and river flow taking into account the convective inertia force and bottom frictional force. The present study is useful exclusively for a river that has rather constant cross-sectional area along the studied tidal reach area. Consider a definition sketch of coordinates and variables used as shown in Figure 1. A coordinate system is adopted so that the x -axis is horizontal along the still water level (SWL), the origin represents the river mouth, and the upstream direction is positive. The z -axis is perpendicular to x with its origin at SWL and is positive above the origin. If there is no lateral inflow, the continuity equation can be written in the form as follows:

$$\frac{\partial \eta}{\partial t} + uS_b + u\frac{\partial \eta}{\partial x} + (h + \eta)\frac{\partial u}{\partial x} = 0 \quad (1)$$

in which η is the instantaneous displacement of water surface above mean water level, u is the instantaneous flow velocity, $h + \eta$ is the water depth, h is the depth of bed below datum and bed slope $S_b = \partial h / \partial x$ of the reach is negative in this formulation.

After applying the Newton's law of motion

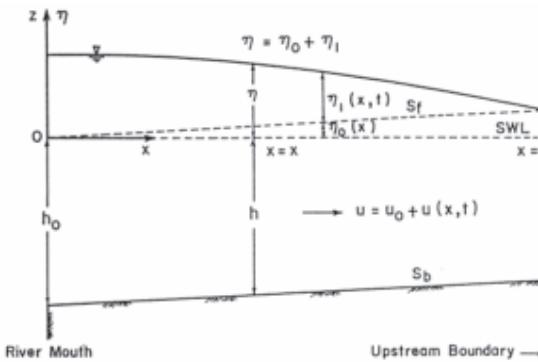


Figure 1. Definition sketch of coordinate and variable.

to one-dimensional flow through an element of water, the equation of motion can be written as follows:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \eta}{\partial x} + g S_f = 0 \quad (2)$$

in which the flow velocity in the river u is made up of a steady component, u_0 created by the discharge of the freshwater and of a time-dependent component, $u(x, t)$, contributed by tide, and g is the gravitational acceleration. The frictional slope S_f for unsteady flow in this case is assumed to have the same form as the steady flow and therefore can be evaluated from the Chezy's equation as:

$$S_f = \frac{u|u|}{C_z^2(h + \eta)} \quad (3)$$

The Chezy's coefficient C_z can be expressed alternatively in terms of the Manning's n or the friction factor f as follows:

$$\frac{g}{C_z^2} = \frac{gn^2}{h^{1/3}} = \frac{f}{2} \quad (4)$$

2. Perturbation method and application

Because the governing equations derived are nonlinear, hyperbolic, partial differential equations, the analytical solution has been solved by applying the perturbation method by Vongvisessomjai and Rojanakamthorn (1989). The perturbation technique or small parameter method is a well-known analytical tool to find approximate solutions to nonlinear problems. Assuming that the certain equations, and possibly the boundary conditions,

depend upon a parameter ε , the general perturbation problem is that of finding the solution for small value of ε , given the linear solution for $\varepsilon = 0$ (the standard system). The parameter ε can then be thought of as a measure of how much the system is perturbed from the standard one that is the linear system. The basic idea of the method is to seek a solution in the form of a power series expansion of the parameter. From the governing Eqs. (1) and (2), it is assumed that the solutions for η and u can be expressed in terms of a power series of the small parameter ε as follows:

$$\eta = \eta_0 + \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \dots \quad (5)$$

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots \quad (6)$$

where u_0 is treated as the freshwater velocity, considered to be constant for a certain period of time. The frictional slope, Eq. (3), is expanded in terms of the small perturbation parameter as follows:

$$S_f = S_{f0} + \varepsilon S_{f1} + \varepsilon^2 S_{f2} + \dots \quad (7)$$

$$\text{where } S_{f0} = -\frac{u_0^2}{C_z^2(h + \eta_0)}$$

$$\text{and } S_{f1} = -\frac{2u_0 u_1}{C_z^2(h + \eta_0)} + \frac{\eta_1 u_0^2}{C_z^2(h + \eta_0)^2}$$

Note that in this study, the perturbation method has been used in order to purposefully separate steady and unsteady components as zeroth and higher orders. This does not mean that the

zeroth order river flow is more significant than the higher order tidal current because the solutions are direct summation of zeroth order and higher order solutions without multiplication with ε or ε^2 as shown in Eqs. (20) and (23). After substituting Eqs. (5) - (7) into the governing Eqs. (1) and (2), a set of two equations can be arranged according to the order of the parameter as follows:

Zeroth order:

$$\frac{\partial \eta_0}{\partial t} + u_0 S_b + u_0 \frac{\partial \eta_0}{\partial x} + (h + \eta_0) \frac{\partial u_0}{\partial x} = 0 \quad (8)$$

$$\frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial x} + g \frac{\partial \eta_0}{\partial x} + g S_{f0} = 0 \quad (9)$$

First order:

$$\frac{\partial \eta_1}{\partial t} + u_1 S_b + u_1 \frac{\partial \eta_0}{\partial x} + u_0 \frac{\partial \eta_1}{\partial x} + (h + \eta_0) \frac{\partial u_1}{\partial x} + \eta_1 \frac{\partial u_0}{\partial x} = 0 \quad (10)$$

$$\frac{\partial u_1}{\partial t} + u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} + g \frac{\partial \eta_1}{\partial x} + g S_{f1} = 0 \quad (11)$$

The nonlinear convective inertia terms in Eq. (10) and bottom friction term in Eq.(11) are linearized by replacing the unknown η_1 and u_1 with known zeroth order solutions of η_0 and u_0 .

Eliminating u_1 from the above two Eqs. (10) and (11) yields the first order governing equation which has been linearized as follows:

$$\frac{\partial^2 \eta_1}{\partial t^2} + 2u_0 \frac{\partial^2 \eta_1}{\partial x \partial t} + (u_0^2 - gh_0) \frac{\partial^2 \eta_1}{\partial x^2} - \frac{2gu_0}{C_z^2 h_0} \frac{\partial \eta_1}{\partial t} - \frac{3gu_0^2}{C_z^2 h_0} \frac{\partial \eta_1}{\partial x} = 0 \quad (12)$$

where: $h_0 = h + \eta_0$

The zeroth order governing eqnations 8 and 9 are linearized by the steady flow condition :

$$u_0 = \text{Constant} \quad (13)$$

$$\text{and } \frac{d\eta_0}{dx} = -S_b = -S_{f0}$$

$$\text{Upon integration and applying boundary condition, } x = 0, \eta_0 = 0 \text{ yields, } \eta_0 = -S_{f0} x \quad (14)$$

The standard form of solution of the first order governing Eq. (12) has been solved by a more general method as shown in Eq. (15) which is the same as that of Ippen and Harleman (1996) with its corresponding velocity, Eq.(16), and their parametric Eqs. (17) and (18), to define k and μ .

$$\eta_1 = a_0 \exp(-\mu x) \cos(\sigma t - kx) \quad (15)$$

$$u_1 = \frac{a_0}{h_0} c_0 \exp(-\mu x) \frac{k_0^2}{k^2 + \mu^2} \left\{ \left[\frac{k}{k_0} - \frac{k^2 + \mu^2}{k_0^2} \left(\frac{u_0}{c_0} \right) \right] \cos(\sigma t - kx) - \frac{\mu}{k_0} \sin(\sigma t - kx) \right\} \quad (16)$$

$$\frac{k}{k_0} = \frac{\sqrt{1 + \left(\frac{\mu}{k_0} \right)^2 - 2 \left(\frac{\mu}{k_0} \right)^2 \left(\frac{u_0}{c_0} \right)^2 + \left(\frac{\mu}{k_0} \right)^2 \left(\frac{u_0}{c_0} \right)^4} - \frac{3g}{C_z^2 k_0 h_0} \left(\frac{\mu}{k_0} \right) \left(\frac{u_0}{c_0} \right)^2 \left[1 - \left(\frac{u_0}{c_0} \right)^2 \right] - \frac{u_0}{c_0}}{1 - \left(\frac{u_0}{c_0} \right)^2} \quad (17)$$

$$\frac{\mu}{k_0} = \frac{g}{C_z^2 k_0 h_0} \left[\frac{3}{2} \frac{k}{k_0} \left(\frac{u_0}{c_0} \right)^2 - \frac{u_0}{c_0} \right] \left/ \left[\frac{k}{k_0} - \frac{k}{k_0} \left(\frac{u_0}{c_0} \right)^2 + \frac{u_0}{c_0} \right] \right. \quad (18)$$

In relations as Eqs. (15) to (18), u_0 is the amplitude of tide at the river mouth, μ is the damping modulus, $\sigma = 2\pi/T$ is the angular velocity, T is the period of tide, $k = 2\pi/L$ is the tide number, L is the length of tide, $k_0 = 2\pi/L_0$ is the tide number at the river mouth, L_0 is the length of tide at the river mouth, and c_0 is the frictionless celerity. For known values of u_0/c_0 and $g/(C_z^2 k_0 h_0)$, in order to define the presumed solution η_1 , Eq. (15), the above Eqs. (17) and (18) have to be used to compute dimensionless tide number k/k_0 and dimensionless damping modulus μ/k_0 by trial and error. Figure 2 shows the computations of both values in variations of Froude number and the term $g/(C_z^2 k_0 h_0)$. The approximate values can be extrapolated from these graphs.

3. Harmonic analysis of tide

Because tide is composed of various constituents which interact simultaneously with the river flow, the resulting records of water levels in the river show rather complicated patterns. Individual interaction of each constituent of tide in each month is gained by the harmonic analysis of the hourly water levels. The characteristics of the tide can then be correlated with the river flow. Four predominant constituents of the tide are analyzed for their characteristics as follows:

1. Principal lunar M_2 with a period of 12.4206 hours
2. Principal solar S_2 with a period of 12.0000 hours
3. Luni-solar declinational K_1 with a period of 23.9346 hours
4. Large lunar declinational O_1 with a period of 25.8194 hours

The formula of the harmonic analysis of tide is as below:

$$\eta_r(t) = \Delta h + \sum_{i=1}^N a_i \sin \left[\frac{2\pi t}{T_i} + \delta_i \right] \quad (19)$$

in which $\eta_r(t)$ is the resultant tide recorded as a function of time t at a particular location and it is composed of N constituents. The periods T_i of the i -constituents are known from the astronomical computations. The mean water level Δh , the amplitudes a_i , and phases δ_i are easily calculated from a discrete hourly water level, η_r . It can be noticed that Δh can represent zeroth order of water level of steady flow and the summation terms of various constituents of tides can represent the first order for unsteady flow.

Analysis and results

From eclectic analysis of all above theoretical considerations, which are results as Eqs. (13) and (15) and harmonic analysis theory as Eq. (19), the analytical form of the water surface fluctuation for various constituents can be consequently developed to be in the form as follows:

$$\eta(x, t) = (-S_{f0}x + \Delta h_0) + \sum_{i=1}^N a_{0i} \exp(-\mu_i x) \cos \left(\frac{2\pi t}{T_i} - k_i x + \delta_{0i} - \frac{\pi}{2} \right) \quad (20)$$

$$\text{where: } S_{f0} = \frac{u_0 |u_0|}{C_z^2 (h + \eta)}$$

in which Δh_0 is the mean water level at estuary and δ_{0i} is the phases δ_i at the estuary obtained from the harmonic analysis of tide. The term $-\pi/2$ is added since the phase angle δ_{0i} is calculated from the harmonic analysis using the sine function as Eq. (19). Accordingly, when the obtained phase angle is

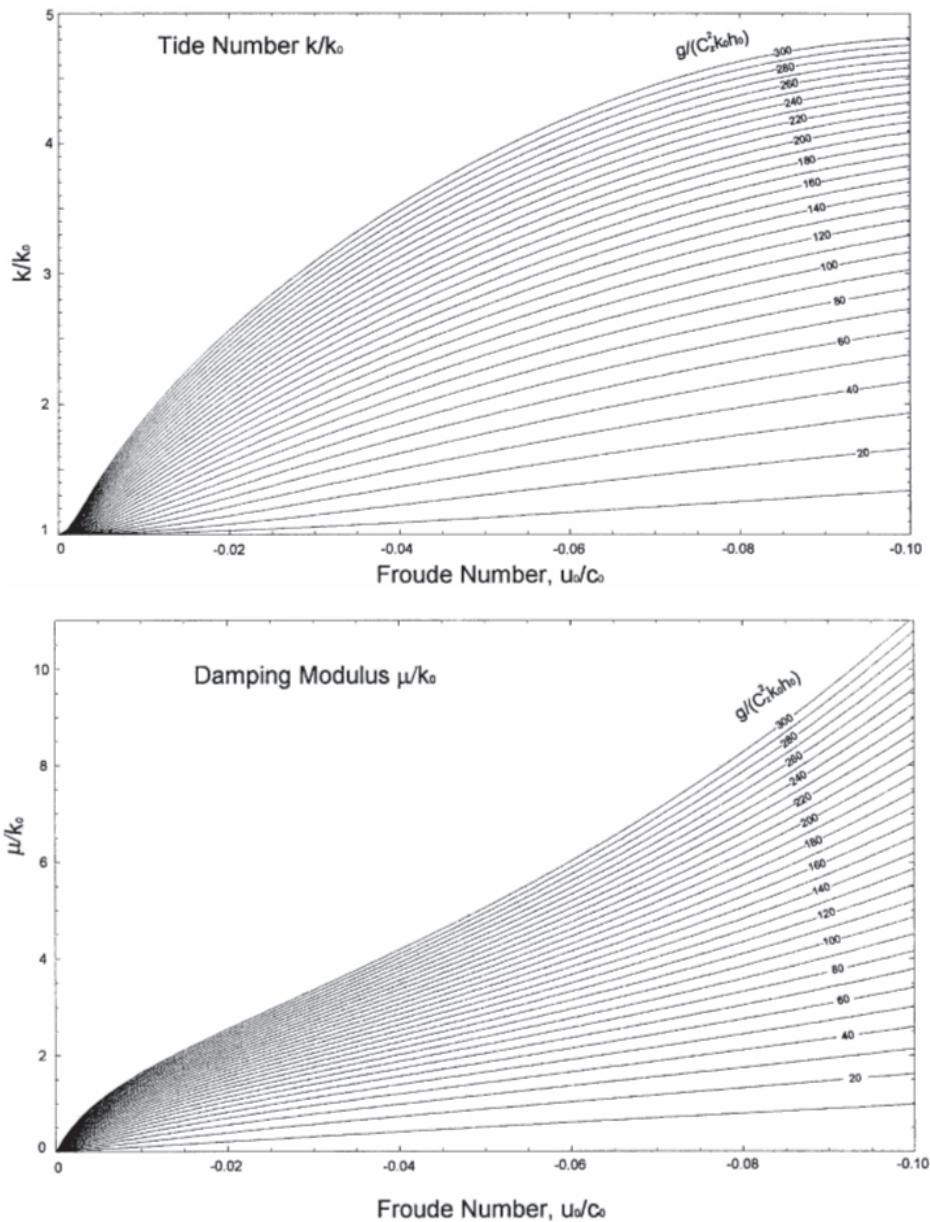


Figure 2. Effect of Froude number u_0/c_0 on tide number k/k_0 and damping modulus μ/k_0 .

used in the cosine function in the Eq. (20), it is then added the term $-\pi/2$ for its conversion. Again, note that in this analysis, the origin represents the river mouth, and the upstream direction is positive. Therefore, the freshwater velocity u_0 is considered negative when it flows from upstream to estuary. By knowing the value of freshwater velocity, the values of tide number k and damping modulus μ for a 1-D river can then be obtained easily from Figure 2.

The tidal velocity Eq. (16) is adjusted to conform with the harmonic analysis of tide Eq.(19) as :

$$u(x,t) = \frac{a_0}{h} c_0 e^{-\mu x} \frac{k_0}{\sqrt{\mu^2 + k^2}} \cos\left(\sigma t - kx + \delta_0 + \alpha - \frac{\pi}{2}\right) \quad (21)$$

By multiplying by effective cross-sectional area, A_e , we obtain the tidal discharge as :

$$Q(x,t) = \frac{a_0}{h} c_0 A_e e^{-\mu x} \frac{k_0}{\sqrt{\mu^2 + k^2}} \cos\left(\sigma t - kx + \delta_0 + \alpha - \frac{\pi}{2}\right) \quad (22)$$

Therefore, one finds the equation of tidal flow in the tide and river flow interaction for various constituents as follows :

$$Q(x,t) = Q_f + A_e \sum_{i=1}^N \frac{a_{0i}}{h} c_0 \exp(-\mu_i x) \frac{k_{0i}}{\sqrt{\mu_i^2 + k_i^2}} \cos\left(\frac{2\pi t}{T_i} - k_i x + \delta_{0i} + \alpha_i - \frac{\pi}{2}\right) \quad (23)$$

where: $\alpha_i = \tan^{-1}\left(\frac{\mu_i}{k_i}\right)$ (24)

in which $Q_f = u_0 A_e$ is the freshwater discharge that is in negative sign when it flows from upstream to estuary.

Case study: the Chao Phraya river

The Chao Phraya river is the most important river in Thailand. It occupies most of northern and central regions of Thailand and its length is about 1,000 km. It drains into the Gulf of Thailand where a strong mixed tide prevails at the estuary. Severe flood in the Chao Phraya delta occurred quite often in the past, for instance, in 1980, 1983, 1995, and 2002. In 1995, flood damage in this whole river was evaluated to be about 12 billion baht. The cross-section along the lower reach of 112 km of this river is somewhat uniform, possessing an average width of 400 m and an average depth of 9.2 m. The one-dimensional flow can then be assumed to be used in this river. Consequently, the theory can be applied properly in the Chao Phraya river. Tides recorded at five stations in this lower reach in 1983 are used in this study. The lower reach of the river and the five stations are illustrated in Figure 3.

The comparisons of water level between the present study (Eq. (20)) and measurements at five stations along the river with respect to time are shown in Figure 4. Figure 5 shows the figuration of the analytical model of water surface fluctuation in variations of both spatial and temporal scales through a three-dimensional plot. The comparisons of water level between the computation and observation with respect to longitudinal distance in various times, three-hour interval from 15 to 16 October 1983, are also displayed in Figure 6.

From Eq. (23), the tidal discharge along the river can then be calculated as shown in Figure 7 together with the results from the numerical model. Note that it is imperative to interpret the output from a numerical model in order to conform with the present study. For example, in DHI software (MIKE11) the results of tidal flow are in the opposite sign of ones from this study. This is because of the mechanism of governing equations. In other words, the governing equations in DHI software are given that downstream direction is positive. It can be readily noticeable regarding this that the average flow (or freshwater flow) from DHI software is positive when it flows from upstream to estuary. Therefore, when comparing to results from the present study, the outputs from numerical model must be adapted into the same-sign system first.

The obtained results, particularly on tidal flows, are very useful for the current AIT research project (2002) on improvement of mathematical water quality models and knowledge transfer on model usage. This research requires a time-series of tidal flows at estuaries from four major rivers draining into the Upper Gulf of Thailand for modeling of hydrodynamics, water quality, eutrophication, and heavy metal by using MIKE21 modules (DHI software). Data of tidal discharges are scarce because of high cost of measurement especially in the lower reach of the river strongly influenced by tides from the sea.

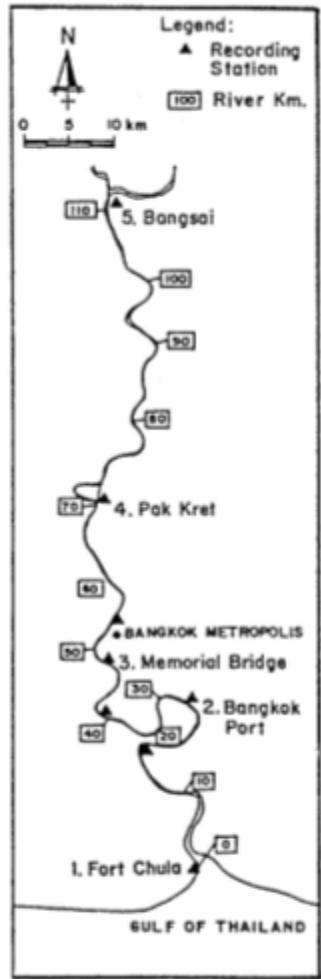


Figure 3. The lower reach of the Chao Phraya river.

The results of the present study are also important for computing the dispersion coefficient, a very important parameter when solving the advective-dispersion equation to predict the unsteady salinity intrusion in an estuary, since it is dependent on the parameters of salinity concentration, salinity gradient, tides, and tidal flow (Hieu, 1993).

Limitation of model

The following assumptions are used in the derivation of the analytical model which require some justifications as follows:

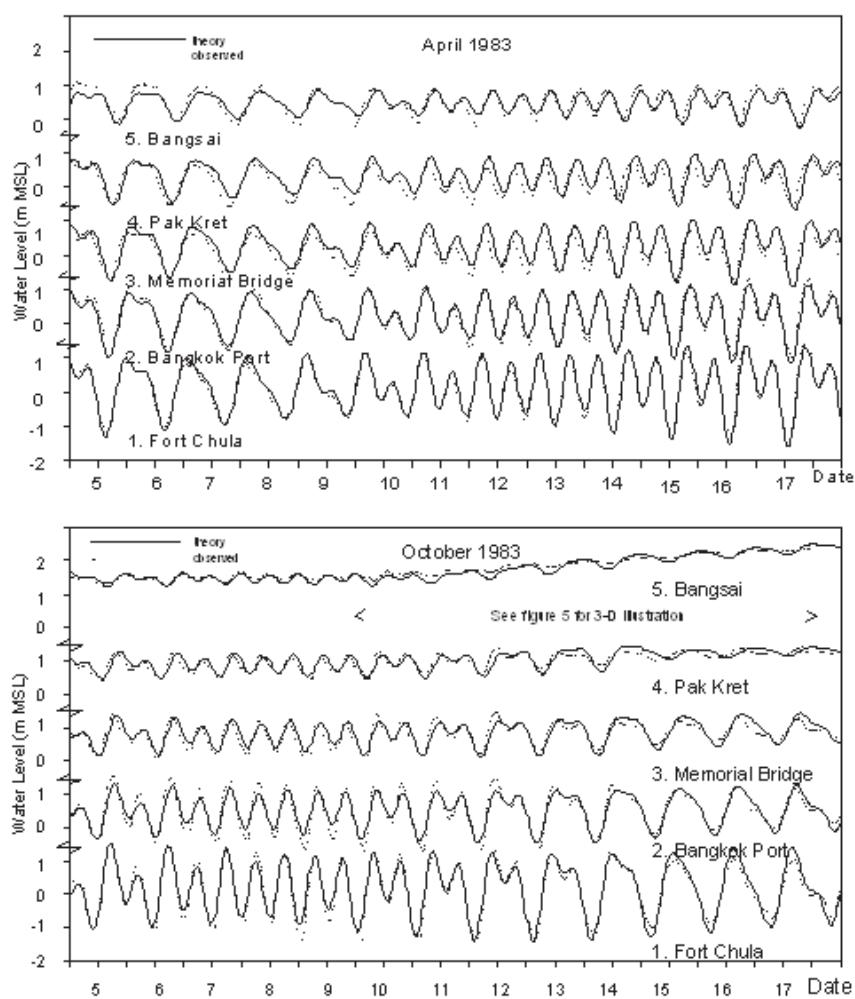


Figure 4. Comparison of water level between theoretical results and observed data.

1. Constant depth and width

The Chao Phraya river is the most important river in Thailand. The Bangkok Port is located at km 28 from the river mouth as shown in Figure 3. Regular maintenance dredging is made of the navigation channel of the Bangkok Port that the water depth of 9.2 m and river width of 400 m are kept almost constant along the lower reach of 112 km of this river.

2. Steady state discharge

Since the Chao Phraya river is a large river with a catchment area of 162,000 km² and a length

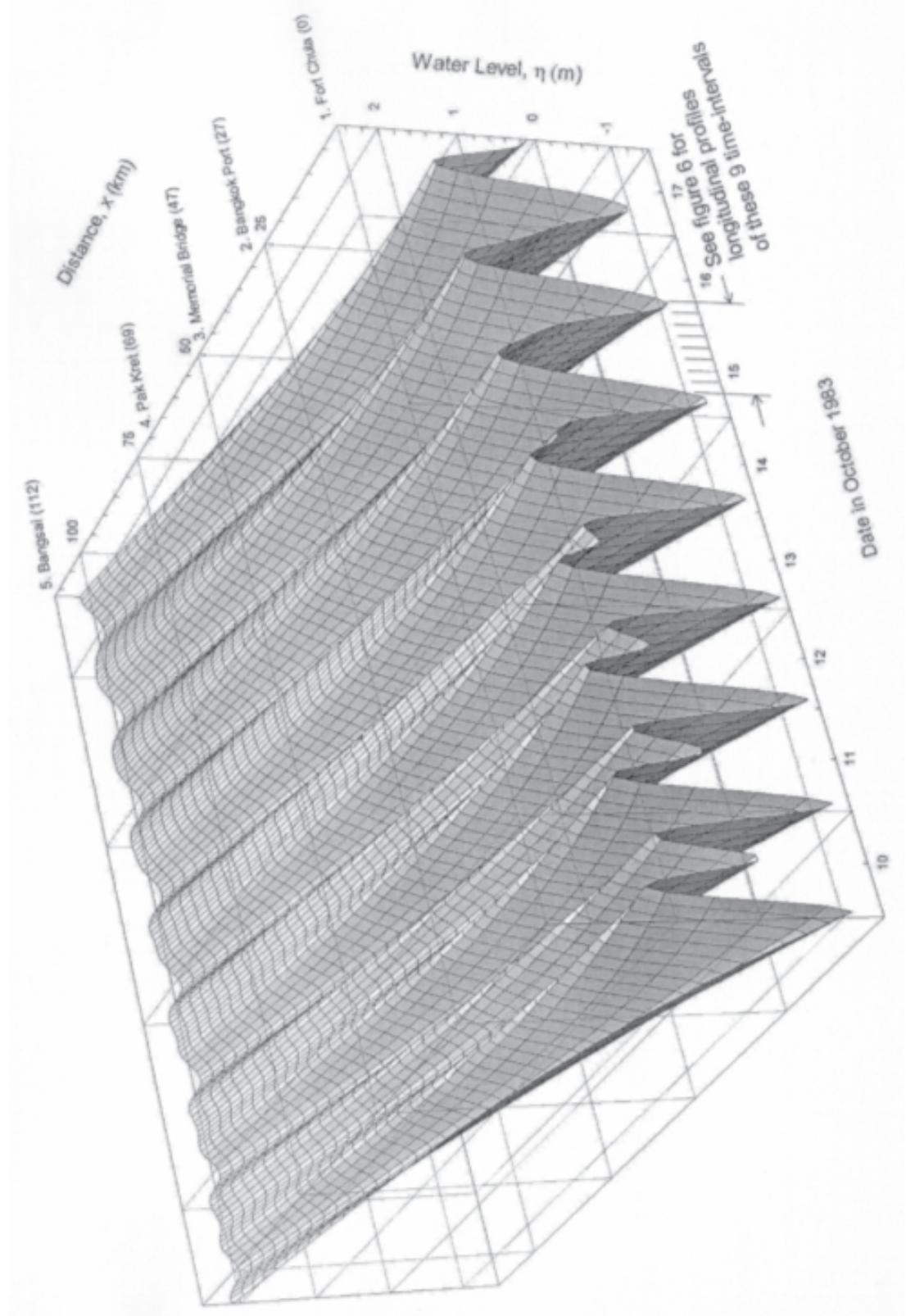


Figure 5. Three-dimensional plot of the analytical model of water surface fluctuation with respect to distance and time.

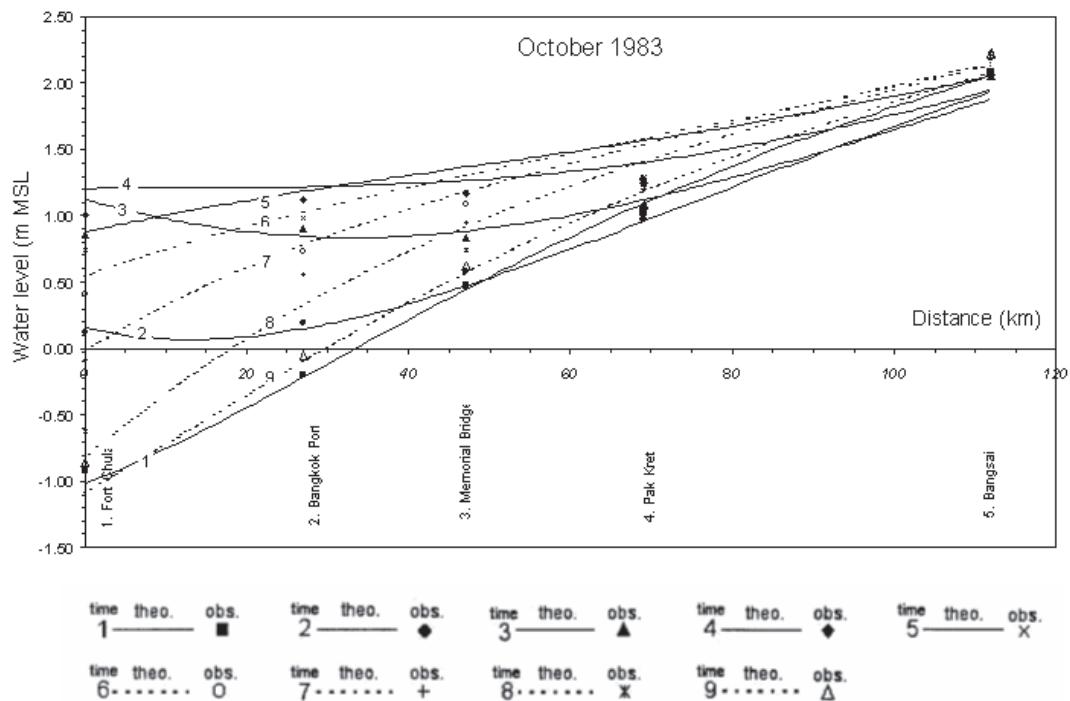


Figure 6. Longitudinal profiles of water level between theoretical computation and measurement on October 15 and 16, 1983 with 3 hour interval.

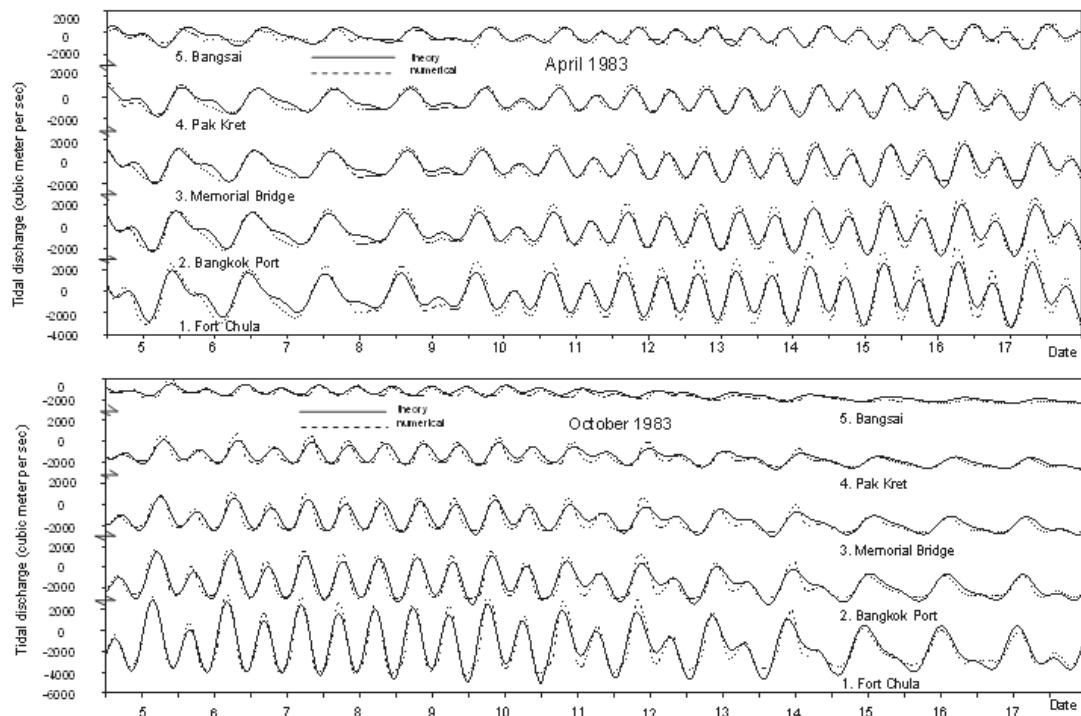


Figure 7. Comparison between theoretical computation and numerical output of tidal discharge along the Chao Phraya river.

of 1,000 km, its discharges change slowly with time that its monthly values can be assumed constant. But in the rainy month of October, with a peak discharge of freshwater, the mean daily discharge varies. Analyses are then made of shorter durations of 15 days and 7.5 days but the results show a similar tendency to those of the monthly duration (Vongvisessomjai and Rojanakamthorn, 1989).

3. High discharge

The perturbation method sets the river flow as the zeroth order solution and the tidal flow as the first order solution which means that the zeroth order river flow is more significant than the first order tidal flow. The model is suitable for rivers which have high discharges. The damping modulus, Eq. (18) as plotted in Figure 2 is zero when the river discharge is zero which under-estimates the actual damping modulus due to the bed roughness of the river.

Conclusions

An analytical model of tide and river flow interaction has been presented. The focus in this paper is on representation of tides and tidal flow at any spatial and temporal scales. The combination of the results from solving governing equations by using perturbation method by Vongvisessomjai and Rojanakamthorn (1989), which is the same as the analytical description of tide with linear frictional force from Ippen and Harleman (1966), and the theory of harmonic analysis could predict water level fluctuation resulting from the interaction of tide and river flow completely and beautifully especially in the rainy season when the river discharge is high. By applying the unsteady continuity equation, the equation of tidal discharge resulting from interaction of tide and river flow for various

constituents was obtained and the tidal flow could be foretold readily and satisfactorily. The present analytical study leads to the easiness in prediction of water level fluctuation and tidal discharge at any distance and time in a one-dimensional river by only knowing the average freshwater velocity along the river, average water depth, effective cross-sectional area, and the value of roughness coefficient together with doing harmonic analysis for the hourly water level records at the river mouth station only. The model was applied into the case study, the Chao Phraya river, and it was found that the results from model usage were satisfactory.

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