ORIGINAL ARTICLE

A study and development of Windows based program of reliability analysis for assessing service life of cracked connections

Wiwat Puatatsananon[1], Kittisak Kuntiyawichai[1] and Suchart Limkatanyu[2]

Abstract

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The development of a Windows based framework to undertake probabilistic fracture mechanics studies is reported. The reliability method used in the program is Monte-Carlo Simulation method. The results of the computation of the program are stress intensity factor, reliability index and probability of failure. The probabilistic studies of cruciform welded joint containing Lack of Penetration (LOP) defect and T-butt geometry containing surface crack at weld toe are performed in both critical crack growth and fatigue problem. The results can be used as an indicator for assuring the safety of this particular type of connection. It can also be used as a design criterion for the connection.

Key words : stress intensity factor, fracture, fatigue life, probabilistic method, Monte-Carlo Simulation

[1]Ph.D.(Structural Engineering), Asst. Prof., Department of Civil Engineering, Faculty of Engineering, Ubonratchathani University, Warin Chamrap, Ubon Ratchathani, 34190 Thailand. [2]Ph.D.(Structural Engineering), Asst.Prof., Department of Civil Engineering, Faculty of Engineering, Prince of Songkla University, Hat Yai, Songkhla, 90112 Thailand.
Corresponding e-mail: kittisak.ubu@gmail.com
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บทคัดย่อ

วิวัฒน์ พัวทัศนานนท์ [1] กิตติศักดิ์ ขันติยวิชัย [1] และ สุชาติ ลิ่มกตัญญู [2]

การศึกษาและพัฒนาโปรแกรมคอมพิวเตอร์สำหรับวิเคราะห์ความน่าเชื่อถือ และอายุการใช้งานของรอยต่อของโครงสร้างที่มีรอยตำหนิ

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บทความนี้นำเสนอโปรแกรมคอมพิวเตอร์ซึ่งสามารถใช้ประมวลผลอยู่ในระบบปฏิบัติการวินโดวส์ (Windows) โดยโปรแกรมคอมพิวเตอร์ดังกล่าวได้รับการพัฒนาขึ้นเพื่อใช้สำหรับประเมินความน่าเชื่อถือของรอยต่อของโครงสร้างที่มีรอยตำหนิปรากฏอยู่โดยใช้วิธีมอนติคาร์โล (Monte Carlo) ซึ่งผลลัพธ์ที่ได้จากโปรแกรมคอมพิวเตอร์จะแสดงค่าของ Stress Intensity Factor (*K*) ค่าดัชนี ความน่าเชื่อถือของโครงสร้าง และค่าความน่าจะเป็นของรอยต่อที่จะเกิดการวิบัติ โดยในบทความนี้ได้ศึกษาความน่าเชื่อถือของรอยต่อของโครงสร้างที่มีรอยตำหนิทั้งในกรณีปัญหาของการวิบัติแบบฉับพลันและปัญหาของการวิบัติเนื่องจากความล้าในรอยต่อ ซึ่งรอยต่อที่พิจารณาในการศึกษานี้มีอยู่ 2 ชนิด คือ รอยต่อรูปไม้กางเขน (cruciform joint) และรอยต่อชนรูปตัวที (T-butt joint) โดยผลลัพธ์ที่ได้จากการศึกษาหาความเชื่อถือของรอยต่อของโครงสร้างที่มีรอยตำหนิดังกล่าวสามารถใช้เป็นดัชนีที่บ่งชี้ถึงค่าความปลอดภัยของรอยต่ออีกทั้งยังมีประโยชน์ในการใช้เป็นข้อมูลสำหรับการออกแบบรอยต่ออีกด้วย

[1]ภาควิชาวิศวกรรมโยธา คณะวิศวกรรมศาสตร์ มหาวิทยาลัยอุบลราชธานี อำเภอวารินชำราบ จังหวัดอุบลราชธานี 34190 [2]ภาควิชาวิศวกรรมโยธา คณะวิศวกรรมศาสตร์ มหาวิทยาลัยสงขลานครินทร์ อำเภอหาดใหญ่ จังหวัดสงขลา 90112

Traditionally, evaluations of structural adequacy have been expressed by safety factors $SF - \frac{C}{D}$, where C is the capacity (i.e. strength) and D is the demand (i.e. load). Whereas this evaluation is quite simple to understand, it suffers from many limitations: it 1) treats all loads equally; 2) does not differentiate between capacity and demand respective uncertainties; 3) is restricted to service loads; and last but not least 4) does not allow comparison of relative reliabilities among different structures for different performance modes. Another major deficiency is that all parameters are assigned a single value in an analysis which is then deterministic. An alternative approach, a probabilistic one, extends the factor of safety concept to explicitly incorporate uncertainties in the parameters. The uncertainties are quantified through statistical analysis of existing data. One engineering discipline which could particularly benefit from such a probabilistic approach is fracture mechanics. Indeed it has been reported that, (Duga *et al*., 1983):

> [The] cost of material fracture to the US [is] \$ 119 billion per year, about 4 percent of the gross national product. The costs could be reduced by an estimated missing \$ 35 billion per year if technology transfer were employed to assure the use of best practice. Costs could be further reduced by as much as \$ 28 billion per year through fracture-related research.

As such, the objective of this paper is to marry those two disciplines, Probability and Fracture Mechanics into a user-friendly computer program which would determine the reliability index of cracked connections with known analytical expressions of stress intensity factors. Both critical and subcritical (fatigue) crack growth will be considered. For each class of problem, we thus seek to determine the reliability index which can be perceived as a universal indicator on the adequacy of a structural component.

Theoretical Background and Literature Survey

Critical Crack Growth

One of the underlying principles of fracture mechanics is that unstable fracture occurs when the stress intensity factor (*K*) reaches a critical value K_{Ic}, also called fracture toughness. K_{Ic} represents the inherent ability of a material to withstand a given stress field intensity at the tip of a crack and to resist progressive tensile crack extension.

Thus a crack will propagate (under pure mode I), whenever the stress intensity factor $\boldsymbol{K_I}$ (which characterizes the strength of the singularity for a given problem) reaches a material constant K_{Ic}. Hence, under the assumptions of linear elastic fracture mechanics (LEFM), at the point of incipient crack growth:

$$K_{Ic} = \beta\sigma\sqrt{\pi a} \qquad (1)$$

Thus for the design of a crack, or potentially crack structures, the engineer would have to decide what design variables can be selected, as only two of these variables can be fixed, and the third must be determined.

Subcritical Crack Growth; Fatigue

When a subcritical crack (a crack whose stress intensity factor is below the critical value) is subjected to either cyclic or fatigue load, or is subjected to a corrosive environment, crack propagation will occur. As in many structures one has to assume the presence of minute flaws (as large as the smallest one which can be detected). The application of repeated loading will cause crack growth. The loading is usually caused by vibrations.

Thus an important question that arises is "how long would it be before this subcritical crack grows to reach a critical size that would trigger failure?" To predict the minimum fatigue life of metallic structures, and to establish safe inspection intervals, an understanding of the rate of fatigue crack propagation is required.

Historically, fatigue life prediction was based on *S-N* curves, Figure 1 (or Goodman's Diagram) using a Strength of Material Approach which did not assume the presence of a crack.

Fatigue crack growth can take place under:

1. Constant amplitude loading (good for testing)
2. Variable amplitude loading (in practice)

Under the constant amplitude loading, empirical mathematical relationships which require the knowledge of the stress intensity factors (SIF), have been established to describe the crack growth rate. Models of increasing complexity have been proposed.

All of these relationships indicate that the number of cycles N required to extend a crack by a given length is proportional to the effective stress intensity factor range ΔK raised to a power n (typically varying between 2 and 9).

The first fracture mechanics-based model for fatigue crack growth was presented by Paris and Erdogan (1963) in the early '60s. It is important to recognize that it is an empirical law based on experimental observations. Most other empirical fatigue laws can be considered as direct extensions, or refinements of this one, given by

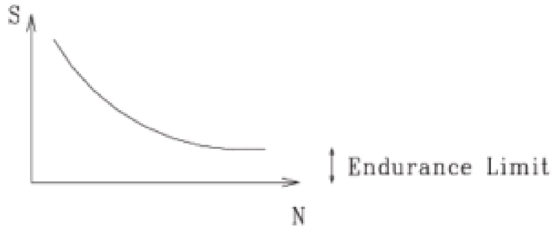

Figure 1. S-N Curve and Endurance Limit

$$\frac{da}{dN} = C(\Delta K)^n \qquad (2)$$

which is a straight line on a log-log plot of $\frac{da}{dN}$ vs ΔK.

$$\Delta K = K_{\max} - K_{\min} = (\sigma_{\max} - \sigma_{\min}) f(g)\sqrt{\pi a} \qquad (3)$$

a is the crack length; N the number of load cycles; C the intercept of line along $\frac{da}{dN}$ and is of the order of 10^{-6} and has units of length/cycle; and n is the slope of the line and ranges from 2 to 10.

Equation (2) can be rewritten as:

$$\Delta N = \frac{\Delta a}{C[\Delta K(a)]^n} \qquad (4)$$

or $$N = \int dN = \int_{a_i}^{a_f} \frac{da}{C[K(a)]^n} \qquad (5)$$

Thus it is apparent that a small error in the SIF calculations would be magnified greatly as n ranges from 2 to 6. Because of the sensitivity of N upon ΔK, it is essential to properly determine the numerical values of the stress intensity factors. However, in most practical cases, the crack shape, boundary conditions, and load are in such a combination that an analytical solution for the SIF does not exist and large approximation errors have to be accepted. Unfortunately, analytical expressions for K are available for only few simple cases. Thus the stress analyst has to use handbook formulas for them (Tada *et al.*, 1973). A remedy to this problem is the usage of numerical methods, of which the finite element method has achieved greatest success.

When compared with experimental data, it is evident that Paris law does not account for:

1. Increase in crack growth rate as $K_{\max}$ approaches K_{Ic}
2. Slow increase in crack growth at $K_{\min} \approx K_{th}$

thus it was modified by Forman *et al.* (1967), Figure 2

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1-R)K_c - \Delta K} \qquad (6)$$

Walker's model (Walker, 1970) is yet another variation of Paris Law which accounts for the stress ratio $R = \frac{K_{\min}}{K_{\max}} = \frac{\sigma_{\min}}{\sigma_{\max}}$

$$\frac{da}{dN} = C\left[\frac{\Delta K}{(1-R)^{(1-m)}}\right]^n \qquad (7)$$

Whereas most methods attempt to obtain numerical coefficients for empirical models which best approximate experimental data, the table look-up method extracts directly from the experimental data base the appropriate coefficients. In a "round-robin" contest on fatigue life predictions, this model was found to be most satisfactory (Miller and Gallagher, 1981).

Reliability Index Definition

The Performance Function F is a function

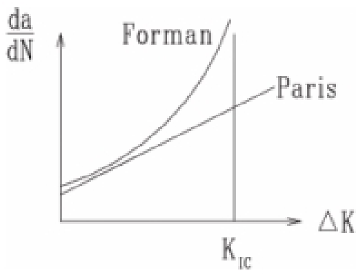

Figure 2. Forman's Fatigue Model

which determines the performance or the state of the system. In general F is a function of one or more variables x_i which describe the geometry, material, loads, and boundary conditions

$$F = F(x_i) \tag{8}$$

and thus F is in turn a random variable with its own probability distribution function, Figure 3. A performance function evaluation typically require a structural analysis, this may range from a simple calculation to a detailed finite element study.

Reliability indices, β are used as a relative measure of the reliability or confidence in the ability of a structure to perform its function in a satisfactory manner. In other words they are a measure of the performance function. Probabilistic methods are used to systematically evaluate uncertainties in parameters that affect structural performance, and there is a relation between the reliability index and risk.

Reliability index is defined in terms of the performance function capacity C, and the applied load or demand D. It is assumed that both C and D are random variables. The safety margin is defined as $Y = C\text{-}D$. Failure would occur if $Y < 0$. If C and D are normal random variables with probability density function $N(\mu_C,\sigma_C)$ and $N(\mu_D,\sigma_D)$ respectively. Y is also a normal random variable with the probability density function $N(\mu_Y,\sigma_Y)$ in which,

$$\mu_Y = \mu_C - \mu_D \tag{9}$$

$$\sigma_Y = \sqrt{\left(\sigma_C^2 + \sigma_D^2\right)} \tag{10}$$

and the reliability index β can be determined by

$$\beta = \frac{\mu_Y}{\sigma_Y} \tag{11}$$

$$= \frac{\mu_C - \mu_D}{\sqrt{\left(\sigma_C^2 + \sigma_D^2\right)}} \tag{12}$$

The probability of failure P_f is equal to the ratio of the shaded area to the total area under the curve in Figure 3. For standard distributions and for β = 3.5, it can be shown that the probability of failure is $P_f = \frac{1}{9{,}091}$ or 1.1×10^{-4}. That is 1 in every 10,000 structural members designed with β = 3.5 will fail because of either excessive load or under-strength sometime in its lifetime. Reliability indices are a relative measure of the current condition and provide a qualitative estimate of the structural performance. Structures with relatively high reliability indices will be expected to perform well. If the value is too low, then the structure may be classified as a hazard. Target values for β are shown in Table 1, and in Figure 4

Monte-Carlo Simulation(MCS)
Methodology

For the general problem in engineering system, the capacity and demand maybe functions of the other random variables. In term of safety of

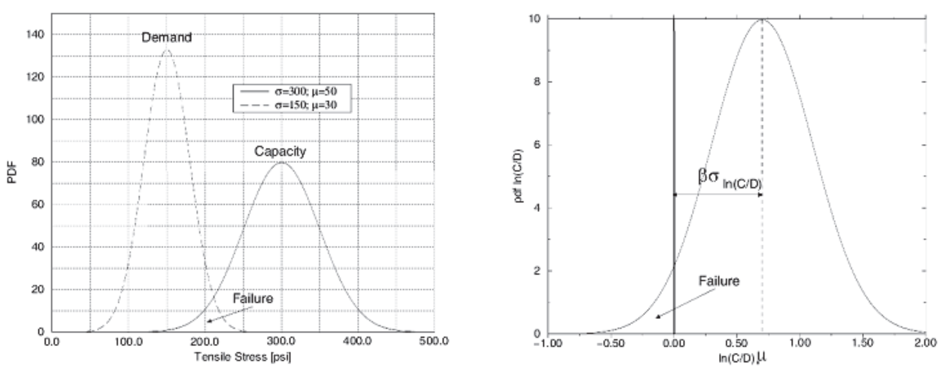

Figure 3. Definition of Reliability Index

Table 1. Selected β values for Steel and Concrete Structures (Ang and Tang, 1984)

Expected Performance	β	Failures
High	5	3/10 million
Good	4	3/100,000
Above Average	3	1/1,000
Below Average	2.5	6/1,000
Poor	2.0	2.3/100
Unsatisfactory	1.5	7/100
Hazardous	1.0	16/100

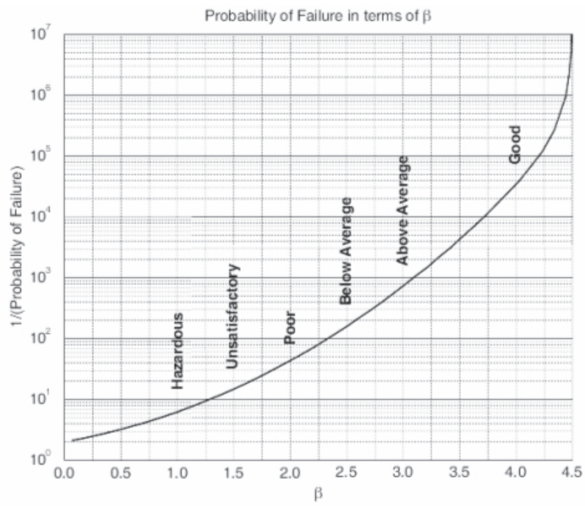

Figure 4. Probability of Failure in terms of β (Ang and Tang, 1984)

margin, the performance function can be expressed as

$$g(C_1, C_2, \ldots, C_n, D_1, D_2, \ldots, D_n) = C(C_1, C_2, \ldots, C_n) - D(D_1, D_2, \ldots, D_n) \quad (13)$$

The failure state will occur when $g < 0$. The reliability index can be calculated by using Monte-Carlo Simulation as the follows.

1. Initialize random number generators.
2. Perform n analysis, for each one:
   - For each variable, determine a random number for the given distribution.
   - Determine the performance function.
   - Analyze, and store the results
3. Count the number of analyses, n_f which performance function indicate failure, the likelihood of structural failure will be $P_f = n_f / n$.
4. The reliability index is then determined by using

$$\beta = \Phi^{-1}(1 - p_f) \quad (14)$$

Analytical expression of SIF of geometries used in this research

In this study, there are 2 selected cracked cases which are cruciform welded joint containing lack of penetration (LOP) defect and T-butt geometry containing surface crack at weld toe, to be considered. Their stress intensity factors can be shown as follows.

Cruciform welded joint containing lack of penetration (LOP) defect

Maddox (1975) defined the magnification of stress intensity factor which would be present for a crack of the same geometry but without the present of the weld. After that many researchers including Lie (1983), Thurlbeck (1991) and Bowness and Lee (1996) have been carried out to evaluate M_k by considering cracks at weld toes. For fillet welds and partially T butt welds, the initial crack may be taken from the unpenetrated region. Frank and Fisher (1979) were the first to introduce the formula for stress intensity factor of the root of cruciform welded joints containing lack of penetration (LOP) as shown in Eq 15.

$$K_I = M_k \sigma_m \left(\pi a \sec \frac{\pi a}{w} \right)^{0.5} \tag{15}$$

where

$$M_k = A_0 + A_1\left(\frac{2a}{w}\right) + A_2\left(\frac{2a}{w}\right)^2$$

$$w = B + 2h$$

$$A_0 = 0.956 - 0.343\left(\frac{h}{B}\right)$$

$$A_1 = -1.219 + 0.6210\left(\frac{h}{B}\right) - 12.220\left(\frac{h}{B}\right)^2 + 9.704\left(\frac{h}{B}\right)^3 - 2.741\left(\frac{h}{B}\right)^4$$

$$A_2 = 1.954 - 7.938\left(\frac{h}{B}\right) + 13.299\left(\frac{h}{B}\right)^2 - 9.541\left(\frac{h}{B}\right)^3 + 2.513\left(\frac{h}{B}\right)^4$$

σ_m is the membrane stress in the loaded member and the limits of validity for this formula are within the following range;

$$\frac{B}{T} = 1,\ 0.2 < \frac{h}{B} < 1.2,\ 0.1 < \frac{2a}{w} < 0.7$$

and dimensions are indicated in Figure 5.

T-butt geometry containing surface crack at weld toe (Brennan *et al.*, 1999)

In Figure 6, the expression for the stress intensity factor is given by

$$K_I = \beta \sigma \sqrt{\pi a} \tag{16}$$

where β is stress intensity calibration factor, a is a crack depth and σ is a nominal stress. The stress calibration intensity factors for tension and bending are functions of plate thickness (T), weld attachment thickness (t), width of weld attachment (L), weld toe radius (ρ),weld angle (α), crack depth (a)

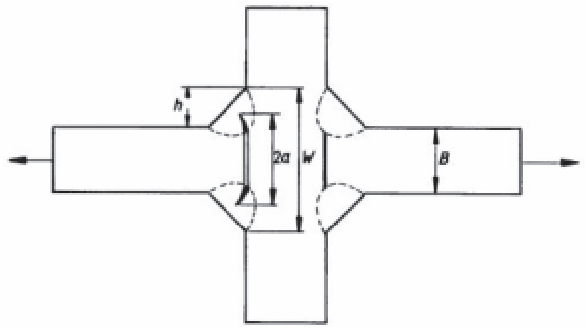

Figure 5. Crack and welded joint geometries (Fisher and Frank, 1979)

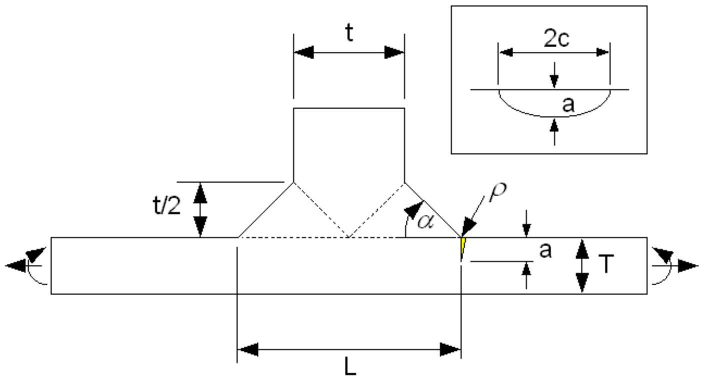

Figure 6. T-butt geometry containing surface crack at weld toe

and half surface crack length (c). They are defined as follows;

- Stress calibration intensity factors for tension is given by

$$\beta_t = 1.03\left(\frac{a}{T}\right)^P \exp\left\{C_0 + C_1\left(\frac{a}{T}\right) + \left(\frac{a}{T}\right)^2\right\} + C_3 + C_4 \qquad (17)$$

where

$$P = -0.365 + 0.207\left(\frac{a}{c}\right)^{0.5} - 0.144\left(\frac{a}{c}\right) + M_P$$

$$C_0 = -0.963 + 1.102\left(\frac{a}{c}\right)^{0.5} - 1.430\left(\frac{a}{c}\right) + M_0$$

$$C_1 = 3.084 - 6.542\left(\frac{a}{c}\right)^{0.5} + 9.023\left(\frac{a}{c}\right) + M_1, \qquad \text{if } \frac{a}{c} < 0.156$$

$$C_1 = 2.913 - 3.245\left(\frac{a}{c}\right)^{0.5} + 1.761\left(\frac{a}{c}\right) + M_1, \qquad \text{if } \frac{a}{c} \geq 0.156$$

$$C_2' = 2.627 - 10.767\left(\frac{a}{c}\right)^{0.5} + 9.553\left(\frac{a}{c}\right), \qquad \text{if } \frac{a}{c} < 0.2$$

$$C_2^{'} = -0.0625 - 0.557\left(\frac{a}{c}\right)^{0.5} + 0.156\left(\frac{a}{c}\right), \qquad \text{if } \frac{a}{c} \geq 0.2$$

if $C_2^{'} > 0.914$ then $C_2^{'} = 0.914$

$$C_2 = C_2^{'} + M_2$$

$$M_P = 0.172 - 0.1550\alpha - 0.0016\frac{T}{P}$$

$$M_0 = 0.284 - 0.1780\alpha - 0.0064\frac{T}{P}$$

$$M_1 = -0.317 + 0.0115\alpha + 0.0099\frac{T}{P}$$

$$M_2 = 0.0045 + 0.206\alpha - 0.0054\frac{T}{P}$$

$$C_3 = -0.45\left\{0.2 - \left(\frac{a}{T}\right)\right\}^{0.409}\left\{1.1 - \left(\frac{a}{c}\right)\right\}^{0.3}\left(\frac{L}{T}\right)^{-0.549}\left[1.0 - \exp - \left\{\frac{0.2 - \left(\frac{a}{T}\right)}{0.15}\right\}^2\right] \quad \text{if } \frac{a}{T} < 0.2$$

$$C_3 = 0 \quad \text{if } \frac{a}{T} \geq 0.2$$

$$C_4 = -2.5\left\{0.03 - \left(\frac{a}{T}\right)\right\}^{1.28}\left\{1.1 - \left(\frac{a}{c}\right)\right\}^{2.285}\alpha^{1.5}\left(\frac{\rho}{T}\right)^{-0.7}\left(\frac{L}{T}\right)^{-0.394}\left[1.0 - \exp - \left\{\frac{0.03 - \left(\frac{a}{T}\right)}{S_4}\right\}^2\right]$$

if $\frac{a}{T} < 0.03$

where $S_4 = 0.006$

but $S_4 = 0.018$ if $\alpha > 0.6109$ and $\frac{\rho}{T} < 0.035$ and $\frac{L}{T} < 0.035$

$$C_4 = 0 \quad \text{if } \frac{a}{T} \geq 0.03$$

α is in radians

- Stress calibration intensity factors for bending
  It is given by

$$\beta_b = 0.96A\ln\left(\frac{a}{T}\right) + C_0 + C_1\left(\frac{a}{T}\right) + C_2\left(\frac{a}{T}\right)^2 + C_3 + C_4 + C_5 \tag{18}$$

where

$$A = -0.388 - 0.958\left(\frac{a}{c}\right)^{0.5} + 1.111\left(\frac{a}{c}\right) + M_A, \qquad \text{if } \frac{a}{c} < 0.1$$

$$A = -0.686 + 0.310\left(\frac{a}{c}\right)^{0.5} + 0.0622\left(\frac{a}{c}\right) + M_A, \qquad \text{if } \frac{a}{c} \geq 0.1$$

$$C_0 = 0.544 - 4.125\left(\frac{a}{c}\right)^{0.5} + 4.018\left(\frac{a}{c}\right) + M_0, \qquad \text{if } \frac{a}{c} < 0.1$$

$$C_0 = -0.645 + 1.111\left(\frac{a}{c}\right)^{0.5} - 0.648\left(\frac{a}{c}\right) + M_0, \qquad \text{if } \frac{a}{c} < 0.1$$

$$C_1 = -2.664 + 22.408\left(\frac{a}{c}\right)^{0.5} - 22.264\left(\frac{a}{c}\right) + M_1, \quad \text{if } \frac{a}{c} < 0.1$$

$$C_1 = 3.860 - 6.128\left(\frac{a}{c}\right)^{0.5} + 2.876\left(\frac{a}{c}\right) + M_1, \qquad \text{if } \frac{a}{c} \geq 0.1$$

$$C_2 = 8.758 - 41.156\left(\frac{a}{c}\right)^{0.5} + 29.768\left(\frac{a}{c}\right) + M_2, \qquad \text{if } \frac{a}{c} < 0.1$$

$$C_2 = -1.648 + 0.926\left(\frac{a}{c}\right)^{0.5} + 0.00393\left(\frac{a}{c}\right) + M_2, \quad \text{if } \frac{a}{c} \geq 0.1$$

$$M_A = 0.597 - 0.649\alpha - 0.0028\frac{T}{\rho}$$

$$M_0 = 1.282 - 1.325\alpha - 0.0077\frac{T}{\rho}$$

$$M_1 = -2.222 + 2.154\alpha + 0.0170\frac{T}{\rho}$$

$$M_2 = 0.789 - 0.621\alpha - 0.0097\frac{T}{\rho}$$

$$C_3 = -0.25\left\{0.25 - \left(\frac{a}{T}\right)\right\}^{0.5}\left\{1.1 - \left(\frac{a}{c}\right)\right\}^{0.16}\alpha^{2.0}\left(\frac{\rho}{T}\right)^{-0.16}\left(\frac{L}{T}\right)^{-0.37}\left[1.0 - \exp-\left\{\frac{0.25 - \left(\frac{a}{T}\right)}{0.15}\right\}^2\right]$$

$$\text{if } \frac{a}{T} < 0.25$$

$$C_3 = 0 \quad \text{if } \frac{a}{T} \geq 0.25$$

$$C_4 = -4.0\left\{0.05-\left(\frac{a}{T}\right)\right\}^{0.565}\left\{1.1-\left(\frac{a}{c}\right)\right\}^{0.3}\alpha^{1.35}\left(\frac{\rho}{T}\right)^{-0.3}\left(\frac{L}{T}-0.455\right)^{0.204}\left[1.0-\exp-\left\{\frac{0.05-\left(\frac{a}{T}\right)}{S_4}\right\}^2\right]$$

if $\frac{a}{T} < 0.05$ and $\frac{L}{T} < 0.455$

where $S_4 = 0.05$

but $S_4 = 0.06$ if $\alpha > 0.6109$ and $\frac{\rho}{T} < 0.04$ and $\frac{L}{T} < 0.035$

$C_4 = 0$, if $\frac{a}{T} \geq 0.05$

$$C_5 = -0.14\left\{\left(\frac{a}{T}\right)-0.35\right\}^{0.098}\left\{1.1-\left(\frac{a}{c}\right)\right\}^{0.862}\alpha^{0.675}\left(\frac{\rho}{T}\right)^{-0.077}\left(\frac{L}{T}\right)^{0.148}\left[1.0-\exp-\left\{\frac{\left(\frac{a}{T}\right)-0.35}{0.2}\right\}^2\right]$$

if $\frac{a}{T} > 0.35$

C5 = 0, if $\frac{a}{T} \leq 0.35$

The limits of validity for these formulae are summarized in the Table 2.

Problem Formulation

In linear elastic fracture mechanics (LEFM) critical crack growth is governed by the following equation

$$K_I = \beta\sigma\sqrt{\pi a} \geq K_{Ic} \quad (19)$$

Table 2. The limits of validity for the stress calibration intensity factors for tension and bending

Geometries	Range
Weld Angle (α)	30º, 45º, 60º
Crack Aspect Ratios (a/c)	$0 \leq a/c \leq 1.0$
Crack Depths (a/T)	$0.01 \leq a/T \leq 1.0$
Weld Toe Radius (ρ/T)	$0.01 \leq \rho/T \leq 0.066$
Attachment Widths (L/T)	$0.3 \leq L/T \leq 4.0$

where K_I is the mode stress intensity factor (SIF), K_{Ic} is the fracture toughness (material property), α the far field stress, a the flaw size[1] in most case a refers to half of crack length, and β a geometry factor.

We define the performance function in terms of demand (D) and supply (S) as:

$$g(S,D) = \text{Supply - Demand} \qquad (20)$$

and if $g(S,D) < 0$ than we have a "failure state", if $g(S,D) > 0$ we have a "safe state", otherwise we have a **limit state** if $g(S,D) = 0$. In the context of LEFM we define the performance function as

$$g(K_{Ic}, K_I) = K_{Ic} - K_I \qquad (21)$$

For the fatigue problem, the failure mode is considered after N cycles of loading have been applied. It is governed by one of the following criteria,

1. $a_n > a_c$ where a_n is the final crack length after the N cycles of loading.

2. $K_{max} \geq K_{Ic}$ where K_{max} is the stress intensity factor after the N cycles of loading due to the maximum amplitude of load σ_{max}.

In this report, the later is used. Therefore, the performance function in this case can be expressed by

$$\begin{aligned} g(K_{Ic}, K_{max}) &= K_{Ic} - K_{max} \\ &= K_{Ic} - \sigma_{max}\beta\sqrt{\pi a_n} \end{aligned} \qquad (22)$$

where σ_{max} is the maximum amplitude of the stress, β is the geometry factor and a_n is the crack length after the N cycles of loading. The failure state is given by $g < 0$.

Critical Crack Growth

For the critical crack growth problem, the methods which is used to evaluating the reliability of the selected cracked structures is and Monte-Carlo Simulation (MCS). In this case the performance function is governed by Equation (21). The random variables and constant values that are used in each selected case can be summarized in the Table 3.

In LEFM problem, to evaluate the reliability of the cracked structures, the Monte-Carlo Simulation can be applied by comparing the value of the stress intensity factor with the value of the fracture toughness of each sample, counting the number of the samples which perform the failure state which is $g < 0$ or $K_I > K_{Ic}$, and then calculating the probability of failure which is equal to the number of the samples which perform the failure state over the total number of the samples that have been tested. For each sample, the process of Monte-Carlo Simulation can be shown as follows.

1) Generate a random number for K_{Ic}, a, and σ corresponding to its distribution.

2) Compute the stress intensity factor. Generally,

$$K_I = \sigma\beta\sqrt{\pi a} \qquad (23)$$

Table 3. Summary of the random variables and constant values which are used in each selected case

Case	Random Variables	Constants	Equation of K_I	Figure
Cruciform welded joint containing Lack Of Penetration (LOP) defect	K_{Ic}, a, σ	B, h	Equation (15)	Figure 5
T-butt geometry containing surface crack at weld toe	K_{Ic}, a, c, ρ, α, σ	T,t	Equation (16)	Figure 6

[1] in most case a refers to half of the crack length

3) Compare K_I with K_{Ic}. If $K_I \geq K_{Ic}$, the sample fails. Count and go to the next sample.

The whole process of Monte-Carlo Simulation can be shown in Figure 7.

Subcritical Crack Growth

In this case, Monte-Carlo Simulation is used for evaluating the probability of failure of the structure. The probability of failure can be calculated from n_f/n in which n is the total number of the samples and n_f is the number of the samples which perform the failure state, in this case, n_f is the number of the samples which fail within the N cycles of loading. Once the probability of failure is obtained, the reliability index can be calculated by using Equation (14). The process of evaluation of reliability of fatigue problem by using Monte-Carlo Simulation can be shown in Figure 8.

From Figure 8, to compute, in this paper, we use Paris law and Forman law as the following.

Paris law

$$\Delta a = C(\Delta K)^n \Delta N \quad (24)$$

Forman Law

$$\Delta a = \frac{C(\Delta K)^n \Delta N}{(1-R)K_{Ic} - \Delta K} \quad (25)$$

where

$$\Delta K = (\sigma_{max} - \sigma_{min})\beta\sqrt{\pi a}$$

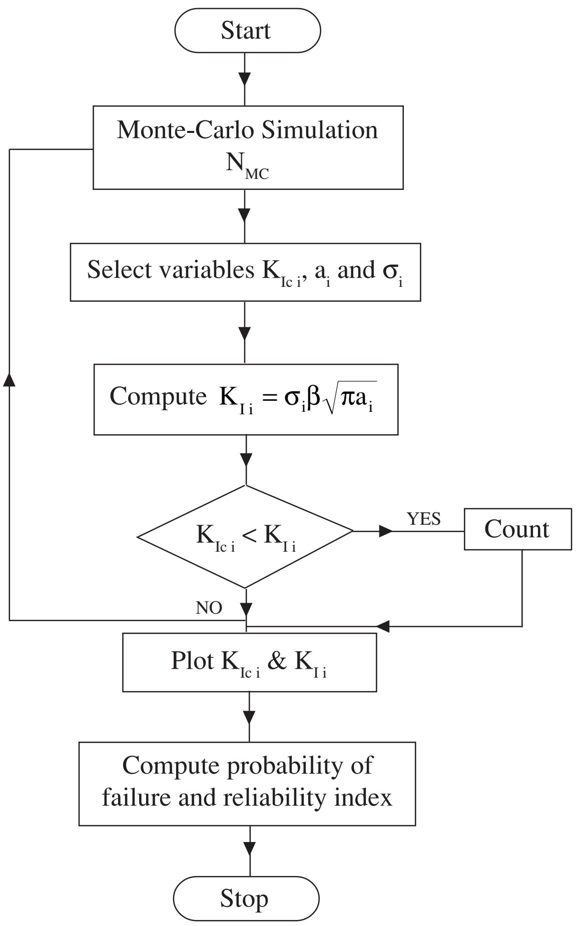

Figure 7. Monte-Carlo Simulation for the LEFM problem

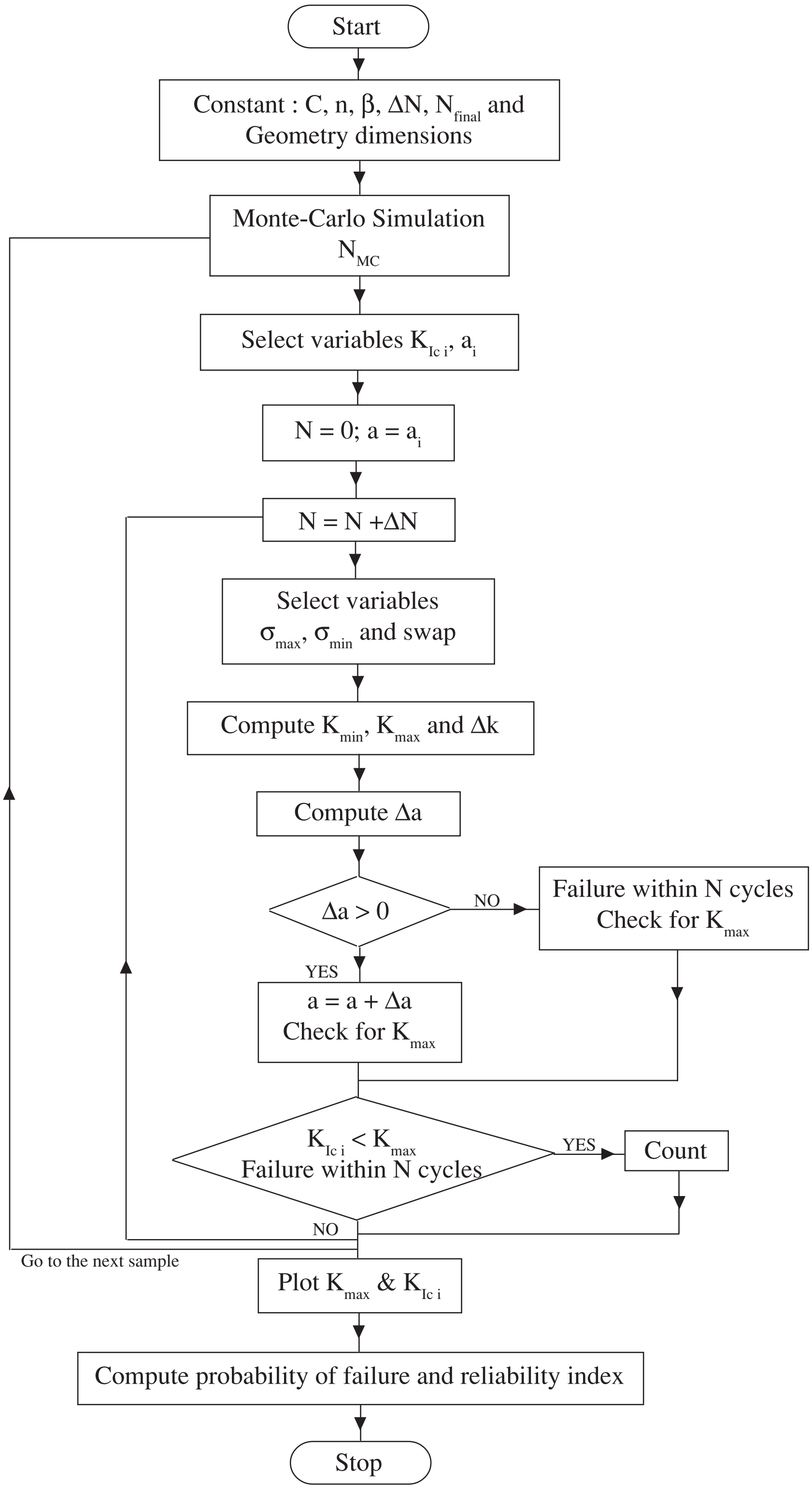

Figure 8. Monte-Carlo Simulation for the fatigue problem

and

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

For Forman law, it has one specific case of failure state when $\Delta a < 0$. It can be shown as follows.

From Equation (25), we obtain

$$\Delta a = \frac{C(\Delta K)^n \Delta N}{(1-R)K_{Ic} - \Delta K}$$

$$= \frac{C(\Delta K)^n \Delta N}{(1-\frac{\sigma_{\min}}{\sigma_{\max}})K_{Ic} - (\sigma_{\max} - \sigma_{\min})\beta\sqrt{\pi a}}$$

$$= \frac{C(\Delta K)^n \Delta N \sigma_{\max}}{(\sigma_{\max} - \sigma_{\min})K_{Ic} - (\sigma_{\max} - \sigma_{\min})\sigma_{\max}\beta\sqrt{\pi a}}$$

$$= \frac{C(\Delta K)^n \Delta N \sigma_{\max}}{(\sigma_{\max} - \sigma_{\min})(K_{Ic} - K_{\max})}$$

If $K_{\max} > K_{Ic}$, it means the sample fails. This causes $\Delta a < 0$.

Implementation

In this paper, two windows applications name "SLA-CC'" (Service Life Assessment for Cruciform Connection) and "SLA-TB'" (Service Life Assessment for T-Butt geometry) are developed by using Microsoft Visual C++ for evaluating the reliability of the cruciform welded joint containing lack of penetration (LOP) defect and T-butt geometry containing surface crack at weld toe, respectively, which can be indicated in term of reliability index and the probability of failure. They can perform the calculation in both critical and subcritical crack growth problem. For both problems, Monte-Carlo Simulation Method is used as the criterion of the calculation. SLA are designed to be dialog based program. Inside the dialog, it has computation part that the user can put input, run the program and look at the result of the computation. When the program runs, the windows of the program will look like in Figure 9 and Figure 10.

Numerical studies

Having developed the computational environment in which the user can readily determine the reliability index and the probability of failure, we now first seek to exploit this environment to conduct a preliminary parametric study which could yield valuable data to the practicing engi-

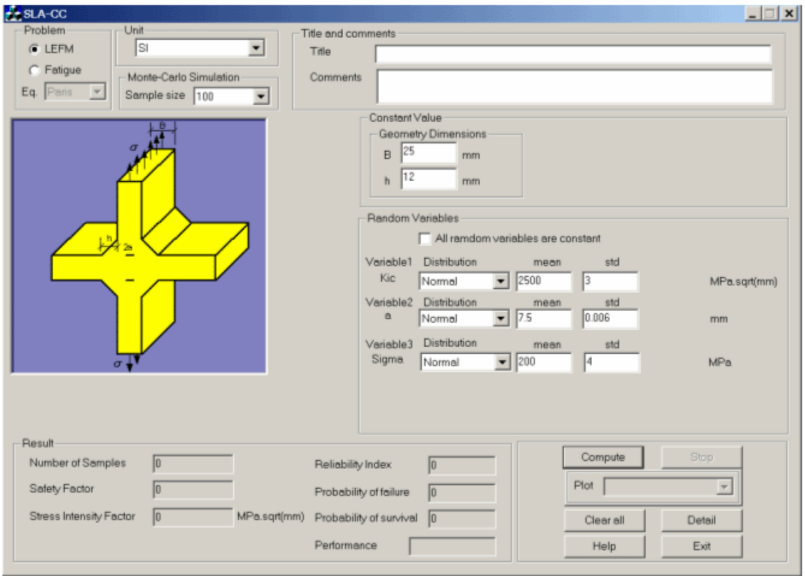

Figure 9. SLA-CC program

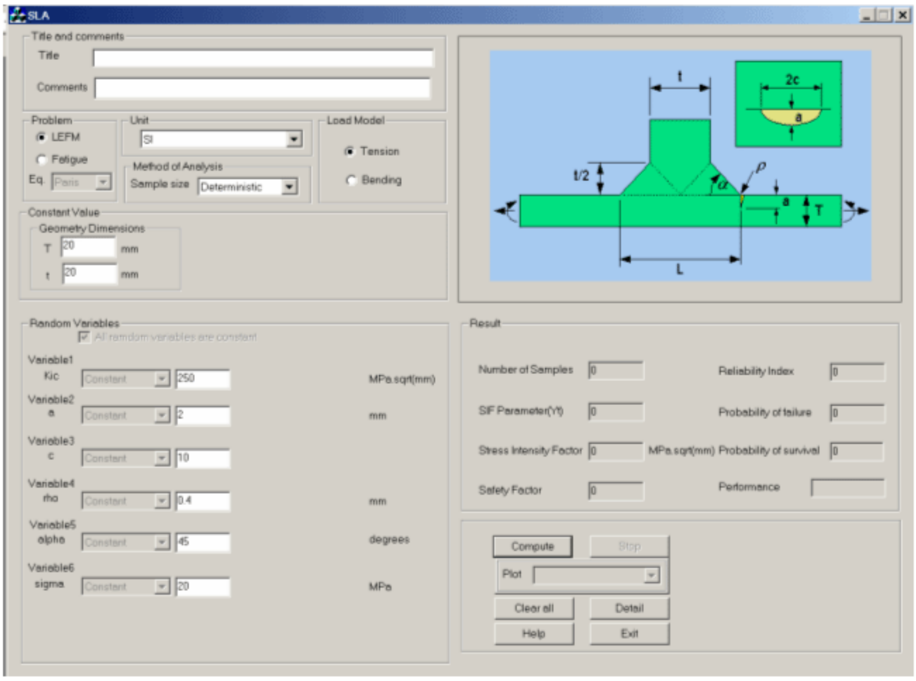

Figure 10. SLA-TB program

neers, and possibly shed some light on the probability of failure trend in terms of the selected variables. Then, the effects of the stress intensity factor range and the variation of the Coefficients of Variation, (COV) of the selected variables on the fatigue life of the selected problem are investigated. Probability of Failure for practical purpose, the SLA program is then used to perform the reliability analysis of the both cracked cases in LEFM problem in order to find their probability of failure trend in terms of the selected variables (K_{Ic}, a and σ). The assumptions and the process to get the probability of failure function in each cracked case can be shown as follows.

- All random variables are assumed to be normal random variates.

- The effect of the variation of each variable in term of its COV on the reliability of the cracked structures has been studies. The reliability analysis of each cracked case where COV of each variable varying with an increment of 5% from 0-20% are performed by using the Monte-Carlo Simulation method with 100,000 samples to determine the probability of failure.

- In each analysis, the mean value of the fracture toughness KIC is normalized to 1. The reliability analyses where the mean values of K_I / K_{Ic} are 0.5, 0.7 and 0.9 are performed for given geometries in each cracked case, i.e.,

  • Cruciform welded joint containing lack of penetration (LOP) defect ; h/B = 0.3, 0.6, 0.9 and 2a/W = 0.2, 0.4, 0.6

  • T-butt geometry containing surface crack at weld toe under tension where $\alpha = 30^{\circ}$, ρ/T = 0.01 and L/T = 1.0 and a/c = 0.25, 0.5, and 0.75

- The results of the probability of failure in each increment of COV of each variable are tabulated in Tables 4 to 9.

The effect of statistical variability of applied stress range, stress intensity factor and size of the initial defects on the fatigue life

The model used in the analysis is as follows; B = T = 1 in., h = 0.5 in., W = B+2h = 2 in and a = 0.2 in. The cruciform welded joint is made of steel with a K_{Ic} value at the service temperature of 60 $ksi\sqrt{in}$. In each case study, the Monte-Carlo simulation with n = 10,000 samples is used for

Table 4. Probability of failure of Cruciform welded joint containing lack of penetration (LOP) defect of the case where h/B = 0.3

Cov			$2a/W$ = 0.2			$2a/W$ = 0.4			$2a/W$ = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.08158	0	0	0.0992	0.00053	0.00081	0.1493
		0.10	0	0.00045	0.17215	0	0.00076	0.18199	0.00047	0.00335	0.21089
		0.15	0	0.00599	0.2477	0	0.00717	0.25218	0.00039	0.01302	0.26528
	0.10	0.05	0	3.00E-05	0.1159	0	0.00048	0.16792	0.04823	0.05073	0.2619
		0.10	0	0.00116	0.18984	0	0.00368	0.22047	0.04837	0.05687	0.2866
		0.15	0	0.00828	0.25115	1.00E-05	0.01478	0.26749	0.04818	0.07273	0.31383
	0.15	0.05	0	0.00028	0.15817	0	0.00744	0.23042	0.13393	0.13578	0.32452
		0.10	0	0.00285	0.21106	0	0.01469	0.25803	0.13406	0.14107	0.33768
		0.15	0	0.01292	0.25959	5.00E-05	0.03068	0.29014	0.13268	0.15766	0.35408
0.10	0.05	0.05	0	0.00256	0.18711	0	0.0033	0.19614	0.00033	0.00688	0.2227
		0.10	0	0.00703	0.23224	1.00E-05	0.00821	0.23847	0.00042	0.01352	0.25659
		0.15	4.00E-05	0.02093	0.27682	6.00E-05	0.02274	0.28045	0.00046	0.03029	0.292
	0.10	0.05	0	0.00359	0.20001	0	0.00766	0.22941	0.0482	0.06131	0.291
		0.10	0	0.00974	0.23974	1.00E-05	0.01515	0.26134	0.04719	0.07225	0.30982
		0.15	0.00012	0.02419	0.28193	0.00021	0.03188	0.29407	0.04738	0.0899	0.3316
	0.15	0.05	1.00E-05	0.00687	0.22061	7.00E-05	0.02048	0.26761	0.13305	0.14956	0.3433
		0.10	2.00E-05	0.01464	0.25247	0.0002	0.03059	0.28811	0.13523	0.15801	0.3545
		0.15	0.00016	0.02972	0.2898	0.00064	0.04782	0.31418	0.13295	0.17038	0.37153
0.15	0.05	0.05	0.00036	0.02674	0.26631	0.00041	0.0281	0.27127	0.0011	0.03325	0.28562
		0.10	0.00085	0.03563	0.28306	0.0009	0.03741	0.28767	0.00163	0.04364	0.30033
		0.15	0.00163	0.05184	0.31208	0.00173	0.05362	0.31439	0.00236	0.06077	0.32273
	0.10	0.05	0.00063	0.02938	0.2734	0.00091	0.03614	0.28979	0.04799	0.09164	0.33463
		0.10	0.0008	0.03844	0.29066	0.00113	0.04542	0.30399	0.04847	0.09963	0.34208
		0.15	0.00174	0.05477	0.31542	0.00221	0.06312	0.32486	0.05091	0.11647	0.35677
	0.15	0.05	0.00069	0.03418	0.27827	0.00149	0.05071	0.30883	0.13449	0.17407	0.37149
		0.10	0.00127	0.04401	0.29679	0.00253	0.06073	0.3235	0.13584	0.17953	0.3827
		0.15	0.00221	0.06061	0.31617	0.00377	0.07768	0.33617	0.13708	0.19401	0.39036

studying the reliability analysis of the cracked structures and all random variables (K_{Ic}, a and σ) are assumed to be normal variables.

Case Study I

SLA program has been used to perform the analysis of the service life in the given geometry with different level of the stress intensity factor range (ΔK = 0.3KIC, 0.4K_{Ic}, 0.5K_{Ic} and K_{min} = 0.1 K_{Ic}). The COV of all random variables are 0.1. The results of probability of failure at each increment of cycles of loading are plotted as shown Figure 11.

The results show that initially, the stress intensity factor range has no effect on the probability of failure of the cracked structures. However, as the number of cycles of loading increase, the wider the stress intensity range is, and the lower the fatigue life of the cracked structures obtained.

Case Study II

The effects of the variation of the COV of the random variables (stress intensity factor, size of the initial defects and applied stress ranges) on the fatigue life of the given cracked structures are

Table 5. Probability of failure of Cruciform welded joint containing lack of penetration (LOP) defect of the case where h/B = 0.6

Cov			2*a*/*W* = 0.2			2*a*/*W* = 0.4			2*a*/*W* = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.08086	0	0	0.09384	0.00053	0.00065	0.13579
		0.10	0	0.00047	0.17161	0	0.00064	0.1791	0.00047	0.00241	0.20258
		0.15	0	0.00591	0.24682	0	0.00678	0.25028	0.00039	0.01124	0.26203
	0.10	0.05	0	4.00E-05	0.11286	0	0.00021	0.15399	0.04823	0.04963	0.24232
		0.10	0	0.0011	0.18807	0	0.00269	0.21245	0.04837	0.05387	0.27388
		0.15	0	0.00778	0.25086	1.00E-05	0.01242	0.26366	0.04817	0.06779	0.306
	0.15	0.05	0	0.00024	0.15371	0	0.00411	0.21339	0.13393	0.13448	0.30769
		0.10	0	0.00242	0.20881	0	0.01014	0.24699	0.13406	0.1376	0.32547
		0.15	0	0.01235	0.25853	2.00E-05	0.02504	0.28361	0.13268	0.15253	0.34798
0.10	0.05	0.05	0	0.00253	0.18652	0	0.00307	0.19294	0.00033	0.00584	0.21514
		0.10	0	0.00709	0.23176	1.00E-05	0.00795	0.23577	0.0004	0.01195	0.2518
		0.15	3.00E-05	0.02103	0.2769	4.00E-05	0.02248	0.27921	0.00042	0.02811	0.28915
	0.10	0.05	0	0.00349	0.19919	0	0.00628	0.22159	0.04818	0.0575	0.27921
		0.10	0	0.00978	0.23856	1.00E-05	0.01348	0.25588	0.04719	0.06774	0.30162
		0.15	0.0001	0.02399	0.28065	0.00015	0.02947	0.29087	0.04726	0.08447	0.32588
	0.15	0.05	1.00E-05	0.00666	0.21773	4.00E-05	0.01593	0.25553	0.13305	0.14549	0.33194
		0.10	2.00E-05	0.01413	0.25103	0.00014	0.02565	0.2791	0.13521	0.15312	0.34687
		0.15	0.00013	0.029	0.28848	0.00044	0.04264	0.30752	0.13284	0.16508	0.36727
0.15	0.05	0.05	0.0004	0.02633	0.26564	0.00044	0.02731	0.26962	0.00103	0.03144	0.28209
		0.10	0.00091	0.03549	0.28308	0.00096	0.03703	0.28685	0.00155	0.04193	0.29727
		0.15	0.00155	0.05226	0.31109	0.00163	0.05332	0.31277	0.00221	0.05881	0.32051
	0.10	0.05	0.00062	0.02881	0.2733	0.00081	0.03347	0.2864	0.04782	0.08691	0.32686
		0.10	0.00084	0.03847	0.28904	0.00103	0.04347	0.29935	0.04828	0.09516	0.33674
		0.15	0.00174	0.05471	0.31529	0.00203	0.0609	0.32347	0.05047	0.11198	0.35292
	0.15	0.05	0.0007	0.034	0.27703	0.00116	0.04578	0.3011	0.13433	0.16932	0.36539
		0.10	0.00116	0.04306	0.29675	0.00197	0.05537	0.31772	0.13556	0.17476	0.37862
		0.15	0.00206	0.05968	0.31583	0.00309	0.07228	0.3313	0.1367	0.1893	0.38734

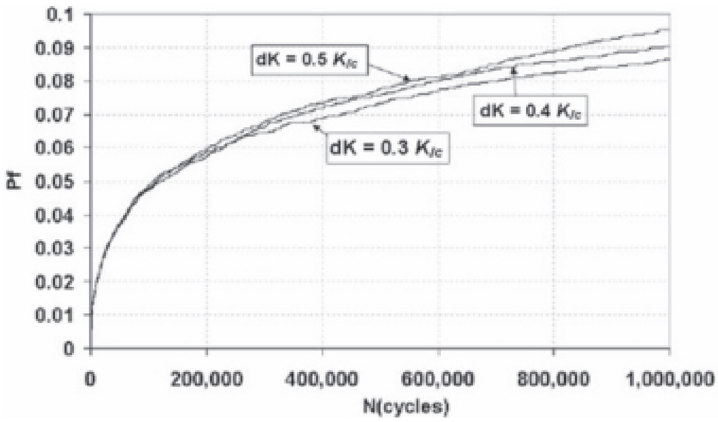

Figure 11. Service life of case B = 25

Table 6. Probability of failure of Cruciform welded joint containing lack of penetration (LOP) defect of the case where h/B = 0.9

Cov			2*a*/*W* = 0.2			2*a*/*W* = 0.4			2*a*/*W* = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.07946	0	0	0.09123	0.00053	0.00065	0.13306
		0.10	0	0.00045	0.17094	0	0.00062	0.17782	0.00047	0.00227	0.20091
		0.15	0	0.00584	0.24634	0	0.00655	0.24998	0.00039	0.01105	0.26162
	0.10	0.05	0	1.00E-05	0.10786	0	0.00014	0.14627	0.04823	0.04954	0.23885
		0.10	0	0.00089	0.1857	0	0.0023	0.20822	0.04837	0.05349	0.27194
		0.15	0	0.00726	0.24942	1.00E-05	0.01154	0.26192	0.04817	0.06702	0.30526
	0.15	0.05	0	0.00015	0.14578	0	0.00302	0.20375	0.13393	0.1343	0.30463
		0.10	0	0.00205	0.20422	0	0.00861	0.24128	0.13406	0.13718	0.32371
		0.15	0	0.01114	0.25615	2.00E-05	0.02273	0.28	0.13268	0.15167	0.34682
0.10	0.05	0.05	0	0.00254	0.18588	0	0.00295	0.19192	0.00033	0.00561	0.21403
		0.10	0	0.00696	0.23117	0	0.00776	0.23499	0.0004	0.01161	0.25108
		0.15	3.00E-05	0.02083	0.27676	4.00E-05	0.02214	0.27889	0.00041	0.02782	0.28874
	0.10	0.05	0	0.00325	0.19664	0	0.00567	0.21759	0.04818	0.05701	0.27751
		0.10	0	0.00948	0.23694	0	0.01276	0.25341	0.04719	0.06712	0.30027
		0.15	9.00E-05	0.02348	0.27998	0.00013	0.02857	0.29017	0.04725	0.08356	0.32564
	0.15	0.05	1.00E-05	0.00611	0.21338	3.00E-05	0.01417	0.25014	0.13305	0.14508	0.33001
		0.10	2.00E-05	0.0131	0.24787	0.00011	0.02335	0.275	0.13521	0.15231	0.34593
		0.15	0.00013	0.02773	0.28668	0.00037	0.04022	0.30501	0.13282	0.16439	0.36739
0.15	0.05	0.05	0.00039	0.02625	0.2651	0.00043	0.02707	0.26899	0.00102	0.03121	0.28153
		0.10	0.00087	0.03551	0.28304	0.00095	0.03679	0.28631	0.00154	0.04163	0.29666
		0.15	0.00154	0.0521	0.31085	0.00163	0.05315	0.31297	0.00217	0.05854	0.32036
	0.10	0.05	0.00062	0.02838	0.27192	0.00077	0.03269	0.28464	0.04781	0.08639	0.32583
		0.10	0.00084	0.03786	0.28789	0.001	0.04262	0.29784	0.04822	0.09458	0.33642
		0.15	0.00173	0.05411	0.31483	0.00197	0.05991	0.32253	0.05037	0.11134	0.353
	0.15	0.05	0.0007	0.03286	0.27456	0.00111	0.04347	0.29823	0.13429	0.16861	0.36461
		0.10	0.00113	0.04213	0.29469	0.00182	0.05344	0.31497	0.13554	0.17426	0.3783
		0.15	0.002	0.05833	0.31438	0.00291	0.07041	0.32927	0.13667	0.18886	0.38779

investigated. Given $\Delta K = 0.5K_{Ic}$ and $K_{\min} = 0.1K_{Ic}$. We define the data sets of the analyses as shown in Table 10.

The results of probability of failure at each increment of cycles of loading are plotted as shown in Figure 12.

We use the performance of unsatisfactory or the probability of failure of the cracked structures = 0.07 to be the state where the connection should be repaired. From Figure 12, the numbers of cycles at which each analysis reaches the given performance are 328900, 12000, 68100 and 6300 cycles, respectively. The results show that the increment of COV of each variable will reduce the service life of the cracked structures. The variation of the COV of the variable which affects on the fatigue life of the given crack structures the most is the COV of the stress range ($\Delta\sigma$). Therefore, the knowledge of dispersion characteristic of each random variable is very important in order to evaluate the service life of the cracked structures accurately.

Conclusions

SLA is a windows application for evaluat-

Table 7. Probability of failure of T-butt geometry containing surface crack at weld toe under tension where and a/c = 0.25

Cov			$2a/W$ = 0.2			$2a/W$ = 0.4			$2a/W$ = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.07798	0	0	0.0901	0	1.00E-05	0.09312
		0.10	0	0.00043	0.16765	0	0.0006	0.1734	0	0.00064	0.17464
		0.15	0	0.00555	0.24303	0	0.00643	0.24453	0	0.00672	0.24334
	0.10	0.05	0	1.00E-05	0.10246	0	0.00014	0.13819	0	0.00023	0.14509
		0.10	0	0.00062	0.1804	0	0.00184	0.19764	0	0.00213	0.19922
		0.15	0	0.00761	0.24716	0	0.01076	0.25417	0	0.01151	0.25219
	0.15	0.05	0	8.00E-05	0.13115	0	0.00148	0.18211	0	0.00208	0.18767
		0.10	0	0.00185	0.18822	0	0.00635	0.21815	0	0.00752	0.22004
		0.15	1.00E-05	0.01034	0.24756	1.00E-05	0.0185	0.26127	1.00E-05	0.02017	0.25873
0.10	0.05	0.05	0	0.00271	0.18558	1.00E-05	0.00303	0.19107	1.00E-05	0.00308	0.19127
		0.10	1.00E-05	0.0075	0.22988	2.00E-05	0.00819	0.23262	2.00E-05	0.00837	0.23245
		0.15	4.00E-05	0.01947	0.27692	6.00E-05	0.02067	0.27808	7.00E-05	0.02104	0.27742
	0.10	0.05	0	0.00325	0.19223	1.00E-05	0.00507	0.20902	1.00E-05	0.00553	0.21006
		0.10	5.00E-05	0.00918	0.23238	6.00E-05	0.01221	0.24254	6.00E-05	0.01303	0.24116
		0.15	7.00E-05	0.02219	0.27684	0.00012	0.02681	0.28119	0.00012	0.02754	0.27916
	0.15	0.05	0	0.00502	0.20506	0	0.01004	0.23323	1.00E-05	0.0116	0.23311
		0.10	0	0.01255	0.23727	3.00E-05	0.01995	0.25568	5.00E-05	0.02138	0.25365
		0.15	5.00E-05	0.02643	0.27316	0.00017	0.03571	0.28251	0.00026	0.03751	0.27791
0.15	0.05	0.05	0.00049	0.02571	0.26153	0.00057	0.02688	0.26404	0.00056	0.02707	0.26364
		0.10	0.00073	0.03557	0.28431	0.00075	0.03653	0.28588	0.00076	0.03671	0.28525
		0.15	0.00142	0.05189	0.30923	0.00143	0.05302	0.30934	0.00146	0.05297	0.30877
	0.10	0.05	0.00051	0.02819	0.26552	0.0006	0.0318	0.27147	0.0006	0.03222	0.27013
		0.10	0.00086	0.03772	0.28277	0.00102	0.04147	0.28746	0.00109	0.04221	0.2854
		0.15	0.00163	0.05226	0.31238	0.00184	0.05624	0.31403	0.00187	0.05675	0.31106
	0.15	0.05	0.00071	0.03119	0.26624	0.00096	0.03914	0.27909	0.001	0.03987	0.27545
		0.10	0.00107	0.0401	0.28379	0.00137	0.04791	0.29207	0.00148	0.04869	0.28809
		0.15	0.00179	0.05602	0.30827	0.00237	0.06474	0.31355	0.00255	0.06535	0.30855

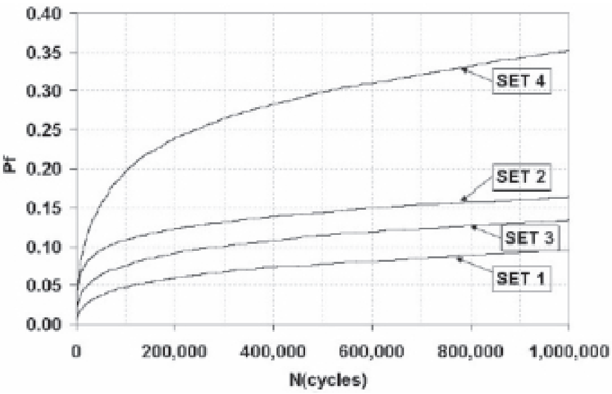

Figure 12. The probability of failure at each increment of cycles of loading of the given geometry in each set of analysis.

Table 8. Probability of failure of T-butt geometry containing surface crack at weld toe under tension where and a/c = 0.5

Cov			2a/W = 0.2			2a/W = 0.4			2a/W = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.07495	0	0	0.08095	0	0	0.08258
		0.10	0	0.0004	0.16539	0	0.0005	0.16824	0	0.00052	0.16841
		0.15	0	0.00544	0.24035	0	0.00592	0.24118	0	0.00611	0.24006
	0.10	0.05	0	0	0.09291	0	4.00E-05	0.11224	0	5.00E-05	0.11595
		0.10	0	0.00055	0.17311	0	0.00103	0.18122	0	0.00123	0.18112
		0.15	0	0.00658	0.24268	0	0.00818	0.24515	0	0.00847	0.24179
	0.15	0.05	0	5.00E-05	0.1132	0	0.00037	0.14409	0	0.00061	0.14632
		0.10	0	0.00127	0.17571	0	0.00291	0.19186	0	0.00325	0.18894
		0.15	1.00E-05	0.0086	0.23767	1.00E-05	0.01281	0.24358	1.00E-05	0.01322	0.23765
0.10	0.05	0.05	0	0.00247	0.18366	0	0.00264	0.18574	0	0.00266	0.18529
		0.10	1.00E-05	0.00747	0.22876	2.00E-05	0.0077	0.22966	3.00E-05	0.00765	0.22912
		0.15	3.00E-05	0.01945	0.27519	3.00E-05	0.02008	0.27503	3.00E-05	0.02011	0.27369
	0.10	0.05	0	0.0029	0.18584	0	0.00372	0.1942	0	0.00387	0.19351
		0.10	5.00E-05	0.00821	0.22745	5.00E-05	0.00976	0.2306	5.00E-05	0.01015	0.22808
		0.15	6.00E-05	0.0211	0.27197	8.00E-05	0.02343	0.27318	6.00E-05	0.02347	0.26926
	0.15	0.05	0	0.00397	0.19228	0	0.00633	0.20639	0	0.00661	0.2035
		0.10	0	0.01074	0.22625	2.00E-05	0.0143	0.23534	2.00E-05	0.01479	0.23055
		0.15	4.00E-05	0.02403	0.26381	0.00011	0.02854	0.2663	0.00014	0.0287	0.25951
0.15	0.05	0.05	0.00054	0.02559	0.26074	0.00051	0.02631	0.26116	0.00054	0.02622	0.26034
		0.10	0.00075	0.0352	0.28292	0.00074	0.03568	0.2833	0.00075	0.03593	0.28227
		0.15	0.00134	0.05148	0.30814	0.00132	0.05171	0.30818	0.00136	0.05164	0.30713
	0.10	0.05	0.00049	0.02715	0.26021	0.00057	0.02892	0.26213	0.00055	0.02886	0.25985
		0.10	0.00078	0.03664	0.27793	0.00084	0.03868	0.27939	0.00084	0.03844	0.27598
		0.15	0.00151	0.05078	0.30758	0.00166	0.05289	0.30792	0.00152	0.05253	0.30383
	0.15	0.05	0.00062	0.02849	0.25609	0.0007	0.03234	0.26049	0.00071	0.03219	0.2553
		0.10	0.00097	0.03677	0.27445	0.00113	0.04053	0.27728	0.00107	0.04015	0.272
		0.15	0.00169	0.05318	0.29979	0.00194	0.05727	0.29972	0.00196	0.05671	0.2925

ing the stress intensity factor and the reliability of the cruciform welded joint containing lack of penetration (LOP) defect and T-butt geometry containing surface crack at weld toe, respectively, in both LEFM and fatigue problems. The reliability methods used in LEFM problem and fatigue problem is Monte-Carlo Simulation method. The results of the computation of the program are stress intensity factor, reliability index and probability of failure. The results can be used for monitoring the performance of the cracked structures at the different levels of stress and crack length. Its performance is then used in the maintenance purpose, i.e., it's time to repair the connection or how much the stress level should be reduced at the given crack length in order to keep the level of satisfied performance of the cracked structures.

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Table 9. Probability of failure of T-butt geometry containing surface crack at weld toe under tension where and a/c = 0.75

Cov			2*a*/*W* = 0.2			2*a*/*W* = 0.4			2*a*/*W* = 0.6		
			K_I / K_{Ic}			K_I / K_{Ic}			K_I / K_{Ic}		
K_{Ic}	a	σ	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
0.05	0.05	0.05	0	0	0.08133	0	0	0.08446	0	0	0.08674
		0.10	0	0.00047	0.1693	0	0.00052	0.17041	0	0.00056	0.17086
		0.15	0	0.00597	0.24201	0	0.00622	0.24171	0	0.00645	0.24163
	0.10	0.05	0	0	0.11156	0	6.00E-05	0.12101	0	7.00E-05	0.12568
		0.10	0	0.00097	0.18433	0	0.00129	0.18763	0	0.0015	0.1881
		0.15	0	0.00795	0.24698	0	0.00886	0.24781	0	0.00952	0.24601
	0.15	0.05	5.00E-05	0.00016	0.14258	0	0.00052	0.15646	0	0.00064	0.1603
		0.10	3.00E-05	0.00208	0.19536	0	0.00315	0.20053	0	0.0039	0.20068
		0.15	6.00E-05	0.01155	0.24875	1.00E-05	0.01391	0.24929	1.00E-05	0.01474	0.24548
0.10	0.05	0.05	0	0.00277	0.18669	0	0.00285	0.18761	0	0.00288	0.18788
		0.10	2.00E-05	0.00773	0.23003	3.00E-05	0.00778	0.23044	3.00E-05	0.00801	0.23038
		0.15	4.00E-05	0.02017	0.27473	4.00E-05	0.02043	0.27489	4.00E-05	0.02061	0.2739
	0.10	0.05	0	0.00373	0.19617	0	0.00417	0.19921	0	0.00451	0.19944
		0.10	4.00E-05	0.00947	0.23227	5.00E-05	0.01017	0.23355	5.00E-05	0.01067	0.23228
		0.15	7.00E-05	0.02329	0.2748	7.00E-05	0.02421	0.27422	7.00E-05	0.02499	0.27293
	0.15	0.05	6.00E-05	0.00545	0.20901	0	0.00684	0.21496	0	0.00748	0.21414
		0.10	6.00E-05	0.01334	0.23793	2.00E-05	0.01565	0.24083	4.00E-05	0.0165	0.23872
		0.15	9.00E-05	0.02732	0.27073	0.00015	0.02977	0.27003	0.00014	0.03054	0.26669
0.15	0.05	0.05	0.00056	0.0262	0.26199	0.00058	0.02638	0.26192	0.00056	0.02651	0.26179
		0.10	0.00078	0.03592	0.28331	0.00077	0.0361	0.28338	0.00079	0.0362	0.28308
		0.15	0.00142	0.05214	0.30809	0.00141	0.05221	0.3083	0.00143	0.05262	0.3073
	0.10	0.05	0.00056	0.02865	0.2643	0.0006	0.02965	0.26478	0.00063	0.02993	0.2636
		0.10	0.0009	0.03788	0.28164	0.0009	0.03883	0.28119	0.00095	0.03909	0.27949
		0.15	0.00156	0.05267	0.30905	0.00156	0.0539	0.30842	0.00158	0.0538	0.3063
	0.15	0.05	0.00078	0.03099	0.26441	0.00078	0.03331	0.26552	0.00084	0.0337	0.26163
		0.10	0.00117	0.04006	0.28102	0.00117	0.04253	0.28117	0.0012	0.04263	0.27738
		0.15	0.00196	0.05631	0.30279	0.00202	0.05878	0.30173	0.00209	0.05861	0.2978

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Table 10. Data sets of the analyses of the effects of the variation of the COV of the random variables on fatigue life of the cracked structures

Analysis	Coefficient of Variation		
	KIC	**a**	**Stress**
Set 1	0.1	0.1	0.1
Set 2	0.15	0.1	0.1
Set 3	0.1	0.15	0.1
Set 4	0.1	0.1	0.15

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