Songklanakarin J. Sci. Technol.
35 (5), 611-614, Sep. - Oct. 2013

SJST SONGKLANAKARIN JOURNAL OF SCIENCE AND TECHNOLOGY
http://www.sjst.psu.ac.th

Original Article

Critical temperature of magnetic superconductors by two-band Ginzburg-Landau approach

Arpapong Changjan[1]* and Pongkaew Udomsamuthirun[2,3]

[1] *Department of Science and Mathematics, Faculty of Science and Technology, Pathumwan Institute of Technology, Pathum Wan, Bangkok, 10330 Thailand.*

[2] *Prasarnmitr Physics Research Unit, Department of Physics, Faculty of Science, Srinakharinwirot University, Watthana, Bangkok, 10110 Thailand.*

[3] *Thailand Center of Excellence in Physics, Commission on Higher Education, Ratchathewi, Bangkok, 10400 Thailand.*

Received 22 October 2012; Accepted 7 June 2013

Abstract

In this research, we study the critical temperature of the two-band magnetic superconductor by Ginzburg-Landau (GL) approach in the absent of external magnetic field. There are four models of assumption used. We find the interband-to-band ratio on critical temperature formulas analytically. The critical temperature ratio decreased as the interband-to-band ratio increased.

Keywords: Ginzburg-Landau theory, two-band magnetic superconductors, critical temperature

1. Introduction

The Ginzburg-Landau (GL) theory is widely used to apply for the magnetic properties of superconductor (Abrikosov, 1957; Gorkov, 1958). Askerzade (2003) studied two-band GL theory and apply to determine the temperature dependence of lower, upper and thermodynamic critical field for non-magnetic superconductors. Askerzade (2005), Udomsamuthirun *et al*. (2006), Minxia and Zi Zhao (2007) studied the upper critical field of anisotropic two-band superconductors by GL theory. The properties of magnetic superconductors were considered by Hampshire (1998; 2001). He studied the GL theory of one-band magnetic superconductors, including the spatial variation and nonlinear magnetic response of in-field magnetic ions and found that ferromagnetism and antiferromagnetism can occur in the superconducting state. To determine the critical temperature $\left(T_c\right)$, Chen *et al.* (2011) derived the formula for the critical temperature of $NbSe_2$ by two-band GL theory and discussed how to properly choose the parameters in the GL theory.

Askerzade (2012) described the physical properties of superconductors in cuprate superconductors, borocarbides, magnesium diboride and oxypnictides. The description of crystal structure, electronic properties, anisotropy, multiband effects and related theoretical models for each group of superconductors was presented.

In the early 1960s, the first experimental observation on two-band superconductivity in metal (Dolgov, 2005) e.g., V, Nb, and Ta hads been proposed, and in 2001, the two-band was also found in MgB_2 .The thermodynamic properties of MgB_2 have been described using two-band Eliashberg theory.

Experimental investigations found FeAs compounds in which superconducting state has been discovered. The theory models of these compounds were discussed on this basis of experimental results (Izyumov and Kurmaev, 2010).

* Corresponding author.
Email address: otto_sinkronity@yahoo.com

The two-band non-magnetic superconductors can be found in borocarbides material. The two-band and multi-band magnetic superconductors are found in Fe-based superconductor (Kamihara *et al.*, 2008). The order parameters in some case of Fe-based superconductor can be reduced into two bands (Mazin *et al.*, 2008) that agreed with ARPES data (Ding *et al.*, 2008). Changjan and Udomsamuthirun (2011) proposed the upper critical field, the lower critical field, and the critical magnetic field ratio to explain the very high upper critical field of the Fe-based superconductor by GL theory.

The critical temperature of two-band magnetic superconductor depends on the temperature-dependent constant of GL equations and analytic formula of T_c is not found. The purpose of this paper is to study the critical temperature of the two-band magnetic superconductor by GL approach in case of absent magnetic field. The 1st GL equations of two-band magnetic superconductor proposed by Changjan and Udomsamuthirun (2011) are used to calculate the equation. Our formulas depend on the temperature dependent constant. There are four models of assumption for the temperature dependent constant i.e. the Chen *et al.* (2011) assumption, the Zhu *et al.* (2008) assumption, the Shanenko *et al.* (2011) assumption, and the Changjan and Udomsamuthirun (2011) assumption.

2. Model and Calculations

We use the 1st GL equations of two-band magnetic superconductor proposed by Changjan and Udomsamuthirun (2011) as

$$\frac{1}{2m_i}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_i+\alpha_i\psi_i+\beta_i\left(\psi_i^*\psi_i\right)\psi_i$$

$$+\varepsilon\psi_j-\varepsilon_i\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_j=0 \qquad (1)$$

Here $i, j=1,2$ and $i\neq j$. There are two order parameters, ψ_1 and ψ_2. The GL free energy for two-band superconductor can be written as (Askerzade, 2003; 2005; Udomsamuthirun *et al.*, 2006; Min-Zia and Zi-Zhao, 2007; Changjan and Udomsamuthirun, 2011),

$$F_{sc}\left[\psi_1,\psi_2\right]=\int d^3r f_{sc}=\int d^3r\left(F_1+F_2+F_{12}+\int H_s dB\right) \qquad (2)$$

with

$$F_{i(i=1,2)}=\frac{1}{2m_i}\left|\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)\psi_i\right|^2+\alpha_i\left|\psi_i\right|^2+\frac{1}{2}\beta_i\left|\psi_i\right|^4 \qquad (3)$$

$$F_{12}=\varepsilon\left(\psi_1^*\psi_2+c.c.\right)+\varepsilon_1\left\{\left(i\hbar\vec{\nabla}-2e\vec{A}\right)\psi_1^*\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)\psi_2+c.c.\right\} \qquad (4)$$

where F_i is the free energy of the separate bands. F_{12} is the interaction term between bands. m_i denotes the effective mass of carriers. ψ_i is the order parameter and $\left|\psi_i\right|^2$ is proportional to the density of carriers, the coefficient α_i depends on the temperature, while coefficient β_i is independent of temperature. The coefficient ε and ε_1 are the interband mixing of two order parameters and their gradients. The first term in Equation 3 accounts for the kinetic energy of the carriers and the lasted term in Equation 4 accounts for the energy stored in the local magnetic fields. $H_s=\frac{B}{\mu_0}-M_{ions}$, $M=M_{sc}+M_{ions}$ and $M_{ions}=\chi H_s$, where $\mu_0 M_{sc}$ is the field produced by the carriers and $\mu_0 M_{ions}$ is the field produced by the ions.

For small change in B-field, a series in B was introduced (Hampshire, 1998)

$$\gamma_0+\gamma_1 B+\gamma_2\frac{B^2}{2\mu_0}=\int\left(B-\mu_0 M_{ions}\right)\frac{dB}{\mu_0}$$

$$-\left(B-\mu_0 M\right)M_{sc}-\left(B-\mu_0 M\right)M_{ions} \qquad (5)$$

Here γ_0, γ_1 and γ_2 are coefficient parameters. We can get the by differentiating above equation twice and taking M_{sc} to be small that

$$\gamma_2=1-\mu_0\frac{dM_{ions}}{dB}-\mu_0\frac{d^2}{dB^2}\left[\left(B-\mu_0 M_{ions}\right)M_{ions}\right]$$

and $\gamma_2=1$ for non-magnetic superconductors. The coefficient parameter γ_1 depends on the gradient of field and $\gamma_1=0$ for the uniform applied field.

In the vicinity of T_c, the magnitude of the order parameters is small and we can neglect by the cubic terms in Equation 1. Consider the absence of fields and gradients (Chen *et al.*, 2011), we get $\alpha_1\alpha_2-\varepsilon^2=0$ at $T=T_c$. If interband interactions are neglected, we can find two independent single-band equations that corresponding to the critical temperature of separate bands. To investigate the critical temperature of two-band magnetic superconductors, the parameters α_1 and α_2 can be approximately in four cases.

Although the GL temperature-dependent parameter (α_1 and α_2) of two-band superconductor can be derived approximately from the microscopic model (Zhitomirsky and Dao, 2004) as a linear function (1-T/T_C), in experimental data of the upper critical field in $NbSe_2$ (Toyota *et al.*, 1976) this model cannot be fitted well in the low temperature region. Then, we assume a non-linear relation of GL temperature-dependent parameters in the four cases. The Zhu *et al.* (2008) assumption, Shanenko *et al.* (2011) assumption and the Changjan and Udomsamuthirun (2011) assumption in order to describe the behavior of the critical temperature.

Case 1: Chen *et al.* (2011) makes the assumption that $\alpha_i=\alpha_{i0}\left(1-\frac{T}{T_{ci}}\right)$ where α_{i0} is the proportionality constant. The critical temperature is given by $T_c=\frac{1}{2}\left[\left(T_{c1}+T_{c2}\right)+\sqrt{\left(T_{c1}-T_{c2}\right)^2+\frac{4\varepsilon^2 T_{c1}T_{c2}}{\alpha_{10}\alpha_{20}}}\right]$. In case $T_{c1}=T_{c2}=$

T_{c0}, we can reduce T_c equation in form $T_c = T_{c0}\left(1-\sqrt{Q}\right)$, where Q is interband-to-band ratio, $Q = \dfrac{\varepsilon^2}{\alpha_{10}\alpha_{20}}$.

Case 2: Zhu *et al.* (2008) makes the assumption that $\alpha_i = \alpha_{i0}\left(1-\left(\frac{T}{T_{ci}}\right)^2 \Big/ 1+\left(\frac{T}{T_{ci}}\right)^2\right)$. The critical temperature is given by $T_c = T_{c1}T_{c2}\sqrt{\dfrac{\left(\alpha_{10}\alpha_{20}-\varepsilon^2\right)}{\left(\alpha_{10}\alpha_{20}+\varepsilon^2\right)\left(T_{c1}^2+T_{c2}^2\right)}}$. In case $T_{c1} = T_{c2} = T_{c0}$, we can reduce T_c equation in form $T_c = T_{c0}\sqrt{\dfrac{1-\sqrt{Q}}{1+\sqrt{Q}}}$.

Case 3: Shanenko *et al.* (2011) makes the the assumption that $\alpha_i = \alpha_{i0}\left[\left(1-\frac{T}{T_{ci}}\right)+\frac{1}{2}\left(1-\frac{T}{T_{ci}}\right)^2\right]$. The critical temperature is given by $T_c = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$, where a = $3\alpha_{10}\alpha_{20}\left[\left(\frac{1}{T_{c1}}+\frac{1}{T_{c2}}\right)^2+\frac{10}{3T_{c1}T_{c2}}\right]$, $b = -12\alpha_{10}\alpha_{20}\left(\frac{1}{T_{c1}}+\frac{1}{T_{c2}}\right)$ and $c = 9\alpha_{10}\alpha_{20}-4\varepsilon^2$. In case $T_{c1} = T_{c2} = T_{c0}$, we can reduce T_c equation in form $T_c = T_{c0}\left(2-\sqrt{1+2\sqrt{Q}}\right)$.

Case 4: Changjan and Udomsamuthirun (2011) modified the assumption of Shanenko *et al.* (2011) by making the assumption that $\alpha_i = \alpha_{i0}\left[p\left(1-\frac{T}{T_{ci}}\right)+\frac{q}{2}\left(1-\frac{T}{T_{ci}}\right)^2\right]$, where p and q are arbitrary constants. This case is general case of Case 1 and Case 3. For $p = q = 1$ this assumption becomes the Shanenko *et al.* (2011) assumption and for $p = 1$ and $q = 0$ this assumption becomes the Chen *et al.* (2011) assumption. In this case, $p = 1$ and $q = -1$ were chosen that the approximate GL temperature-dependent parameter can be written in exponential form as α_i = $\alpha_{i0}\left[\left(1-\frac{T}{T_{ci}}\right)-\frac{1}{2}\left(1-\frac{T}{T_{ci}}\right)^2\right] \approx \alpha_{i0}\ln(2-\frac{T}{T_{ci}})$ for $\left(1-\frac{T}{T_{ci}}\right)<1$.

Then α_i can be approximated that $\alpha_i \approx \alpha_{i0}\ln\left(2-\frac{T}{T_{ci}}\right)$.

The critical temperature is then given by $T_c = \left(T_{c1}+T_{c2}\right)+\sqrt{\left(T_{c1}+T_{c2}\right)^2+T_{c1}T_{c2}\left(e^Q-4\right)}$. For $T_{c1} = T_{c2} = T_{c0}$, we can reduce T_c equation into the form of $T_c = T_{c0}\left(2-e^{\sqrt{Q}}\right)$.

3. Results

The effect of α_i and ε on the critical temperature ratio $\left(\frac{T_c}{T_{c0}}\right)$ is shown in Figure 1. It shows that the values of the critical temperature ratio decrease linearly with an increase in the constant Q in Case 1 and decrease non-linearly in other cases. Q is the ratio of interaction band parameter and separate band parameter. When Q goes to zero $T_c = T_{c0}$ in all cases. In Case 4, lets consider at zero critical temperature $\left(\frac{T_c}{T_{c0}}=0\right)$ and $\alpha_{10} = \alpha_{20} = \alpha$ we get $\frac{\varepsilon}{\alpha} \le \ln 2$. We find that the possibility ratio of interband mixing of two order parameters (ε) and temperature dependent constant (α) of two-band magnetic superconductor is less than or equal to 0.693. The value of critical temperature decreased as the constant Q increased and it is inversely proportional to the interband mixing of two order parameters. The experimenter can use these models for calculation the critical temperature of two-band magnetic superconductors. The assumption of Changjan and Udomsamuthirun (2011) can better fit the critical temperature than the other models because there are two constants that can be modified.

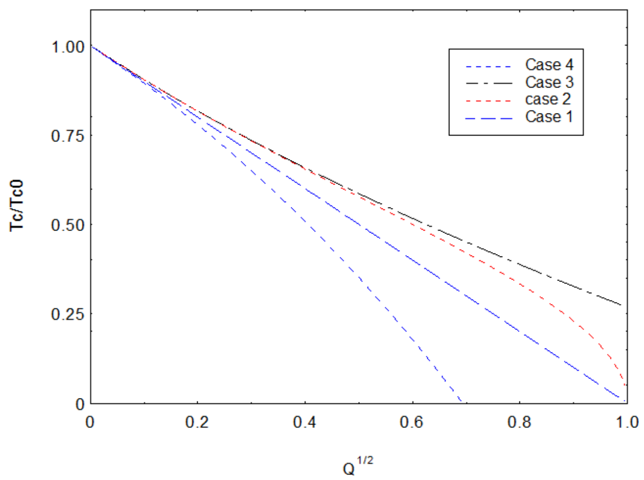

Figure 1. Critical temperature ratio versus α_i and ε, where $Q = \dfrac{\varepsilon^2}{\alpha_{10}\alpha_{20}}$

4. Discussions

In this paper, we calculate the critical temperature in two-band magnetic superconductors analytically. The critical temperature formulas are dependent on α. To investigate the critical temperature of two-band magnetic superconductors, the parameters α_1 and α_2 can be approximated in four cases: the Chen *et al*. (2011) assumption, the Zhu *et al*. (2008) assumption, the Shanenko *et al.* (2011) assumption, and the Changjan and Udomsamuthirun (2011) assumption. We found that the value of critical temperature ratio do decrease with an increase in the constant Q for all cases. The critical temperature is inversely proportional to the interband mixing of two order parameters. The assumption of Changjan and Udomsamuthirun (2011) can fit well the critical temperature than the other models because there are two constants that can be modified.

Acknowledgements

The authors would like to thank Prof. S. Yoksan for useful discussions. The financial support from the Pathumwan Institute of Technology, Srinakharinwirot University and Thailand Center of Excellence in Physics (ThEP), Commission Higher on Education are acknowledged.

References

Abrikosov, A.A. 1957. On the Magnetic Properties of Superconductors of the Second Group. Soviet Physics, Journal of Experimental and Theoretical Physics (JETP). 5, 1174-1182.

Askerzade, I.N. 2003. Ginzburg–Landau theory for two-band s-wave superconductors: application to non-magnetic borocarbides $LuNi_2B_2C$, YNi_2B_2C and magnesium-diboride MgB_2. Physica C. 397, 99-111.

Askerzade, I.N. 2005. Anisotropy of the upper critical field in MgB2: the two-band Ginzburg-Landau theory. Journal of Experimental and Theoretical Physics (JETP) Letter. 81, 717-720.

Askerzade, I.N. 2012. Unconventional superconductors: anisotropy and multiband effects, Springer, New York, U.S.A., pp.1-26.

Changjan, A. and Udomsamuthirun, P. 2011. The critical magnetic field of anisotropic two-band magnetic superconductors. Solid State Communications. 151, 988-992.

Changjan, A.and Udomsamuthirun, P. 2011. Critical magnetic field ratio of anisotropic magnetic superconductors. Physica C. 471, 23-25.

Chen, L., Zuo, J., Lu, Y. and Huang, H. 2011. Two-band calculations on the upper critical field of superconductor $NbSe_2$. Physica C. 471, 1591-1594.

Ding, H., Richard, P., Nakayama, K., Sugawara, K., Arakane, T., Sekiba, Y., Takayama, A., Souma, S., Sato, T., Takahashi, T., Wang, Z., Dai, X., Fang, Z., Chen, G.F., Luo, J.L. and Wang, N.L. 2008. Observation of Fermi-surface–dependent nodeless superconducting gaps in Ba0.6K0.4Fe2As2. Europhysics Letter. 83(4), 47001.

Dolgov, O.V., Kremer, R.K., Kortus, J., Golubov, A.A. and Shulga, S.V. 2005. Thermodynamics of two-band superconductors: The case of MgB_2. Physical Review B. 72, 024504.

Gorkov, L.P. 1958. On the energy spectrum of superconductors. Soviet Physics, Journal of Experimental and Theoretical Physics (JETP). 7, 505-508.

Hampshire, D.P. 1998. Ferromagnetic and antiferromagnetic superconductivity. Physica C. 304, 1-11.

Hampshire, D.P. 2001. The non-hexagonal flux-line lattice in superconductors. Journal of Physics: Condensed Matter. 13, 6095-6113.

Izyumov, Y. and Kurmaev, E. 2010. High-Tc Superconductors Based on FeAs, Springer, New York, U.S.A., pp. 1-3.

Kamihara, Y., Watanabe, T., Hirano, M. and Hosono, H. 2008. Iron-Based Layered Superconductor $La[O_{1-x}F_x]FeAs$ (x=0.05-0.12) with Tc = 26 K. Journal of the American Chemical Society. 130(11), 3296–3297.

Mazin, I.I., Singh, D.J., Johannes, M.D. and Du, M.H. 2008. Unconventional sign-reversing superconductivity in $LaFeAsO_{1-x}Fx$. Physics Review Letter. 101, 057003.

Min-Zia, L. and Zi-Zhao, G. 2007. The upper critical field in two-band layered superconductors. Chinese Physics. 16, 826-833.

Shanenko, A.A., Milosevic, M.V., Peeters, F.M. and Vagov, A.V. 2011. Extended Ginzburg-Landau Formalism for Two-Band Superconductors. Physical Review Letter. 106, 047005.

Toyota, N., Nakatsuji, H., Noto, K., Hoshi, A., Kobayashi, N., Muto, Y., and Onodera, Y. 1976. Temperature and Angular Dependences of Upper Critical Fields for the Layer Structure Superconductor $2H\text{-}NbSe_2$. Journal of Low Temperature Physics. 25, 485-499.

Udomsamuthirun, P., Changjan, A., Kumvongsa, C. and Yoksan, S. 2006. Hc2 of anisotropy two-band superconductors by Ginzburg–Landau approach. Physica C. 433, 62-66.

Zhitomirsky, M.E. and Dao, V.H. 2004. Ginzburg-Landau theory of vortices in a multigap superconductor. Physical Review B. 69, 054508.

Zhu, X., Yang, H., Fang, L., Mu, G. and Hai-Hu, W. 2008. Upper critical field, Hall effect and magnetoresistance in the iron-based layered superconductor $LaFeAsO_{0.9}F_{0.1-\delta}$ Superconductor Science and Technology. 21, 105001.