ORIGINAL ARTICLE

Mathematical model of MMR inversion for geophysical data

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Abstract

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In this paper, we present an analysis of the solution to a number of geophysical inverse problems which are generally non-unique. The mathematical inverse problem that arises is commonly ill-posed in the sense that small changes in the data lead to large changes in the solution. We conduct the inversion algorithm to explore the conductivity for the ground structure. The algorithm uses the data in the form of magnetic field measurements for magnetometric resistivity (MMR). The inversion example is performed to investigate the conductivity ground profile that best fits the observed data. The result is compared with the true model and discussed to show the efficiency of the method. The model for the inversion example with the apparent conductivity and the true conductivity are plotted to show the convergence of the algorithm.

Key words : magnetometric resistivity, MMR., Magnetic field

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บทคัดย่อ

สืบสกุล อยู่ยืนยง

แบบจำลองทางคณิตศาสตร์ของการค้นหาข้อมูลทางธรณีฟิสิกส์ด้วยวิธีสภาพต้านทานแมกรีโทเมตริก

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งานวิจัยชิ้นนี้นำเสนอการวิเคราะห์หาคำตอบของปัญหาย้อนกลับทางธรณีฟิสิกส์ซึ่งคำตอบไม่ได้มีเพียงหนึ่งเดียว ปัญหาย้อนกลับนี้เป็นปัญหาที่อ่อนไหว เมื่อข้อมูลเปลี่ยนแปลงเล็กน้อย มีผลทำให้คำตอบเปลี่ยนแปลงอย่างมาก งานวิจัยชิ้นนี้นำเสนอกระบวนวิธีการหาสภาพนำไฟฟ้าของโครงสร้างแผ่นดิน กระบวนวิธีการนี้ใช้ข้อมูลที่อยู่ในรูปสนามแม่เหล็กที่วัดได้จากวิธีสภาพต้านทานแมกนีโทเมตริก ตัวอย่างการค้นหาสภาพนำไฟฟ้าถูกแสดงในรูปกราฟเปรียบเทียบกับค่าสภาพนำไฟฟ้าจริงและค่าสภาพนำไฟฟ้าปรากฏ โดยแสดงให้เห็นถึงประสิทธิภาพของกระบวนวิธีการ

ศูนย์วิจัยวิทยาการคณนา ภาควิชาคณิตศาสตร์ คณะวิทยาศาสตร์ มหาวิทยาลัยศิลปากร อำเภอเมือง จังหวัดนครปฐม 73000

The recent geophysical literature includes many works on development and application of inversion techniques. It is a topic of widespread active research such as the works conducted by Backus and Gilbert (1967, 1968), Jupp and Vozoff (1975), Yooyuanyong and Siew (1998), Yooyuanyong (2000), Yooyuanyong and Chumchob (2000), Yooyuanyong and Siew (2000), Yooyuanyong *et al.* (2005), and Yooyuanyong and Sripanya (2005). The motivation for this study is to determine the mathematical techniques that may have applications for mapping the ground structure in different parts of Thailand.

In this paper, an analysis of the solution to a number of geophysical inverse problems which are generally non-unique (see Backus and Gilbert (1967, 1968), Jupp and Vozoff (1975), Yooyuanyong *et al.* (2005)) is presented. The mathematical inverse problem that arises is commonly ill-posed in the sense that small changes in the data lead to large changes in the solution. Following the method performed by Jupp and Vozoff (1975), we conduct the algorithm to explore the conductivity for the continuous ground structure. The method uses the data in the form of magnetic field measurements for magnetometric(MMR) survey methods. The simplified inverse problem is to find the conductivity profile of the ground that best fit the observed data. The synthesis data are employed to show the robustness of the algorithm.

Analysis of the inverse problem

The p data values $d_1, d_2, ...d_p$ corresponding to p sample points, or instrument reading, are written as the vector

$$\vec{d} = [d_1, d_2, d_3, \dots, d_p]^T.$$

In our example, for the MMT data $d_i, i = 1,2,3,...,p$ are the magnetic fields at the source-receiver spacing. The restricted earth models are determined by q free parameters, which we write as the vector

$$\vec{x} = [x_1, x_2, x_3, \dots, x_q]^T.$$

In our example, the investigated parameter is the conductivity profile of the ground. The forward problem generates a set of model data for each setting of x. This is denoted as a vector function by

$$\vec{g}(\vec{x}) = [g_1(\vec{x}), g_2(\vec{x}), g_3(\vec{x}), \dots, g_p(\vec{x})]^T.$$

Here, $\vec{g}(\vec{x})$ is the value predicted by the model and corresponds to the observation $\vec{d}$. The inverse problem determines values of $\vec{x}$ such that $\vec{g}(\vec{x})$ matches $\vec{d}$ in some sense, which in this paper, is the minimum of the sum of square error between model and data:

$$F(\bar{x}) = \min \sum_{i=1}^{p} (d_i - g_i)^2 .$$

Inversion process

Iterative method is a common tool for practical inversion. The iterative method successively improves a current model until the error measure is small and the parameters are stable with respect to reasonable changes in the model. Following the method discussed by Jupp and Vozoff (1975), we expand $\bar{g}(\bar{x})$ about $\bar{x}$ in a Taylor series expansion

$$\bar{g}(\bar{x}+\delta\bar{x}) = \bar{g}(\bar{x}) + \bar{J}\delta\bar{x} + \vec{R}(\bar{g},\delta\bar{x}),$$

where $\bar{J} = \left|\frac{\partial g_i}{\partial x_i}\right|$ is the Jacobian matrix of the vector function $\bar{g}(\bar{x})$, $\delta\bar{x}$ is the vector of small change of the investigated parameters, i = 1,2,3, ...,p and j = 1,2,3,...,q. The remainder term $\vec{R}$ depends on $\bar{g}(\bar{x})$. If $\bar{g}(\bar{x})$ is a linear function, then is exactly zero and

$$\bar{g}(\bar{x}+\delta\bar{x}) = \bar{g}(\bar{x}) + \bar{J}\delta\bar{x}.$$

An amount of $\delta\bar{x}$ can be calculated by solving the linear least-squares problem

$$\min\left\|\bar{\in} - \bar{J}\delta\bar{x}\right\|,$$

where $\overset{\rho}{\in} = \overset{\rho}{d} - \overset{\rho}{g}$. In matrix form, we may write

$$\bar{\in} \;=\; \bar{J}\delta\bar{x},$$

and the vector of small change of the investigated parameter can be derived by using the generalized Gauss method as

$$\bar{J}\delta\bar{x} = \bar{J}^{-1}\,\bar{\in}. \qquad (1)$$

The $\delta\bar{x}$ will be used to improve the $\bar{x}$ model. The matrix $\bar{J}$ as mention above now can be derived for our geometric model which is layered and assumed to be a function of depth only. The cylindrical coordinate system is used with the positive downward to the ground.

Magnetic field due to a semi-infinite source in a 1-dimensional ground structure

A semi-infinite vertical wire carries an exciting current I and terminates at the electrode Q. The electrode Q is deliberately placed at the interface $z = z_s$ of layer s and layer s+1 where s is a positive integer less than N-1 as shown in Figure 1. Each layer has conductivity as a function of depth, $\sigma_j(z)$ with thickness h_j.

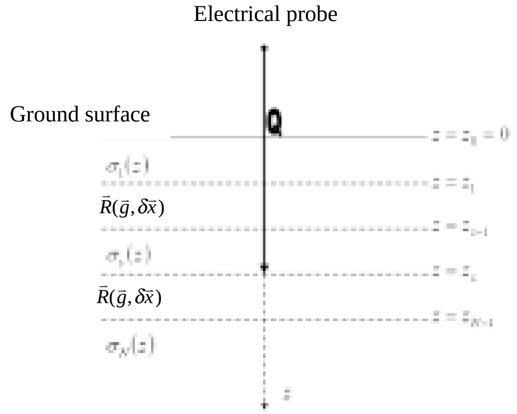

Figure 1. Geometric model of the ground structure

The Maxwell's equations can be used to determine the magnetic field intensity $\vec{H}$ as

$$\nabla \times \vec{E} = \vec{0}, \qquad (2)$$

and

$$\nabla \times \vec{H} = \sigma\vec{E}, \qquad (3)$$

where $\vec{E}$ is the electric field intensity, $\vec{H}$ is the magnetic field intensity and σ is the conductivity of the medium. Using (2) and (3), we have

$$\nabla \times \frac{1}{\sigma}\nabla \times \vec{H} = \vec{0}. \qquad (4)$$

Since the problem is axi-symmetric, and $\vec{H}$ has only an azimuthal component in cylindrical coordinate system (r,ϕ,z). For simplicity, we use H to represent the azimuthal component in the following derivations. Expanding equation (4) yields

$$\frac{1}{\sigma}\frac{\partial^2 H}{\partial z^2}+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial H}{\partial z}+\frac{1}{\sigma}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}(rH)+\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\frac{\partial}{\partial r}(rH)\right)+\frac{\partial}{\partial r}\left(\frac{1}{\sigma}\right)\frac{1}{r}\frac{\partial}{\partial r}(rH)=0. \tag{5}$$

Since σ is a function of depth z only, the above equation becomes

$$\frac{\partial^2 H}{\partial z^2}+\sigma\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial H}{\partial z}+\left(\frac{\partial^2 H}{\partial r^2}+\left(\frac{1}{r}\right)\frac{\partial H}{\partial r}\right)-\frac{1}{r^2}H=0. \tag{6}$$

Taking the Hankel transform defined by

$$\tilde{H}(\lambda,z)=\int_0^\infty rH(r,z)J_1(\lambda r)dr,$$

where J_1 is the Bessel function of the first kind of order one, to the equation (6) and we have

$$\frac{\partial^2 \tilde{H}}{\partial z^2}+\sigma\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\frac{\partial \tilde{H}}{\partial z}-\lambda^2\tilde{H}=0. \tag{7}$$

The solution to equation (7) is

$$\tilde{H}(\lambda,z)=Ae^{m_1 z}+Be^{m_2 z},$$

where A and B are arbitrary constants, and m_1 and m_2 are denoted by

$$m_1=-\frac{\sigma}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)+\frac{1}{2}\sqrt{\sigma^2\left\{\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right\}^2+4\lambda^2},$$

$$m_2=-\frac{\sigma}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)-\frac{1}{2}\sqrt{\sigma^2\left\{\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right\}^2+4\lambda^2},$$

and σ is the conductivity of the medium. Since the current probe has z_s long and the magnetic is generated azimuthal around the probe which can be expressed by the Ampere law as

$$\tilde{H}(\lambda,z)=\frac{I}{2\pi\lambda},$$

thus, the solution to equation (7) now is

$$\tilde{H}(\lambda,z)=Ae^{m_1 z}+Be^{m_2 z}+\frac{I}{2\pi\lambda}.$$

The arbitrary constants A and B can be obtained from the boundary conditions that at the probe on the ground surface

$$\sigma(z)E(r,z)|_{z=0}=0,$$

and the continuity of the magnetic field at the interface of the layer.

Anisotropic layered earth model

In this section, since the stratified models are often relevant and can usually be applied to real geoelectric structure, the two-layered earth model will be considered. Thus, following the derivation in the previous section, we obtain the magnetic field in the overburden as

$$H(r,z)_{over} = \int_0^\infty \left[Ae^{m_1 z} - Ae^{m_2 z} + \frac{I}{2\pi\lambda} \right] \lambda J_1(\lambda r) d\lambda, \quad 0 \le z \le z_s, \tag{8}$$

The magnetic field in the host medium, after applying the boundary conditions as $z \to \infty, H \to 0$, can be written as

$$H(r,z)_{host} = \int_0^\infty \left[De^{m_4(z-z_s)} \right] \lambda J_1(\lambda r) d\lambda, \quad z \ge z_s, \tag{9}$$

where $m_4 = -\frac{\sigma_{host}}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right) - \frac{1}{2}\sqrt{\sigma_{host}^2 \left\{ \frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right) \right\}^2 + 4\lambda^2}$,

and σ_{host} is the conductivity of the host medium. The arbitrary constants A and D can be obtained from the boundary conditions that at the probe on the ground surface

$$\sigma(z)E(r,z)|_{z=0} = 0,$$

and the continuity of the magnetic field at the interface of the first and second layered which give

$$A = \frac{I}{2\pi\lambda\left[-e^{m_1 z_s} + e^{m_2 z_s} + \frac{e^{m_1 z_s}}{m_4}\left(z_s\left(\frac{\partial m_1}{\partial z}\right) + m_1 \right) - \frac{e^{m_2 z_s}}{m_4}\left(z_s\left(\frac{\partial m_2}{\partial z}\right) + m_2 \right) \right]},$$

and

$$D = \frac{I}{2\pi\lambda}\left[\frac{e^{m_1 z_s} - e^{m_2 z_s}}{-e^{m_1 z_s} + e^{m_2 z_s} + \frac{e^{m_1 z_s}}{m_4}\left(z_s\left(\frac{\partial m_1}{\partial z}\right) + m_1 \right) - \frac{e^{m_2 z_s}}{m_4}\left(z_s\left(\frac{\partial m_2}{\partial z}\right) + m_2 \right)} + 1 \right],$$

where σ now is the conductivity of the overburden. The values of $\frac{\partial m_1}{\partial z}$ and $\frac{\partial m_2}{\partial z}$ can be determined by

$$\frac{\partial m_1}{\partial z} = -\frac{2}{2\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + \frac{1}{2\sigma}\frac{\partial^2 \sigma}{\partial z^2} + \frac{\left[-\frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^3 + \frac{2}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right) \right]}{4\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}},$$

$$\frac{\partial m_2}{\partial z} = -\frac{2}{2\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + \frac{1}{2\sigma}\frac{\partial^2 \sigma}{\partial z^2} - \frac{\left[-\frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^3 + \frac{2}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right) \right]}{4\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}}.$$

Derivation of jacobian matrix for MMR method

The elements of the Jacobian matrix as mention in section 3 can be derived by using equations (8) and (9) and can be written in the form of

$$\bar{J}_{ij}^{over}(r_i, z_j) = \int_0^\infty \left[\frac{\partial A}{\partial \sigma} e^{m_1 z_j} + A z_j e^{m_1 z_j} \frac{\partial m_1}{\partial \sigma} - \frac{\partial A}{\partial \sigma} e^{m_2 z_j} - A z_j e^{m_2 z_j} \frac{\partial m_2}{\partial \sigma}\right] \lambda J_1(\lambda r_i) d\lambda, \quad (10)$$

and

$$\bar{J}_{ij}^{host}(r_i, z_j) = \int_0^\infty \left[\frac{\partial D}{\partial \sigma_{host}} e^{m_4 z_j} + D z_j e^{m_4 z_j} \frac{\partial m_4}{\partial \sigma_{host}}\right] \lambda J_1(\lambda r_i) d\lambda, \quad (11)$$

where

$$\frac{\partial D}{\partial \sigma} = \frac{\partial A}{\partial \sigma}\left(e^{m_1 z_s} - e^{m_2 z_s}\right) + A\left(z_s e^{m_1 z_s} \frac{\partial m_1}{\partial \sigma} - z_s e^{m_2 z_s} \frac{\partial m_2}{\partial \sigma}\right)$$

$$\frac{\partial A}{\partial \sigma} = \frac{I}{2\pi\lambda}\left[-e^{m_1 z_s} + e^{m_2 z_s} + \frac{e^{m_1 z_s}}{m_4}\left(z_s \frac{\partial m_1}{\partial z} + m_1\right) - \frac{e^{m_2 z_s}}{m_4}\left(z_s \frac{\partial m_2}{\partial z} + m_2\right)\right]^{-1} \times$$

$$\{-z_s e^{m_1 z_s} \frac{\partial m_1}{\partial \sigma} + z_s e^{m_2 z_s} \frac{\partial m_2}{\partial \sigma} + \frac{z_s e^{m_1 z_s}}{m_4} \frac{\partial m_1}{\partial \sigma}\left(z_s \frac{\partial m_1}{\partial z} + m_1\right)$$

$$-\frac{e^{m_1 z_s}}{m_4^2} \frac{\partial m_4}{\partial \sigma}\left(z_s \frac{\partial m_1}{\partial z} + m_1\right) + \frac{e^{m_1 z_s}}{m_4}\left(z_s \frac{\partial^2 m_1}{\partial \sigma \partial z} + \frac{\partial m_1}{\partial \sigma}\right)$$

$$-\frac{z_s e^{m_2 z_s}}{m_4} \frac{\partial m_2}{\partial \sigma}\left(z_s \frac{\partial m_2}{\partial z} + m_2\right) + \frac{e^{m_2 z_s}}{m_4^2} \frac{\partial m_4}{\partial \sigma}\left(z_s \frac{\partial m_2}{\partial z} + m_2\right)$$

$$-\frac{e^{m_2 z_s}}{m_4}\left(z_s \frac{\partial^2 m_2}{\partial \sigma \partial z} + \frac{\partial m_2}{\partial \sigma}\right)\},$$

$$\frac{\partial m_1}{\partial \sigma} = -\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right) + \frac{\sigma}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2}\right) + \frac{\sigma\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)^2 - \sigma^2\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2}\right)\right)}{2\sqrt{\sigma^2\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)^2 + 4\lambda^2}},$$

$$\frac{\partial m_2}{\partial \sigma} = -\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right) + \frac{\sigma}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2}\right) - \frac{\sigma\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)^2 - \sigma^2\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2}\right)\right)}{2\sqrt{\sigma^2\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\right)\right)^2 + 4\lambda^2}},$$

$$\frac{\partial m_4}{\partial \sigma} = -\frac{1}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right) + \frac{\sigma_{host}}{2}\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2_{host}}\right) - \frac{\sigma_{host}\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right)\right)^2 - \sigma^2_{host}\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right)\right)\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma^2_{host}}\right)\right)}{2\sqrt{\sigma^2_{host}\left(\frac{\partial}{\partial z}\left(\frac{1}{\sigma_{host}}\right)\right)^2 + 4\lambda^2}},$$

$$\frac{\partial^2 m_1}{\partial \sigma \partial z} = \frac{1}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^2 - \frac{1}{2\sigma^2}\left(\frac{\partial^2 \sigma}{\partial z^2}\right)$$

$$+\frac{\left[2\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}\left(\frac{3}{\sigma^4}\left(\frac{\partial \sigma}{\partial z}\right)^3 - \frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right)\right) + \left(\frac{1}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^3 - \frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right)\left(\frac{-\frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^2}{\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}}\right)\right)\right]}{4\left(\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2\right)},$$

and $$\frac{\partial^2 m_2}{\partial \sigma \partial z} = \frac{1}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^2 - \frac{1}{2\sigma^2}\left(\frac{\partial^2 \sigma}{\partial z^2}\right)$$

$$-\frac{\left[2\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}\left(\frac{3}{\sigma^4}\left(\frac{\partial \sigma}{\partial z}\right)^3 - \frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right)\right) + \left(\frac{1}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^3 - \frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)\left(\frac{\partial^2 \sigma}{\partial z^2}\right)\left(\frac{-\frac{2}{\sigma^3}\left(\frac{\partial \sigma}{\partial z}\right)^2}{\sqrt{\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2}}\right)\right)\right]}{4\left(\frac{1}{\sigma^2}\left(\frac{\partial \sigma}{\partial z}\right)^2 + 4\lambda^2\right)}.$$

The apparent conductivity calculated by MMR method

In this paper, we use the apparent conductivity conducted by Chen and Oldenburg (2006) for MMR method to start the iteration process. The apparent conductivity conducted by Chen and Oldenburg (2006) is denoted by

$$\sigma_{apparent} = \frac{a}{\left[\frac{\mu_0 I}{2\pi r H}\left\{\frac{2}{\sqrt{1+\left(\frac{r}{h}\right)^2}} - \frac{\alpha}{\sqrt{1+\left(\frac{r}{2h}\right)^2}}\right\} - 1\right]}, \tag{12}$$

where μ_0 is the magnetic permeability of free space, r is the radial distance between the current electrodes, a is the conductivity of ground surface which is assumed to be known from the measurement, h is denoted the depth of the electrode which is deliberately placed at, $z = h$, I is the electric current injected to the ground and α is denoted by

$$\alpha = \begin{cases} 1.0 & \text{if } \frac{r}{h} < 0.2, \\ \hat{\alpha}(x,y) & \text{otherwise} \quad x = \log_{10}\left(\frac{r}{h}\right), \; y = \log_{10}\left(\frac{\sigma_1}{a}\right). \end{cases}$$

σ_1 is the conductivity of the ground at the end of the electrode, $z = h$, and

$$\hat{\alpha}(x,y) = \left(-0.023 - 0.055y + 0.101y^2\right)$$
$$+\left(-0.012 - 0.029y + 0.005y^2 + 0.087y^3\right)x$$
$$+\left(0.023 + 0.080y - 0.124y^2\right)x^2$$
$$+\left(0.021 + 0.064y - 0.136y^2\right)x^3$$

Numerical experiments

We, firstly, start to consider and analyze the behavior of magnetic field response from the ground at different depths. The pattern of the field hopefully may imply the ground structure. The forward problems are performed to compute the magnetic field by using the known geometric models. The half-space of two layered earth will be considered and used in our model and the magnetic field data from the injection of 1 ampere of DC probe source which 1 meter depth perpendicular to the ground surface are computed and plotted as shown in Figure 2. The conductivity of the two layered earth are 1 S/m. for overburden and 0.2 S/m. for the host. The behavior that the curves of magnetic field perform rarely oscillates and seems to be meaningless for direct interpretation for the ground structure.

In our inverse model example, we simulate the magnetic field data from the injection of 1 ampere of DC probe source which 1 meter depth perpendicular to the ground surface. The conductivity of the ground is denoted by $\sigma(z) = \begin{cases} e^{(\ln_{0.2})z,\; 0 \le z \le 1} \\ 0.2,\; z \ge 1 \end{cases}$. The magnetic field is used to calculate the apparent conductivity by using Chen and Oldenburg formula. The graph of apparent conductivity is plotted and shown in Figure 3.

The matrixes $\bar{J}$ are constructed and the vector $\bar{g}$ can be calculated by using the forward model with initial guess for the conductivity $\sigma(z)$. The Chave algorithm (Chave, 1983) is used to compute the infinite integration under Fortran 90 code on Pentium IV, PC machine. The sensitivity vector $\delta\sigma(z)$ can be determined to improve the initial guess of $\sigma(z)$ by using equation (1). The inversion is made to investigate the conductivity $\sigma(z)$ by using MMR method. The speed of con-

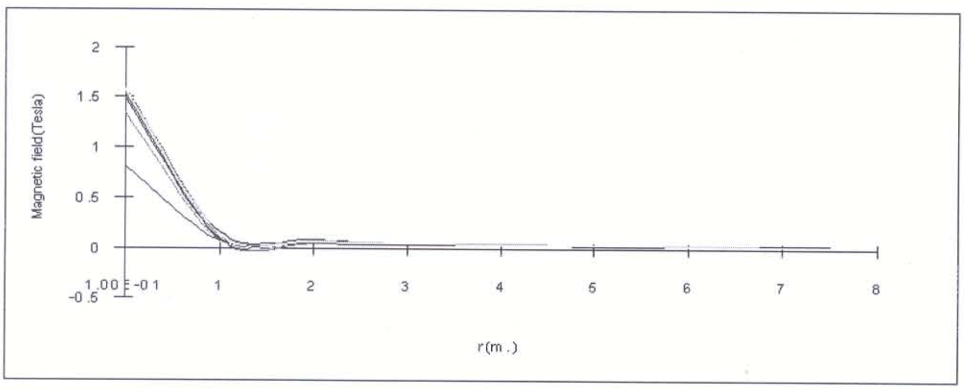

Figure 2. The behavior of magnetic field against *r* at different depth z = 0, 0.1, 0.2,..., 1.0 m.

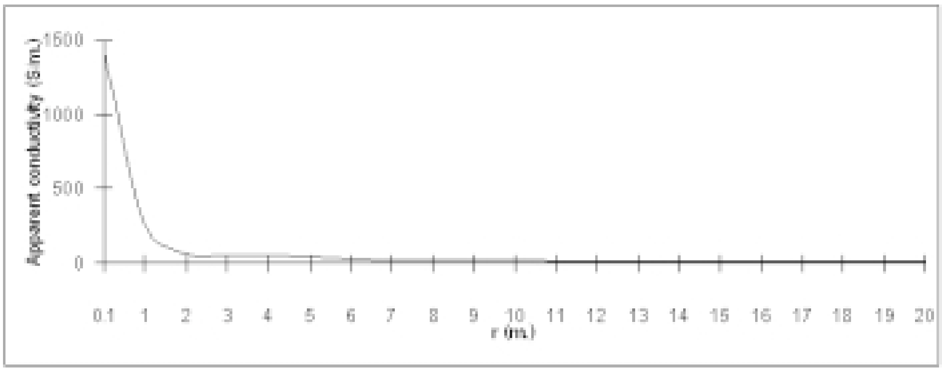

Figure 3. The graph of apparent conductivity against *r*

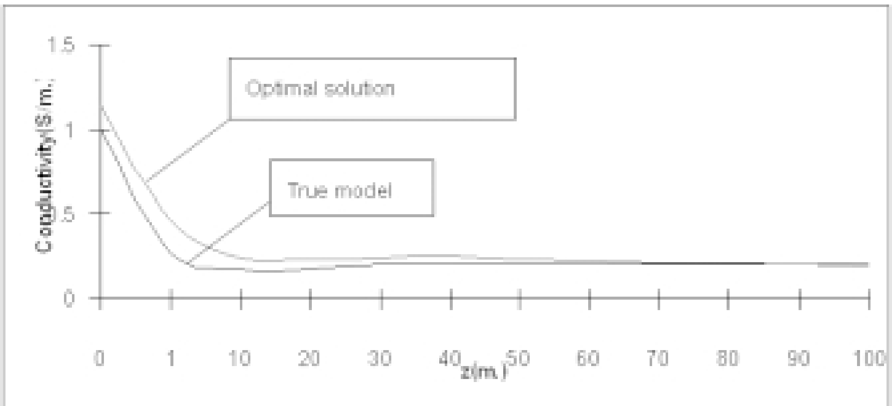

Figure 4. Graph of optimal solution compares with the true model

vergence of inversion process is 12 iterations that best fit the observed data. This record is good as same as the work done by Yooyuanyong and Sripanya (2005). However, we can see that the speed of convergence of our algorithm is slower than the algorithms conducted by Vozoff and Jupp (1975) and Yooyuanyong *et al.* (2005). This leads to our future works to analyze and compare the structure of algorithms. The optimal solution for conductivity of the ground is very close to the true one and is shown in Figure 4.

Conclusions

In this paper, we perform both of mathematical forward and inverse modeling to explore the ground structure. The Maxwell's equations are used as our governing equations. The magnetic fields computed from the forward problem are considered to investigate their relations with depth. Unfortunately, the behavior of the magnetic fields does not show any significant in pattern for identifying the structure of the ground as the curves of magnetic field smoothly decay and rarely oscillate so we need a more complicated method to explore the earth structure.

In the inversion example, the algorithm to invert the conductivity of the ground is presented. The linearized inverse theory is employed to construct the matrix which is used to iteratively obtain the conductivity profile from the starting model. The iterative scheme employs a smoothing filter which aims to reduce high frequency oscillations and to keep the conductivity structure realistic. The inversion of magnetometric resistivity method has been used and performed to give a fast speed of convergence.

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