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Original Article

Compressive modulus of adhesive bonded rubber block

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Abstract

The present study examined the effect of a thin adhesive layer on the modulus of an elastic rubber block bonded between two plates. The plates were assumed to be rigid, both in extension and flexure, and subjected to vertical compression loading. The Gent's approach was used to obtain the analytic deformations of the rubber and adhesive. The analytic deformations were then validated with the finite element model. There was a good agreement between both methods. The modulus of the bonded rubber block, defined as effective modulus, was then studied. The effective modulus was increased by the factor $(1+ (a/2t)^2(6G_r h/G_a t+1)^{-1})$, which is composed of the shape factor of the rubber block *(a/2t, ratio of the bonded and unbonded areas),* and the shear stiffness factor *($G_r h/G_a t$, ratio of modulus and thickness of rubber and adhesive).* The effective modulus does not depend on either factors, when the shear stiffness of the joint is high or $G_r h/G_a t \geq 10$.

Keywords: bonded rubber block, plates, mechanical properties, analytical modeling, finite element analysis

1. Introduction

An elastic rubber block is widely used as an engineering component for load bearing (Gent 1992). Generally, the rubber block is bonded to steel plates and is assumed to have perfect bonding and to be rigid in extension and flexure to provide vertical stiffness. When the elastic rubber block is compressed due to load carrying, the rubber expands laterally. When lateral expansion is restricted, the vertical stiffness of the bonded rubber block is increased and the rubber is assumed to be incompressible (Koh and Lim, 2001). The surface interaction varies depending on the shear stiffness of the adhesive layer, which alters the stiffness of the elastic rubber block. An understanding of the effect of shear modulus and the thickness of the adhesive layer bonding between the two surfaces will facilitate the design of a more suitable adhesive.

The Gent's approach (Gent and Meinecke, 1970) is used to determine the compressive stiffness of a rubber block bonded to rigid plates (Tsai and Lee, 1998). This approach assumes that a) the rubber is in the horizontal plane, remains planar and the vertical lines become parabolic, b) the normal stress components in all directions are equal to the pressure, and c) rubber with a Poisson's ratio exactly equal to 0.5 is assumed to be incompressible. This method is generally accepted to determine the vertical stiffness of a bonded rubber block. The stiffness derived using this method has been verified and there is a good agreement when compared with a finite element analysis.

In this paper, we propose to use the Gent's approach to study the effect of a thin adhesive layer on the deformations, pressure, stress, and vertical stiffness of an elastic bonded rubber block. The Gent's approach was used to determine the analytic solutions of the displacements. The deformations of the rubber and adhesive were also validated by finite element analysis. To present the effect of the thin adhesive layer in the finite element analysis, the spring elements were modeled by the thin adhesive layer analysis (TALA) method (Dechwayukul *et al*., 2003).

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2. Analytic Methods

2.1 Rubber block bonded with thin adhesive layer

We consider an infinite strip of the rubber block in a rectangular Cartesian coordinate *x-z* as shown in Figure 1. The rubber layer has a width of *a* and a thickness of *t*. The bottom and top of the rubber layer are bonded to rigid and stiff plates by a thin adhesive that has a width of *a* and a thickness of *h*. The materials are assumed to be isotropic and linear elastic**.** In this study, it is assumed that the adhesive layer is thin compared to the thickness of rubber. The ratio of *h/t* is about 0 to 0.1 .

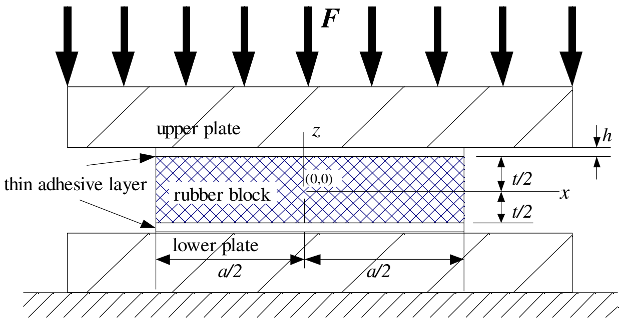

Figure 1. Thin rubber block bonded with thin adhesive layers

The lower plate is constrained in all directions. The upper plate is constrained in the *x*-direction and subjected to uniform vertical compressive load *F*; thus then it is free to translate only in the -*z* direction with the small displacement of Δ. In this case, the assumptions from Gent's approach are used, and it is also assumed that there is no normal strain in the vertical direction for the adhesive layer, because the adhesive is very thin and stiff throughout the thickness. Thus, the displacement functions of the rubber in the *z*-direction can be denoted as $w_r(z)$ when $-t/2 \le z \le t/2$. Also, the displacement functions of the adhesive in the *z*-direction can be denoted as $w_a(z)$, when $t/2 \le z \le t/2+h$ and $-(t/2+h) \le z \le -t/2$. The displacement conditions in the vertical direction are;

$$w_r(t/2) = w_a(t/2) = w_a(t/2+h) = -\Delta \qquad (1)$$

$$w_r(-t/2) = w_a(-t/2) = w_a(-(t/2+h)) = 0 \qquad (2)$$

The displacement functions in the lateral direction (*x*-direction) are $u_r(x, z)$ for the rubber and $u_a(x, z)$ for the adhesive. At arbitrary *x*, we can illustrate the deformations in the lateral direction as shown in Figure 2. It is assumed that there is perfect bonding between the thin adhesive layer and the plates, and the thickness of the adhesive layers is thin; thus, $u_a(x, z)$ is linear at $t/2 \le z \le (t/2+h)$ and $-(t/2+h) \le z \le -t/2$. The $u_r(x, z)$ is parabolic at $-t/2 \le z \le t/2$.

Considering only $z \ge 0$, because of the symmetrical geometry of the displacement in the *x*-direction, and to satisfy those conditions, the displacement functions are:

$$u_a(x,z) = \frac{u_{a0}(x)}{h}\left(\frac{t}{2} + h - z\right) \qquad (3)$$

$$u_r(x,z) = u_{r0}(x)\left(1 - 4\left(\frac{z}{t}\right)^2\right) + u_{a0}(x) \qquad (4)$$

Because of the incompressibility of the rubber layer, the sum of the normal strains in the *x* and *z* directions is zero:

$$\frac{\partial u_r(x,z)}{\partial x} + \frac{\partial w_r(z)}{\partial z} = 0 \qquad (5)$$

Substitute (4) into (5), then integrate and apply the conditions (1) and (2);

$$w_r(z) = \frac{\partial u_{r0}(x)}{\partial x}\left(\frac{4}{3}\frac{z^3}{t^2} - z\right) - \frac{\partial u_{a0}(x)}{\partial x} z - \frac{\Delta}{2} \qquad (6)$$

and,

$$\Delta = \frac{\partial u_{r0}(x)}{\partial x}\left(\frac{2t}{3}\right) + \frac{\partial u_{a0}(x)}{\partial x} t \qquad (7)$$

Because $w_r(z)$ is independent of *x*, the terms of $\partial u_{r0}(x)/\partial x$ and $\partial u_{a0}(x)/\partial x$ are constant in (6) and (7). To determine these terms, it is necessary to consider the equilibrium of forces in the *x*-direction for the adhesive and rubber layers. The state of stress dominant in the rubber layer is pressure (*p*) and shear stress (τ_r). Now it is assumed that there is only shear stress (τ_a) in the thin adhesive layer. Considering the equilibrium of a long strip at any *x* through *-t/2* to *t/2*, it is known that;

$$\tau_a = -G_a \frac{u_{a0}(x)}{h} \qquad (8)$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_r}{\partial z} = G_r\left(\frac{\partial^2 u_r(x,z)}{\partial^2 z} + \frac{\partial^2 w_r(z)}{\partial x \partial z}\right) \qquad (9)$$

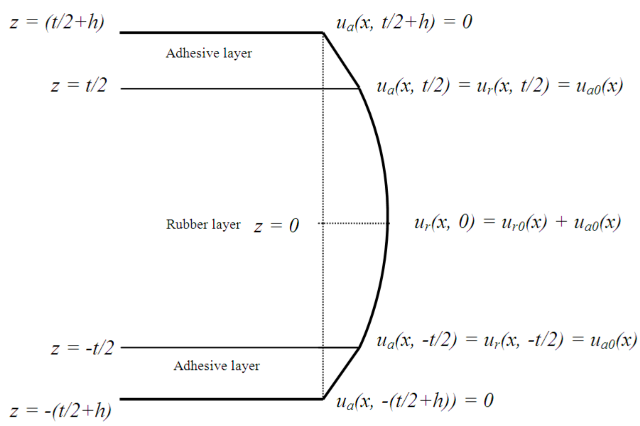

Figure 2. The deformation of the rubber and adhesive layers in the x-direction

$$\int_{-t/2}^{t/2} G_r \left(\frac{\partial^2 u_r(x,z)}{\partial^2 z} + \frac{\partial^2 w_r(z)}{\partial x \partial z}\right) dz = \int_{-\tau_a}^{\tau_a} d\tau \quad (10)$$

By substituting (4), (6), and (8) into (10), where G_a and G_r are the shear modulus of the adhesive and rubber, respectively, we found that;

$$u_{a0}(x) = 4u_{r0}(x)\left(\frac{G_r}{G_a}\right)\left(\frac{h}{t}\right) \quad (11)$$

Substitute (11) into (7);

$$\frac{\partial u_{r0}(x)}{\partial x} = \left(\frac{\Delta}{t}\right)\left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1} \quad (12)$$

$$\frac{\partial u_{a0}(x)}{\partial x} = 4\left(\frac{G_r}{G_a}\right)\left(\frac{h}{t}\right)\left(\frac{\Delta}{t}\right)\left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1} \quad (13)$$

Substitute (12) and (13) into (6);

$$w_r(z) = \Delta\left[\left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1}\left(\frac{4}{3}\right)\left(\frac{z}{t}\right)^3 - \left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1}\left(1 + \frac{4G_r}{G_a}\frac{h}{t}\right)\left(\frac{z}{t}\right) - \frac{1}{2}\right] \quad (14)$$

To determine $u_r(x, z)$ and $u_a(x, z)$, we have to determine $u_{a0}(x)$ and $u_{r0}(x)$ by integrating (12) and (13) and applying the conditions of $u_{a0}(0) = u_{r0}(0) = 0$. These give;

$$u_a(x,z) = \Delta\left[4\left(\frac{G_r}{G_a}\right)\left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1}\left(\frac{x}{t}\right)\left(\frac{1}{2} + \frac{h}{t} - \frac{z}{t}\right)\right] \quad (15)$$

$$u_r(x,z) = \Delta\left[\left(\frac{2}{3} + 4\frac{G_r}{G_a}\frac{h}{t}\right)^{-1}\left(\frac{x}{t}\right)\left(1 - 4\left(\frac{z}{t}\right)^2 + 4\frac{G_r}{G_a}\frac{h}{t}\right)\right] \quad (16)$$

Considering Equations (14) to (16), in the case of perfect bonding, we can assume that the shear modulus of the adhesive *(G_a)* is high compared to the shear modulus of the rubber *(G_r)*. Then, those equations are simplified to

$$w_r(z) = \Delta\left[2\left(\frac{z}{t}\right)^3 - \frac{3}{2}\left(\frac{z}{t}\right) - \frac{1}{2}\right] \quad (17)$$

$$u_a(x,z) = 0 \quad (18)$$

$$u_r(x,z) = \Delta\left[\frac{3}{2}\left(\frac{x}{t}\right)\left(1 - 4\left(\frac{z}{t}\right)^2\right)\right] \quad (19)$$

Considering Equations (14) to (16), in the case of lubrication, we can assume that the shear modulus of the adhesive *(G_a)* is zero. Those give;

$$w_r(z) = \Delta\left[-\left(\frac{z}{t}\right) - \frac{1}{2}\right] \quad (20)$$

$$u_a(x, \pm(t/2 + h)) = 0;\; u_a(x, \pm t/2) = u_r(x) \quad (21)$$

$$u_r(x) = \Delta\left(\frac{x}{t}\right) \quad (22)$$

For the case of lubrication between the rubber and plates, we found that $u_r(x, z)$ is independent of z or can be defined as $u_r(x)$. The adhesive displacement $u_a(x, z)$ becomes zero at $z = \pm(t/2+h)$ and equal to $u_r(x)$ at $z = \pm t/2$.

2.2 Validation of analytic solutions

The analytic solutions shown in Equations (14)-(16) were validated by the Finite Element Method (FEM). The commercial finite element code ABAQUS (ABAQUS 1998) was used for the validation. The FEM model was composed of two parts as shown in Figure 1. The rubber layer with the shape factor of $a/2t = 5$ is created in half width, because of the symmetry. It is meshed into a 2-D plane strain (CPE8H; 8-node bi-quadratic, hybrid with linear pressure). In the finite element model, the rubber layer is assumed linear-elastic material and the Poisson's ratio is very close to 0.5. The thin adhesive layers bonding between the plates and rubber are created with a thickness ratio of $h/t = 0.1$. To represent the restriction of the thin adhesive layer, the TALA method is used. This method uses spring elements to simulate the modulus and thickness of the thin adhesive layer in terms of stiffness. In this study, the shear modulus ratio of the rubber and adhesive $G_r/G_a = 0.5$ was used. At the upper plate, the rubber block is compressed with a uniform vertical displacement of $-\Delta/t = 0.001$.

2.3 Derivation of the modulus of the bonded rubber block

The compression force of the bonded rubber block is given by the sum of F_1 and F_2 (Banks *et al*., 2002). F_1 is the homogeneous compression force, which is obtained when the rubber block is compressed between fully lubricated surfaces. F_2 is the force required to keep points at the original position in the planes of the bonding surface due to shear deformation. To derive F_1, it is assumed that the deformation of the rubber in the long strip is zero or plane strain (Timoshenko and Goodier, 1987). Equation (22) and Hook's law are applied and used when the Poisson's ratio (n) of the rubber is 0.5 and the modulus of the rubber is $E_r=3G_r$ for an isotropic and incompressible material.

$$\varepsilon_x = \frac{\partial u_r}{\partial x} = -\frac{\nu}{E_r}(\nu + 1)\sigma_z \quad (23)$$

$$F_1 = \sigma_z A = -\frac{4}{3}E_r\frac{a}{t}\Delta \quad (24)$$

To derive F_2, the pressure is obtained from an equivalent condition as shown in Equation (9) and applied with a boundary condition of pressure at the edges of the rubber at zero or $p(\pm a/2)=0$. It is found that:

$$p(x) = -\frac{G_r}{\left(\frac{1}{6} + \frac{G_r h}{G_a t}\right)}\left(\frac{\Delta}{t}\right)\left[\left(\frac{x}{t}\right)^2 - \left(\frac{a}{2t}\right)^2\right] \quad (25)$$

The $p(x)$ is positive when the rubber is under compression. Using the same principle, we can determine the shear stress in the rubber layer;

$$\tau_r = \frac{-2G_r}{(\frac{1}{6}+\frac{G_r h}{G_a t})}(\frac{\Delta}{t})(\frac{x}{t})(\frac{z}{t}) \tag{26}$$

Thus,

$$F_2 = \int_{-a/2}^{a/2} p(x)dx = -\frac{a^3}{t^3}\frac{G_r}{(\frac{6G_r h}{G_a t}+1)}\Delta \tag{27}$$

The compressive force is given by the sum of Equations (24) and (27)

$$F = F_1 + F_2 = 4\frac{E_r}{3}\frac{a}{t}\Delta[1+(\frac{a}{2t})^2(\frac{6G_r h}{G_a t}+1)^{-1}] \tag{28}$$

Equation (28) can be written into the effective modulus (E_c) of the bonded rubber block:

$$\frac{E_c}{E_r} = \frac{4}{3}[1+(\frac{a}{2t})^2(\frac{6G_r h}{G_a t}+1)^{-1}] \tag{29}$$

3. Results and Discussion

3.1 Validation of the analytic solutions

The displacement functions as shown in Equations 14 to 16 were validated using finite element analysis. Figure 3 presents the vertical displacement through the thickness of the rubber layer at x=0. The displacement is normalized by Δ, shown in the y-axis, and the vertical position is normalized by t, shown in the x-axis. The maximum vertical displacement is at the upper plate (at z/t = 0.5), where the rubber block is compressed, and the displacement becomes zero at the lower plate, where the block is constrained. At the mid-plane (at z/t = 0), the vertical displacement is half the maximum of the vertical displacement. The plot in Figure 3 shows that there is good agreement between FEM and the analytic solutions for vertical displacement.

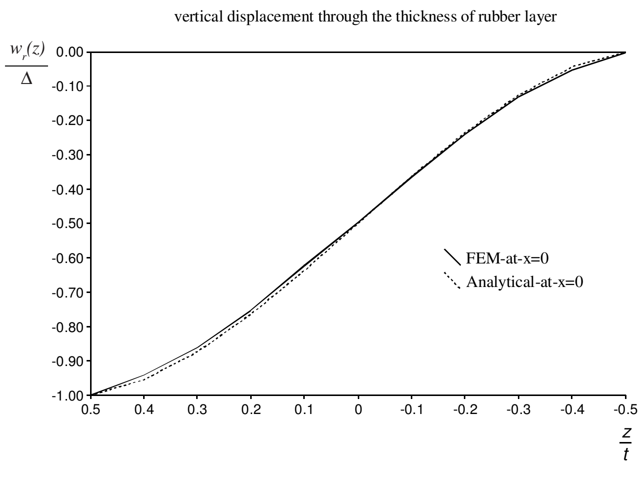

Figure 3. FEM and analytical validation of vertical displacement $w_r(z)/\Delta$ through the thickness of the rubber layer at $x = 0$

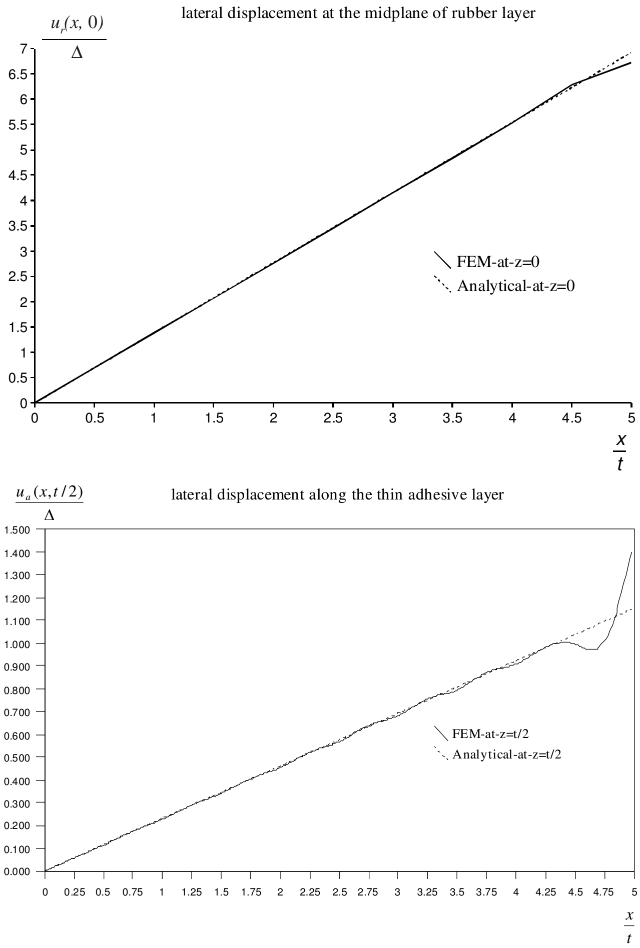

Figure 4. (a) FEM and analytical validation of the lateral displacement $u_r(x,0)/\Delta$ at the mid-plane of the rubber layer at z = 0 and (b) FEM and analytical validation of the lateral displacement $u_a(x,t/2)/\Delta$ along the thin adhesive layer at z = t/2

Figure 4 a) and b) present the lateral displacements in the rubber layer at the mid-plane and in the adhesive layer bonding between the upper plate and rubber, respectively. The lateral displacement is normalized by Δ shown in the y-axis, and the vertical position is normalized by t shown in the x-axis. The lateral displacement increases linearly from x/t = 0 to the unbonded surface of the rubber block (x/t = 5). The maximum lateral displacement occurs at the mid-plane of the rubber block.

The plots revealed that there is agreement for the lateral displacement between the FEM and analytic solutions at $0 \le x/t \le 4.5$. The discrepancies apparently occur near the free or unbonded surface ($4.5 \le x/t \le 5$), because this point is close to the free surfaces of the rubber and bonding area, which is the singularity point.

3.2 Effect of thin adhesive layer on the bonded rubber block

The plots of Equation (29) in Figure 5 show the effect of the adhesive on the effective modulus of the bonded

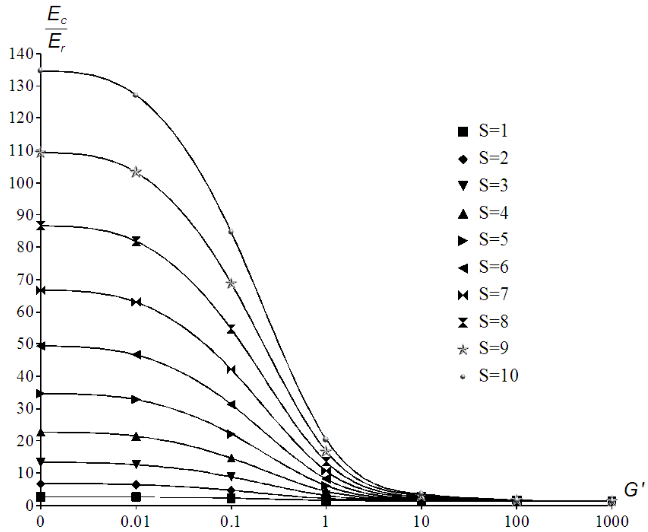

Figure 5. Effect of the adhesive layer ratio *(G′)* on the effective modulus (E_c/E_r) for different *S* (shape factor)

rubber block. S is the shape factor for the long strip, which is *a/2t* (bonded area/free area), and the shear stiffness factor is $G' = G_r h/G_a t$ (ratio of modulus and thickness of rubber and adhesive). In this study, $G_r h/G_a t$ is varied from 0, which is a perfect bonding condition, to 1000. The shape factor (S) is varied from 1 to 10.

4. Discussion and Conclusions

The shear modulus and thickness of the adhesive layer are taken into account to analyze the deformations of the bonded rubber block. The Gent's approach is used to determine the analytic displacement solutions. Equations (14) to (16) are the deformations of the rubber and adhesive, which were validated by the finite element analysis. There is an agreement for the displacement between the finite element analysis and analytic solutions. There were small discrepancies in the lateral deformations of the rubber and adhesive at the corner of the mating surfaces between the rubber and bonding area, due to singularity at the corners.

The analytic solutions indicate that the shear stiffness factor $G_r h/G_a t$, ratio of the modulus, and thickness of the rubber and adhesive on the same area affect the vertical and lateral deformations, pressure, and shear in the rubber under compressive load carrying. The stiffness factor allows for the bonding surface to resist lateral and vertical deformations and to alter pressure and shear stress in the rubber. In the case of a lubricated surface, it is assumed that there is no adhesive, $G_a=0$ or $G_r h/G_a t \to \infty$. When the rubber is compressed, the rubber along the interface area is not constrained and fully moves in the lateral direction. To maintain constant volume due to the incompressibility of the rubber, the pressure is then constant, and there is no shear stress in the rubber. In the case of perfect bonding, it is assumed that there is very stiff adhesive, $G_a \to \infty$ or $G_r h/G_a t=0$. When the rubber is compressed, the rubber near the interface area is fully constrained in the lateral direction, whereas the rubber in other areas is not, which then creates shear stress and a pressure gradient in the rubber.

The modulus of the bonded rubber block defined as effective modulus (E_c) is increased by the factor of $(1+(a/2t)^2(6G_r h/G_a t+1)^{-1})$, which is composed of the shape factor *(a/2t)* and the shear stiffness factor ($G_r h/G_a t$). When there is perfect bonding or $G_r h/G_a t=0$ at the interface, the effective modulus is highest, because the full constraint at the interface retards the rubber expansion. The effective modulus of the rubber gradually decreases and converts to a value of *4/3Er*, as the interface is more compliant in the lubricated condition or $G_r h/G_a t \to \infty$. The shape factor also increases the effective modulus. The effective modulus depends only on the shape factor or the adhesive layer when the shear stiffness of the joint is high or $G_r h/G_a t \geq 10$.

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