ORIGINAL ARTICLE

Effect of material uncertainties on dynamic response of segmental box girder bridge

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Abstract

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The main objective of this paper was to investigate the effect of material uncertainties on dynamic response of segmental box girder bridge subjected to a moving load, in this case a rapid passing trains. Literatures concerned with the design of segmental box girder bridge, the application of finite element analysis to model the segmental box girder bridge, and the minimum requirement for structural conditions of the bridge were described and discussed in detail. A series of finite element analysis was carried out using SAP2000 Nonlinear software. The effect was investigated by varying the Modulus of Elasticity by 5%, 10% and 15%. The results were then compared with the case of assumed uniform property which had already been checked for model accuracy using the Standard prEN 1991-2.

The results showed that, for the uniform case, the dynamic responses of the bridge gave the highest response at the resonance speed. When considering the non-uniform material properties (non-uniform case), the effect of material uncertainties appeared to have an effect on both displacement and acceleration responses. Nonetheless, the dynamic factor provided in the design code was sufficient for designing the

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segmental box girder bridge with either uniform or non-uniform material properties for the train speeds considered in this study.

Key words : segmental box girder bridge, dynamic response, rapid passing train, material uncertainties

บทคัดย่อ

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ผลกระทบของความไม่แน่นอนของคุณสมบัติของคอนกรีตต่อการตอบสนองทางพลศาสตร์ของสะพาน แบบ Segmental Box Girder

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จุดประสงค์หลักของงานวิจัยเพื่อศึกษาผลกระทบของความไม่แน่นอนของคุณสมบัติของคอนกรีตต่อการตอบสนองทางพลศาสตร์ของสะพานแบบ segmental box girder ภายใต้แรงแบบเคลื่อนไหว (Moving load) อันเนื่องมาจากขบวนรถไฟที่วิ่งด้วยความเร็วสูง โดยในบทความจะประกอบด้วยทฤษฎีที่เกี่ยวข้องกับหลักการออกแบบสะพานแบบ segmental box girder เทคนิคการจำลองสะพานแบบ segmental box girder โดยใช้ระเบียบวิธีเชิงตัวเลข (finite element method) และหลักเกณฑ์การตรวจสอบโครงสร้างสะพานชนิดนี้ โดยการศึกษาในครั้งนี้ได้ใช้โปรแกรม SAP2000 Nonlinear ผลกระทบของความไม่แน่นอนของคุณสมบัติของคอนกรีตจะถูกศึกษาโดยใช้การแปรผันของค่าโมดูลัสยืดหยุ่นของคอนกรีตที่ 5 10 และ 15% ตามลำดับ

ผลการศึกษาแสดงให้ทราบว่าในกรณีที่คุณสมบัติของคอนกรีตสม่ำเสมอ การตอบสนองของสะพาน (ค่าการแอ่นตัว และค่าความเร่ง) มีค่าที่สูงที่สุดเมื่อความเร็วของรถไฟเท่ากับความเร็ววิกฤติ สำหรับกรณีที่พิจารณาถึงความไม่สม่ำเสมอของคุณสมบัติของคอนกรีตพบว่ามีผลต่อการตอบสนองทั้งค่าการแอ่นตัวและค่าความเร่ง แต่อย่างไรก็ตาม ค่าตัวคูณพลศาสตร์ที่กำหนดในข้อกำหนดการออกแบบโครงสร้างสะพานแบบ segmental box girder นั้นสามารถครอบคลุมถึงผลของแรงกระทำแบบพลศาสตร์ และความไม่แน่นอนของคุณสมบัติของคอนกรีต

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For thousands of years, human has used bridges to cross any obstructions from traveling (e.g. river, road). Several types of bridges have been used and various types of bridge construction techniques have been created (Xanthakos, 1994). Novel and innovative construction techniques, however, are still needed. External-prestressed precast-concrete segmental hollow box-girder bridge is one of the major developments in bridge construction techniques over the last few years. This development was due to the high demands on economical design, high durability, and fast and versatile construction. The great advantage of this construction technique has rendered them the more desirable structures for many large elevated highways and railway bridges, especially in Southeast Asia region. As opposed to classical monolithic structures, a segmental bridge consists of "small" pieces stressed together by external tendons as shown in Figure 1.

With regards to railway bridges, one of the main design issues is related to the dynamic (moving) loadings, for which basic solutions were described by Timoshenko, Young, and Weaver (1974), and fully discussed by Fryba (1972, 1996). Most engineering design codes for railway bridges have followed the approach using the dynamic amplification factor proposed in UIC (1979), FS (1997), IAPF (2001), and prEN 1991-2 (2002). This approach takes into account the dynamic effects

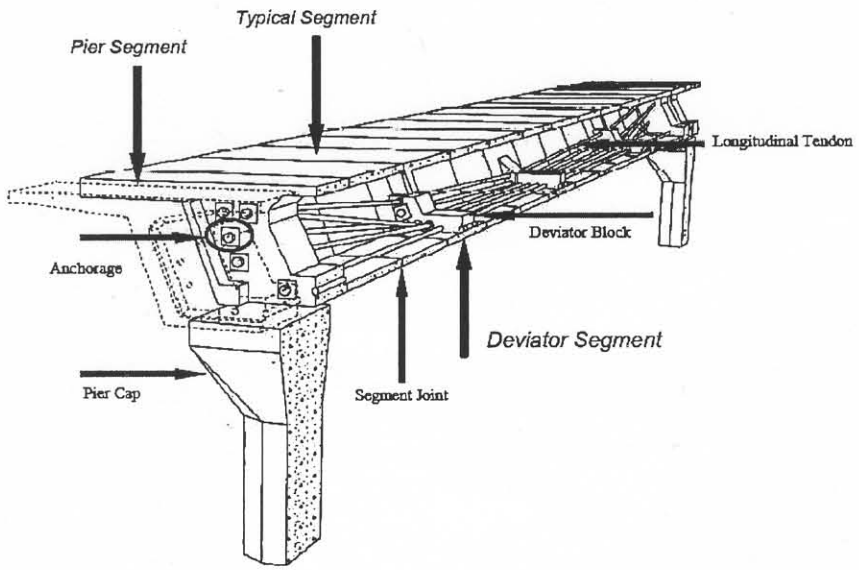


Figure1. An assembly of segmental box girder bridge.

of a single moving load and yields a maximum dynamic amplification factor of $\varphi = 132\%$ for a track without irregularities. In addition, this factor has been used to modify the forces computed based on static assumption. However, this approach does not include the possibility of the variation of material property of each segmental box due to casting process.

As such, the objective of this study was to investigate the effect of material uncertainties on dynamic responses of the segmental box girder bridges subjected to a rapid passing train. First, direct dynamic integration methods were applied to such simple span bridges in order to gain improved knowledge regarding their dynamic behavior. Then, the effects of material uncertainties were investigated by replacing the uniform modulus of elasticity with the varying ones. Monte-Carlo Simulation technique (MCS) was employed to generate a random set of modulus of elasticity. The results were then compared with the case for assumed uniform property. Finally, the effects of material uncertainties on dynamic responses of the segmental box girder bridge were discussed and summarized with regards to the existing design codes.

Literature survey

The following sections provide theoretical backgrounds on the Finite Element (FE) technique for analyzing beam under moving loads, design specification for the structural responses due to the passage of rolling stock, and Monte-Carlo Simulation technique (MCS). The first two topics are required for analyzing and validating beam type structures under a moving load. The last one deals with random number generation, which was used to generate the random material property.

Finite element technique for analyzing beams under moving loads

The dynamic behavior of segmental bridges can be analyzed systematically by means of the finite element technique. Basically, this technique performs a (semi-) discretisation in spatial co-ordinates. This technique can be applied to any type of structures, and can account for both linear and nonlinear types of behaviors (Yang *et al.*, 1997). However, only linear elastic behavior is considered in this study. The finite element discretisation (Bathe, 1992) results in the discrete N-degree of freedom (N-DOF) system of algebraic equations:

$$M\ddot{d} + C\dot{d} + Kd = f(t) \quad (1)$$

where M, C, K are the mass, damping, and stiffness matrices respectively; f(t) is the load vector (from moving loads); and d is the vector of nodal displacements.

In order to integrate these equations in the temporal-coordinate, the modal analysis and reduction technique leading to a reduced number of significant eigen-modes are performed (Chopra, 2001). The modal analysis results in uncoupled algebraic equations, which can be integrated by any standard time-integration schemes such as the Newmark's method (Newmark, 1959).

The simplest procedure to represent the passing train is to apply load-pulse time histories at each node, depending on the arrival time and the spatial discretisation. Therefore, in order to simulate the moving load, one may apply forces and moments, which are a function of time, to all the nodes in the finite element mesh of the whole structure. As shown in Figure 2, a concentrated force moves with velocity V from node 1 to node n of the finite element mesh, which is composed of n nodes and n-1 beam elements.

When a beam is subjected to a concentrated force P, the forces applied on all the nodes of the other beams are equal to zero. The value of the force at the node in the element subjected to a concentrated force is a function of time as shown in Figure 3.

By ignoring moments at both ends of each element ($f_2^{(s)}(t)$ and $f_4^{(s)}(t)$) in Figure 3, a simple linear interpolation for the forces (see Figure 4) would allow the whole procedure to be generalized as followed:

$$f_1^{(s)}(t) = P\left(1 - \frac{x}{l}\right) \quad (2)$$

$$f_3^{(s)}(t) = P\left(\frac{x}{l}\right) \quad (3)$$

The time (t), at which a concentrated force moves with velocity V from node 1 to node i on the beam can be found from the following equation:

$$t_i = \frac{(i-1)\Delta x}{V}, \quad i = 1,2,\ldots,n \quad (4)$$

where Δx is the element length (x_i - x_{i-1}) and V is the train velocity.

Design specification for structural response due to passage of rolling stock

The response of the bridge to the actual rolling stock depends on the train velocity and the

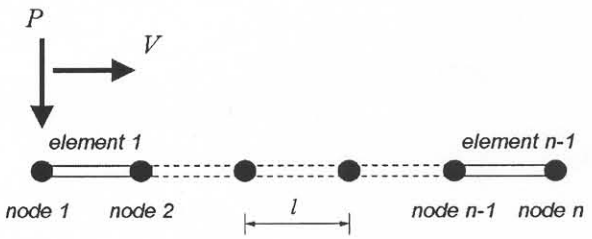


Figure 2. A beam subjected to a concentrated force P moving with velocity V.

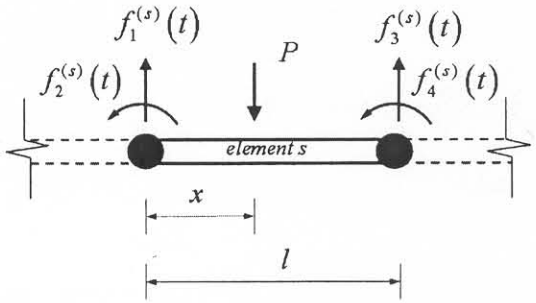


Figure 3. The equivalent forces of the element s subjected to a concentrated force P.

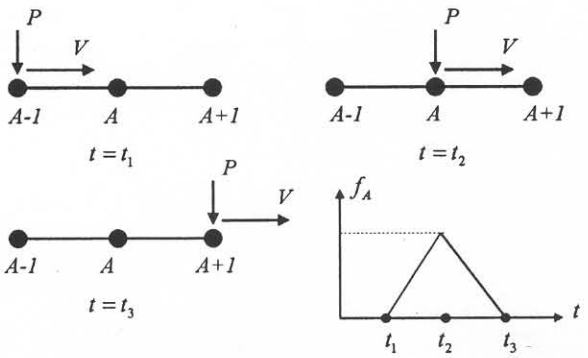


Figure 4. Definition of a load at node A for moving load P in the finite element model (Wu *et al.*, 2000).

Figure 5. Schematic of end rotation measurement.

natural frequency of the structure. In order to check the dynamic characteristics of the bridge, the following safety criteria shall be satisfied (according to the EuroCode: prEN1991-2, (2002)):

1. Vertical acceleration of the bridge $\leq$ 0.35g.

2. The variation of cant induced $\leq$ 0.4 mm/m by transverse rotation of the bridge (upon a base of 3 m longitudinally). This can be calculated from Equation 5.

$$t = \frac{S \times \theta}{L} = S \times \frac{\left(\frac{T \times L}{G \times J}\right)}{L} \tag{5}$$

where t is the cant induced; S is an eccentricity of the rail; T is torsion; L is element length; G is shear modulus of elasticity; and J is torsional constant.

3. End rotation shall not be exceeded.

For ballasted and slab track, the end rotation of the bridge at the expansion joints (for actual traffic loads multiplied by the relevant dynamic factor):

$$\theta = \left[\frac{8 \times 10^{-3}}{h(m)} radians\right] \tag{6}$$

$h(m)$: The distance between the top of rail and the centre of the bridge bearings.

Monte-Carlo Simulation (MCS)

The main key in the application of Monte-Carlo Simulation is the generation of appropriate random numbers for a given distribution of random numbers (Nowak and Collins, 2000; Ang and Tang, 1975; Ang and Tang, 1984). For each random variable, the generation process can be achieved by the following procedures:

1. Generate a uniformly distributed random number between 0 and 1.0.

2. Use the inverse transformation method to transform the uniformly distributed random number to a corresponding random number with a given distribution.

For the inverse transformation method, it can be shown graphically in Figure 6.

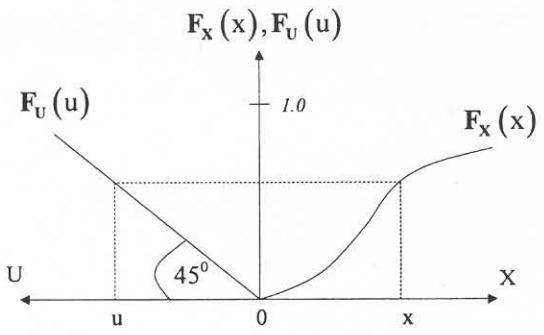


Figure 6. Inverse transformation method.

From Figure 6, suppose U is a standard uniform variate with a uniform PDF between 0 and 1.0 and X is a random variate with its CDF $F_x(x)$. If u is a value of the variate U, the cumulative probability of is equal to u (see Figure 7).

$$F_u(u) = u \tag{7}$$

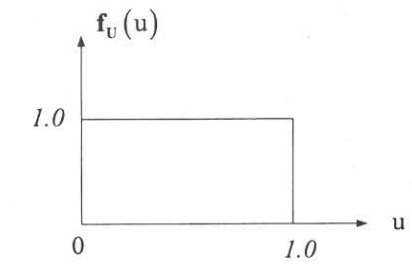
Therefore, for the variate X, at the cumulative probability u, the value of X can be calculated from

$$x = F_x^{-1}(u) \tag{8}$$

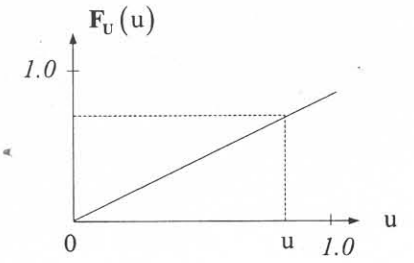
which means if $(u_1, u_2, u_3, ..., u_n)$ is the set of values from U, the corresponding set of values of $(x_1, x_2, x_3, ..., x_n)$ from X is obtained from

$$x_i = F_x^{-1}(u_i) \tag{9}$$

This transformation method can be used most effectively when the inverse of CDF of the random variable X can be expressed analytically.



(a) PDF of U



(b) CDF of U

Figure 7. PDF and CDF of standard uniform variate.

Dynamic behavior of segmental box girder bridge due to passage of high-speed train

This section is mainly concerned with the dynamic response of a single span segmental box girder bridge subjected to train loading using the finite element technique. The finite element software package SAP2000 Nonlinear (2000) was employed in this study. The dynamic characteristics of a typical span of segmental box girder bridge, especially the frequency response and potential resonance, were investigated. The structures were excited by a series of train loading with different speed, i.e. 100, 150 and 174 km/hr (resonance speed), respectively. The safety criteria including acceleration limit, the variation of cant induced by transverse rotation of the bridge, and end rotation were measured.

Model descriptions

Bridge

A typical span of segmental box girder bridge was modeled in this study. The bridge which had a span length of 35.5 m and consisted of 13 segmental boxes is shown in Figure 8.

Three-dimensional frame element available in SAP2000 Nonlinear was used to model the bridge. These elements had 12 degrees of freedom. Each element was rigidly attached to each other (rigid connection). The cross sectional properties of each segment are summarized in Table 1.

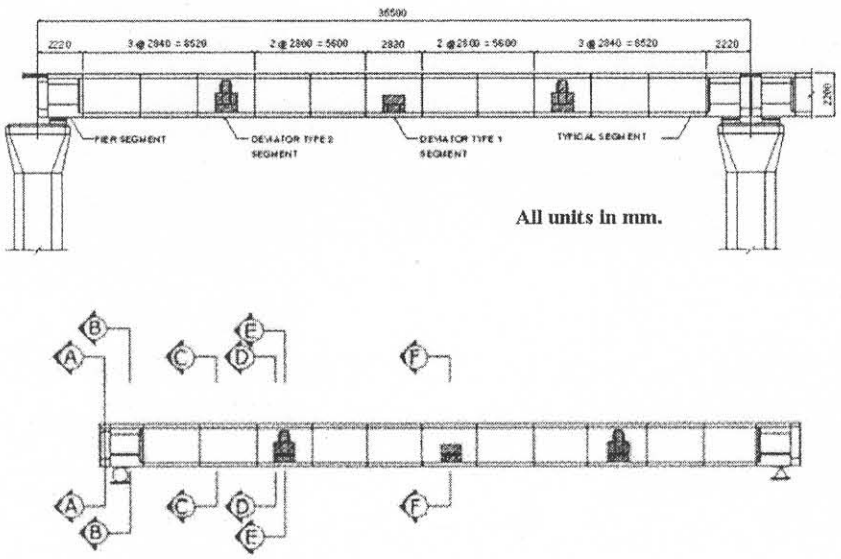


Figure 8. Schematic of a typical span of segmental box girder bridge.

Table 1. Cross section properties of segmental box.

	A (m^2)	J (m^4)	I_{22} (m^4)	I_{33} (m^4)
Section A-A	6.215	0. 4866	44.632	2.327
Section B-B	10.204	8.490	53.361	5.433
Section C-C	5.751	7.969	32.667	4.033
Section D-D	7.900	9.640	35.742	5.044
Section E-E	12.250	13.289	41.766	5.564
Section F-F	7.177	9.386	35.715	4.797

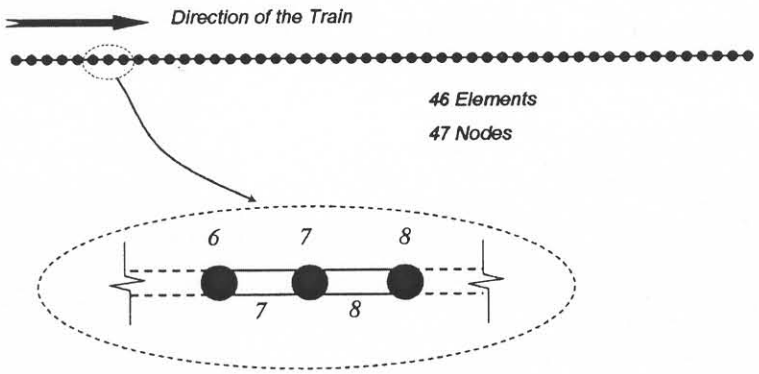


Figure 9. Schematic of the finite element model of a typical span of segmental box girder bridge.

Material properties used for the FE model are summarized below:

Concrete compressive strength	400 ksc
Young's Modulus	3,020,000 T/m^2
Poisson's Ratio	0.2

The finite element mesh of a typical span of segmental box girder bridge consists of 46 elements and 47 nodes as shown in Figure 9.

Railway Track

The standard gauge was employed in this project. The arrangement of the track is shown in Figure 10.

Vehicle

The train used in this study has 4 cars with equal axle load of 19 tons. The total length of the train is 68.35 m as shown in Figure 11.

Analysis of results

Checking safety criteria of the bridge

First of all, the safety criteria of the bridge were checked as follows:

- Vertical acceleration of the bridge at the mid-span of the bridge was monitored using finite element analysis. The obtained values must be less than 0.35g. The results are shown in Table 2. Both train speeds satisfied this criterion.
- The variation of cant induced by transverse rotation of the bridge can be obtained using Equation 5. The obtained values are less than the limit value (0.4 mm/m)
- End rotation at both supports shall not exceed the value calculated from Equation 6 as follows;

The end rotation Limit

$$\theta = \frac{8 \times 10^{-3}}{3.32} = 0.00241 \ \ radian$$

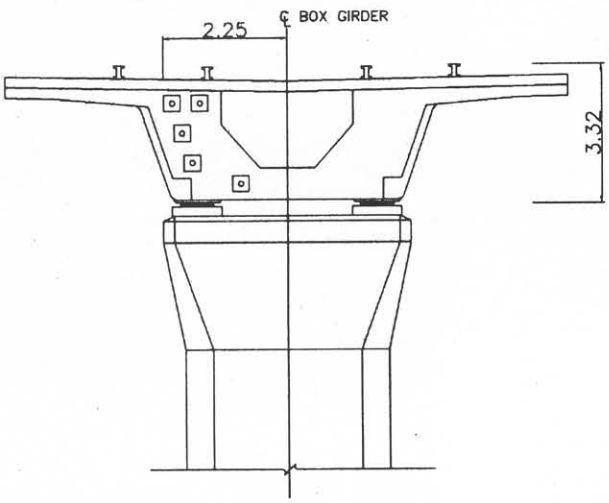


Figure 10. Schematic of the track arrangement.

Figure 12 shows an example of end rotation response at both supports due to passage of the train at $V = 100$ km/hr. The opposite sign of response indicates the opposite direction of end rotation.

The results of end rotation response at the support due to the train speeds of 100 and 150 $\frac{km}{hr}$ are shown in Table 3.

From Table 3, it can be seen that the maximum end rotation of the bridge was within the limit.

Dynamic response of a typical span of segmental box girder bridge under uniform material properties (uniform case)

For this part of the study, the Young's modulus of the concrete was assumed to be uniform for all elements. Three train speeds, V, i.e. 100, 150 and 174 km/hr (the resonance speed), were considered. The dynamic responses of the bridge due to passage of the rapid train were monitored in terms of displacement and acceleration responses as shown in Figures 13 and 14. Figure 15 compares the maximum displacement and maximum acceleration responses at the mid-span of the bridge (uniform case).

As observed in Figures 13 and 14, the simulation results show that the resonance speed gives a higher response than those obtained from the case of 150 km/hr and 100 km/hr, respectively. At the resonance speed, the maximum displacement is around 0.007208 m and the maximum acceleration is about 1.521 m/s^2, as shown in Figure 15. Therefore, it is reasonable to conclude that the velocities considered in this study gave a

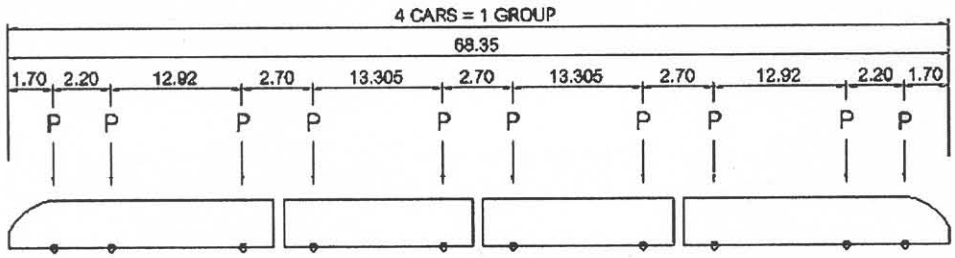
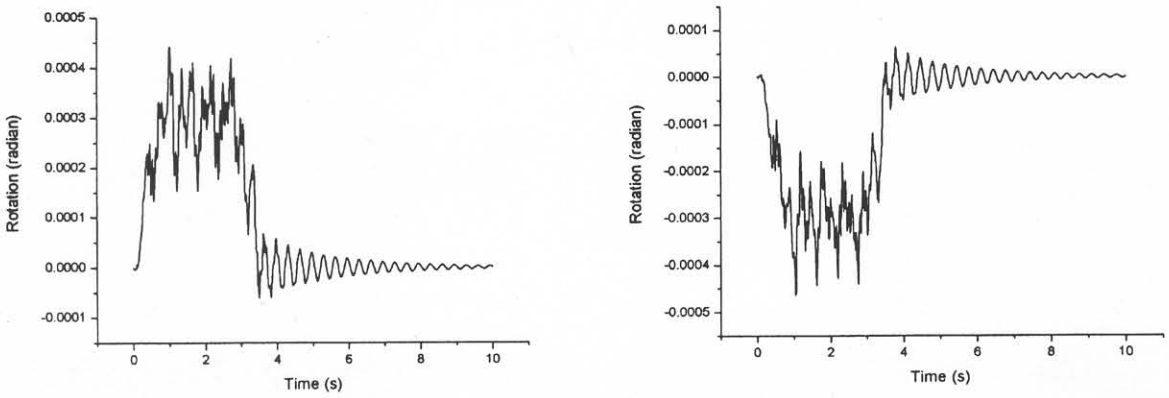


Figure 11. Schematic representation of the train used in this study.

Table 2. Checking of maximum acceleration at the mid-span.

Train Speed $(\frac{km}{hr})$	Max. Acc. $(\frac{m}{s^2})$	Max. Acc. (g)	Check A < 0.35g
100	0.7098	0.07235	OK
150	0.7214	0.07354	OK



(a) Left support (maximum rotation = 0.0004416 rad.) (b) Right support (maximum rotation = 0.0004646 rad.)

Figure 12. Schematic end rotation at both supports of the bridge (V=100 $\frac{km}{hr}$).

Table 3. Checking end rotation at the supports.

Train Speed ($\frac{km}{hr}$)	End supports	Max. Theta from SAP2000 Nonlinear (radian)	Theta Limit (radian)	Check status
100	Left	0.0004416	0.00241	OK
	Right	0.0004646	0.00241	OK
150	Left	0.0004515	0.00241	OK
	Right	0.0004480	0.00241	OK

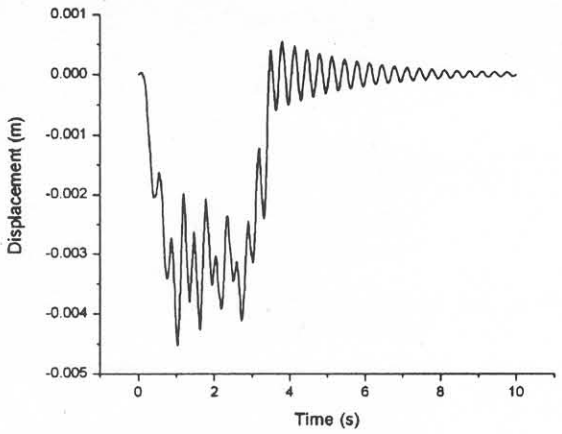
response almost 50% lower than that of the resonance speed.

Dynamic response of a typical span of segmental box girder bridge under non-uniform material properties (non-uniform case)

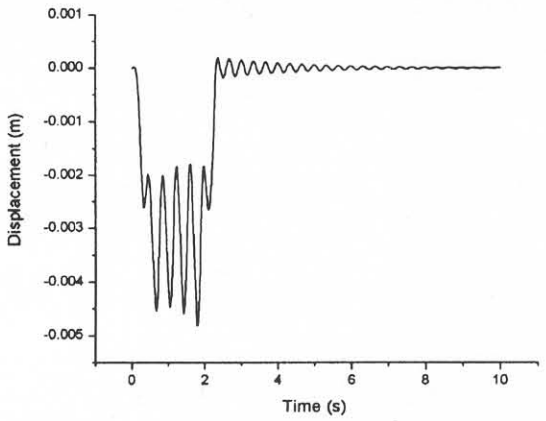
For this part of the study, the material property, i.e. the Young's modulus, was assumed to vary throughout the girder span. Monte Carlo Simulation technique (MCS) was used to generate the random property of material by assuming that the Young's modulus had normal distributions. Three variations were investigated: 5, 10 and 15% deviation. For each case of variation, 200 samples were generated and analyzed under two train speeds (100 and 150 km/hr).

By using SAP2000 Nonlinear software, the dynamic responses of each sample were obtained. The dynamic response parameters monitored in this part were the same as the uniform case, i.e. displacement and acceleration responses. By extracting only the maximum response from each simulation, the histogram of the response at 100 km/hr can be plotted as shown in Figures 16 and 17. The distribution of the histogram has a normal distribution, which is similar to the distribution of material property. Distribution of the response at 150 km/hr is also similar to that obtained from the case of 100 km/hr. Comparison of the distribution due to different variation of material properties is summarized in Tables 4 and 5.

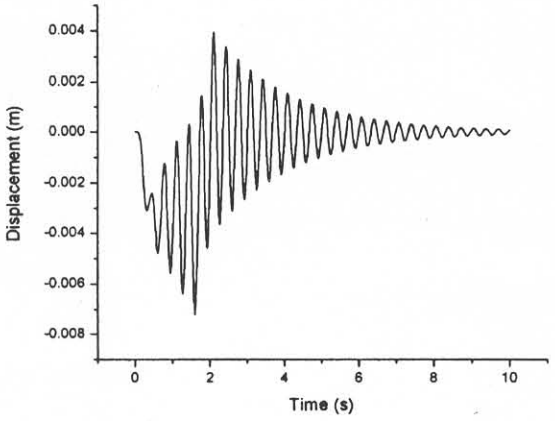
From Table 4, it is reasonable to conclude that the uncertainty of Young's modulus of the concrete to the dynamic response of span can be



(a) V=100 $\frac{km}{hr}$ (maximum displacement = 0.004529 m)



(b) V=150 $\frac{km}{hr}$ (maximum displacement = 0.004823 m)



(c) V=174 $\frac{km}{hr}$ (resonance speed) (maximum displacement = 0.007208 m)

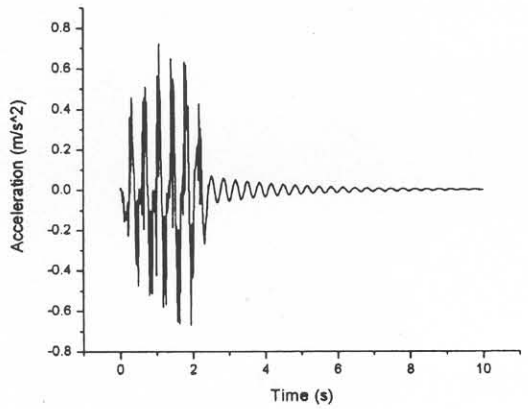
Figure 13. Displacement responses at the mid-span of the bridge (uniform case).

attributed as following;

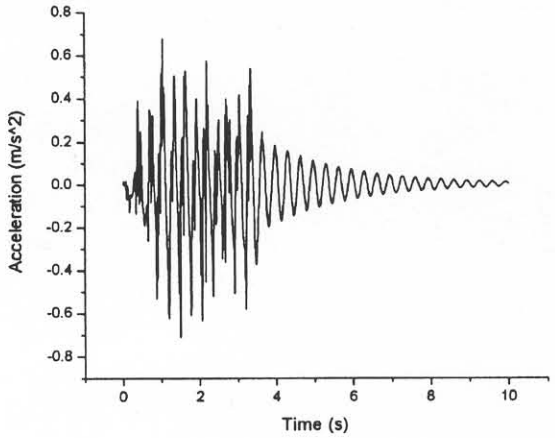
1. The distribution of the response increases with increasing variation of Young's modulus. The highest response occurs when the velocity of the train and material variation are 150 km/hr and 15%, respectively. The maximum displacement response is around 1.26 of the mean case. This value is the dynamic factor, which is actually covered by the design code. Therefore, by using the dynamic amplification factor provided by the existing design code, the effects of dynamic loads and material uncertainties on the responses of the segmental box girder bridge are sufficiently accounted.

2. By increasing the train speed, the effect of material uncertainties on dynamic response of the bridge becomes more significant.

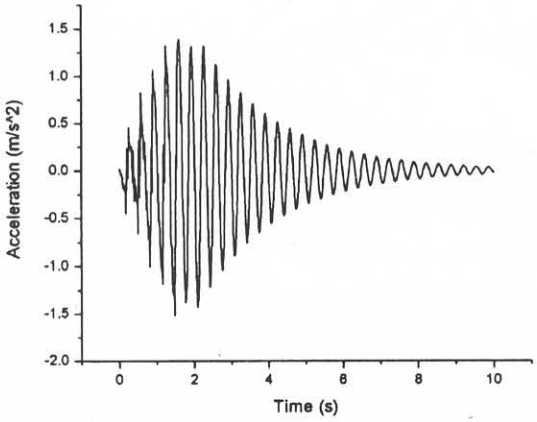
In the case of acceleration response, the effects of material uncertainties on the acceleration response of the bridge show a similar trend as the case of displacement response, as summarized in Table 5. The maximum acceleration response (0.8116 m/s^2) occurs when the velocity of the train and material variation are 150 km/hr and 15%, respectively. This value is still within the safety requirement of the existing design code (0.35g). Therefore, by using the dynamic amplification factor provided by the design code, the



(a) V=100 $\frac{km}{hr}$ (maximum acceleration = 0.7098 $\frac{m}{s^2}$)

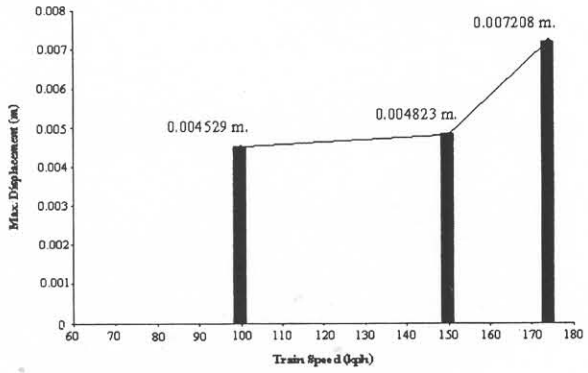


(b) V=150 $\frac{km}{hr}$ (maximum acceleration = 0.7214 $\frac{m}{s^2}$)

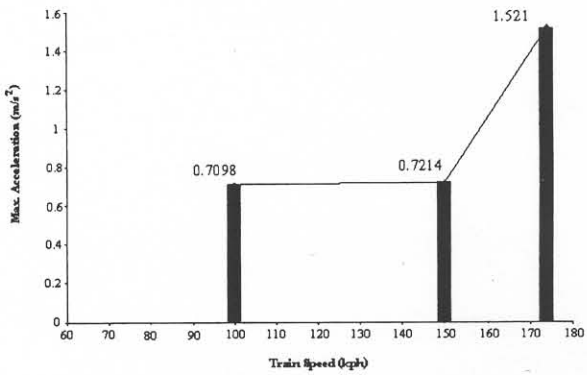


(c) V=174 $\frac{km}{hr}$ (resonance speed) (maximum acceleration = 1.521 $\frac{m}{s^2}$)

Figure 14: Acceleration responses at the mid-span of the bridge (uniform case).

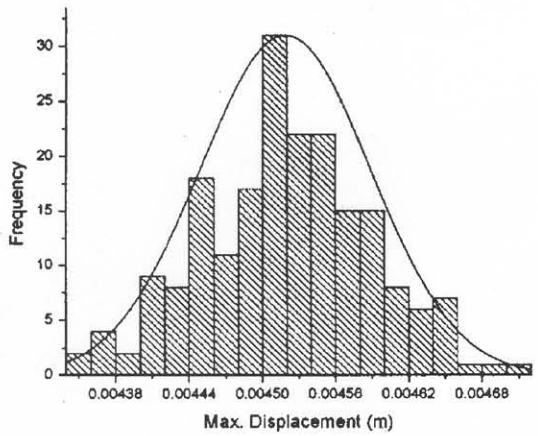


(a) Maximum displacement

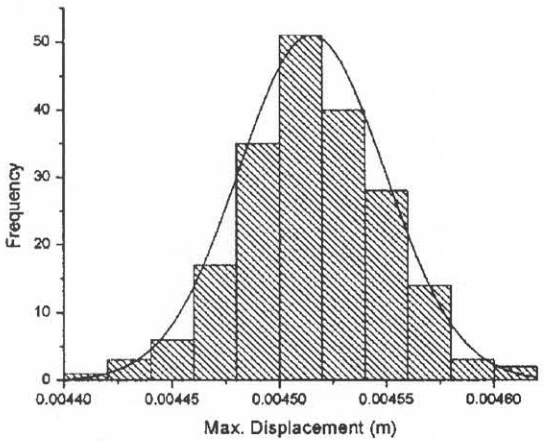


(b) Maximum acceleration

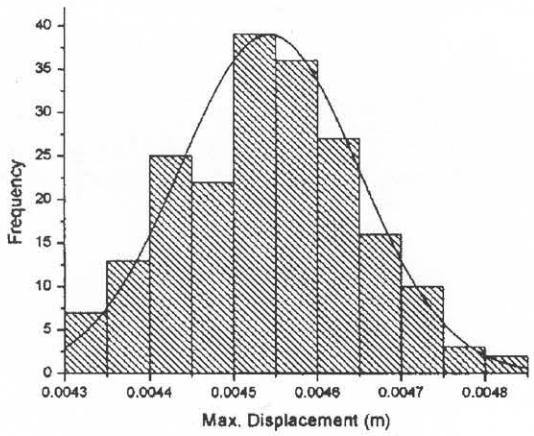
Figure 15. A comparison of maximum displacement and maximum acceleration responses at the mid-span of the bridge (uniform case).



(a) 5% variation

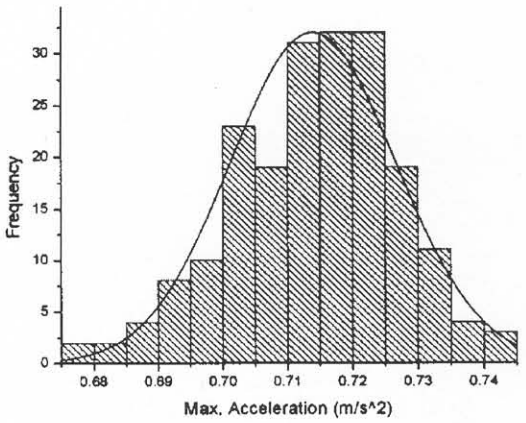


(b) 10% variation

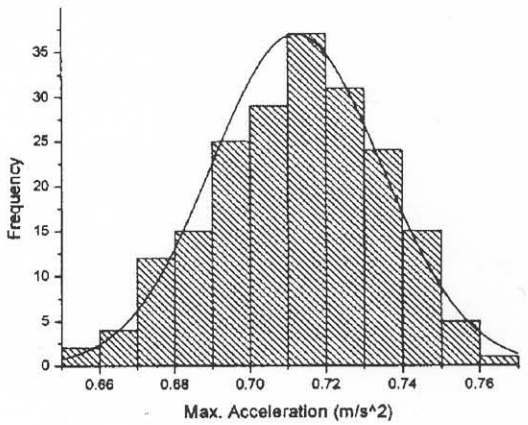


(c) 15% variation

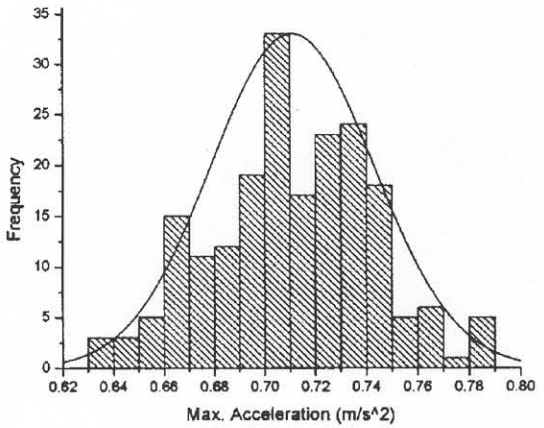
Figure 16. Histograms of the maximum displacement at the mid-span, V=100 $\frac{km}{hr}$ with 5, 10 and 15% variation.



(a) 5% variation



(b) 10% variation



(c) 15% variation

Figure 17. Histograms of the maximum acceleration at the mid-span, V=100 $\frac{km}{hr}$ with 5, 10 and 15% variation.

Table 4. Comparison of the distribution of displacement response due to different variation of material properties.

Train Speed (kph)	Variation of material property	Min. Displacement (m)	Mean Displacement (m)	Max. Displacement (m)	Different between Min. and Mean response (%)	Different between Mean and Max. (%)
100	Uniform	-	0.004529	-	-	-
	5%	0.004413	0.004515	0.004604	2.26	1.93
	10%	0.004344	0.004519	0.004705	3.87	3.95
	15%	0.004316	0.004542	0.004840	4.98	6.16
150	Uniform	-	0.004823	-	-	-
	5%	0.004506	0.004787	0.005083	5.87	5.82
	10%	0.004301	0.004813	0.005439	10.68	11.52
	15%	0.004299	0.004817	0.005444	10.71	11.52

Table 5. Comparison of the distribution of acceleration response due to different variation of material properties.

Train Speed (kph)	Variation of material property	Min. Acceleration (m/s^2)	Mean Acceleration (m/s^2)	Max. Acceleration (m/s^2)	Different between Min. and Mean response (%)	Different between Mean and Max. (%)
100	Uniform	-	0.7098	-	-	-
	5%	0.6788	0.7139	0.7418	4.92	3.76
	10%	0.6555	0.7122	0.7658	7.96	7.00
	15%	0.6301	0.7108	0.7891	11.35	9.92
150	Uniform	-	0.7214	-	-	-
	5%	0.6582	0.7144	0.7614	7.87	6.17
	10%	0.6054	0.7172	0.8111	15.62	11.58
	15%	0.6052	0.7176	0.8116	15.64	11.58

effects of dynamic loads and material uncertainties on the responses of the segmental box girder bridge are also sufficient for the acceleration response.

Conclusions

This paper presented the effect of material uncertainties of concrete on the dynamic response of segmental box girder bridge using the finite element software SAP2000 Nonlinear. The analyses deal with the material properties, i.e. uniform material properties (uniform case) and non-uniform material properties (non-uniform case) of the bridge. For the uniform case, the dynamic responses of the bridge gave the highest response at the resonance speed (V=174 km/hr) because of the resonance phenomena. When considering the non-uniform material properties (non-uniform case), the effect of material uncertainties appears to have an effect on both displacement and acceleration response. There is an important evidence from this study that the dynamic factor provided in the design code is sufficient for designing the segmental box girder bridge containing either uniform or non-uniform material properties for the train speeds considered in this study.

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