Songklanakarin J. Sci. Technol.
37 (4), 465-475, Jul. - Aug. 2015

http://www.sjst.psu.ac.th

Original Article

On $(\in,\in\vee q_k)$-intuitionistic (fuzzy ideals, fuzzy soft ideals) of subtraction algebras

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Received: 27 February 2015; Accepted: 29 April 2015

Abstract

The intent of this article is to study the concept of an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal and $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of subtraction algebras and to introduce some related properties.

2010 AMS Classification: 06F35, 03G25, 08A72.

Keywords: Subtraction algebras, $(\in,\in\vee q_k)$-intuitionistic fuzzy (soft) subalgebras, $(\in,\in\vee q_k)$-intuitionistic fuzzy (soft) ideals.

1. Introduction

Schein (1992) cogitated the system of the form $(X;\circ,\backslash)$, where X is the set of functions closed under the composition "$\circ$" of functions (and hence $(X,\circ)$ is a function semigroup) and the set theoretical subtraction "$\backslash$" (and hence $(X,\backslash)$ is a subtraction algebra in the sense of Abbot (1969). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. He suggested a problem concerning the structure of multiplication in a subtraction semigroup. It was explained by Zelinka (1995), and he had solved the problem for subtraction algebras of a special type known as the "atomic subtraction algebras". The notion of ideals in subtraction algebras was introduced by Jun *et al.* (2005). For detailed study of subtraction algebras see (Ceven and Ozturk, 2009 and Jun *et al.*, 2007).

The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh (1965). Since then the notion of fuzzy sets is actively applicable in different algebraic structures. The fuzzification of ideals in subtraction algebras were discussed in Lee and Park (2007). Atanassov (1986) introduced the idea of intuitionistic fuzzy set, which is more general one as compared to a fuzzy set.

Bhakat and Das (1996) introduced a new type of fuzzy subgroups, that is, the $(\in,\in\vee q)$-fuzzy subgroups. Jun *et al.* (2011) introduced the notion of $(\in,\in\vee q_k)$-fuzzy subgroup. In fact, the $(\in,\in\vee q_k)$-fuzzy subgroup is an important generalization of Rosenfeld's fuzzy subgroup. Shabir *et al.* (2010) characterized semigroups by $(\in,\in\vee q_k)$-fuzzy ideals, also see Shabir and Mahmood (2011, 2013).

Molodtsov (1999) introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that are free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, work on the soft set theory is progressing rapidly. Maji *et al.* (2002)

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described the application of soft set theory to a decision-making problem. Maji *et al.* (2003) also studied several operations on the theory of soft sets. Maji *et al.* (2001b, 2004) studied intuitionistic fuzzy soft sets.

The notion of fuzzy soft sets, as a generalization of the standard soft sets, was introduced by Maji (2001a), and an application of fuzzy soft sets in a decision-making problem was presented. Ahmad *et al.* (2009) have introduced arbitrary fuzzy soft union and fuzzy soft intersection. Aygunoglu *et al.* (2009) introduced the notion of fuzzy soft group and studied its properties. Yaqoob *et al.* (2013) studied the properties of intuitionistic fuzzy soft groups in terms of intuitionistic double t-norm. Jun *et al.* (2010) introduced the notion of fuzzy soft BCK/BCI-algebras and (closed) fuzzy soft ideals, and then derived their basic properties. Williams and Saeid (2012) studied fuzzy soft ideals in subtraction algebras. Recently, Yang (2011) have studied fuzzy soft semigroups and fuzzy soft (left, right) ideals, and have discussed fuzzy soft image and fuzzy soft inverse image of fuzzy soft semigroups (ideals) in detail. Liu and Xin (2013) studied the idea of generalized fuzzy soft groups and fuzzy normal soft groups.

In this article, we study the concept of $(\in, \in \vee q_k)$-intuitionistic fuzzy (soft) ideals of subtraction algebras. Here we consider some basic properties of $(\in, \in \vee q_k)$-intuitionistic fuzzy (soft) ideals of subtraction algebras.

2. Preliminaries

In this section we recall some of the basic concepts of subtraction algebra which will be very helpful in further study of the paper. Throughout the paper X denotes the subtraction algebra unless otherwise specified.

Definition 2.1 (Jun *et al.*, 2005) A non-empty set X together with a binary operation "-" is said to be a subtraction algebra if it satisfies the following:

(S_1) $x-(y-x)=x$,

(S_2) $x-(x-y)=y-(y-x)$,

(S_3) $(x-y)-z=(x-z)-y$, for all $x,y,z \in X$.

The last identity permits us to omit parentheses in expression of the form $(x-y)-z$. The subtraction determines an order relation on $X: a \le b \Leftrightarrow a-b=0$, where $0=a-a$ is an element that does not depends upon the choice of $a \in X$. The ordered set $(X;\le)$ is a semi-Boolean algebra in the sense of Abbot (1969), that is, it is a meet semi lattice with zero, in which every interval $[0,a]$ is a Boolean algebra with respect to the induced order. Here $a \wedge b = a-(a-b)$; the complement of an element $b \in [0,a]$ is $a-b$ and is denoted by b'; and if $b,c \in [0,a]$; then

$b \vee c = (b' \wedge c')' = ((a-b)\wedge(a-c)) = a-(a-b)-((a-b)-(a-c))$.

In a subtraction algebra, the following are true see (Jun *et al.*, 2005) :

$(a1)$ $(x-y)-y=x-y$,

$(a2)$ $x-0=x$ and $0-x=0$,

$(a3)$ $(x-y)-x=0$,

$(a4)$ $x-(x-y)\le y$,

$(a5)$ $(x-y)-(y-x)=x-y$,

$(a6)$ $x-(x-(x-y))=x-y$,

$(a7)$ $(x-y)-(z-y)\le x-z$,

$(a8)$ $x \le y$ if and only if $x=y-w$ for some $w \in X$,

$(a9)$ $x \le y$ implies $x-z \le y-z$ and $z-y \le z-x$, for all $z \in X$,

$(a10)$ $x,y \le z$ implies $x-y = x\wedge(z-y)$,

$(a11)$ $(x\wedge y)-(x\wedge z)\le x\wedge(y-z)$,

$(a12)$ $(x-y)-z=(x-z)-(y-z)$.

Definition 2.2 (Jun *et al.*, 2005) A non-empty subset A of a subtraction algebra X is called an ideal of X, denoted by $A \lhd X$: if it satisfies:

$(b1)$ $a-x \in A$ for all $a \in A$ and $x \in X$,

$(b2)$ for all $a,b \in A$, whenever $a \vee b$ exists in X then $a \vee b \in A$.

Proposition 2.3 (Jun *et al.*, 2005) A non-empty subset A of a subtraction algebra X is called an ideal of X, if and only if it satisfies:

$(b3)$ $0 \in A$,

$(b4)$ for all $x \in X$ and for all $y \in A$, $x-y \in A \Rightarrow x \in A$

Proposition 2.4 (Jun *et al.*, 2005) Let X be a subtraction algebra and $x,y \in X$. If $w \in X$ is an upper bound for x and y, then the element $x \vee y = w-((w-y)-x)$ is the least upper bound for x and y.

Definition 2.5 (Jun *et al.*, 2005) Let Y be a non-empty subset of X then Y is called a subalgebra of X if $x-y \in Y$, whenever $x,y \in Y$.

Definition 2.6 (Lee and Park, 2007) Let f be a fuzzy subset of X. Then f is called a fuzzy subalgebra of X if it satisfies:

(FS) $f(x-y)\ge \min\{f(x), f(y)\}$, whenever $x,y \in X$.

Definition 2.7 (Lee and Park, 2007) A fuzzy subset f is said to be a fuzzy ideal of X if satisfies:

(FI1) $f(x-y)\ge f(x)$,

(FI2) If there exists $x \vee y$, then $f(x \vee y)\ge \min\{f(x), f(y)\}$, for all $x,y \in X$.

3. $(\in,\in\vee q_k)$-intuitionistic fuzzy ideals

In this section we will discuss some properties related to $(\in,\in\vee q_k)$-intuitionistic fuzzy ideals of subtraction algebras.

Definition 3.1 An intuitionistic fuzzy set A in X is an object of the form $A=\{(x,\mu_A(x),\gamma_A(x)):\ x\in X\}$, where the function $\mu_A\ :\ X\to[0,1]$ and $\gamma_A\ :\ X\to[0,1]$ denote the degree of membership and degree of non-membership of each element $x\in X$, and $0\le\mu_A(x)+\gamma_A(x)\le1$ for all $x\in X$. For simplicity, we will use the symbol $A=(\mu_A,\gamma_A)$ for the intuitionistic fuzzy set $A=\{(x,\mu_A(x),\gamma_A(x)):\ x\in X\}$. We define $0(x)=0$ and $1(x)=1$ for all $x\in X$.

Definition 3.2 Let X be a subtraction algebra. An intuitionistic fuzzy set $A=\{(x,\mu_A(x),\gamma_A(x)):\ x\in X\}$, of the form

$$x_{(\alpha,\beta)}y=\begin{cases}(\alpha,\beta) & if\ y=x\\ (0,1) & if\ y\ne x,\end{cases}$$

is said to be an intuitionistic fuzzy point with support x and value (α,β) and is denoted by $x_{(\alpha,\beta)}$. A fuzzy point $x_{(\alpha,\beta)}$ is said to intuitionistic belongs to (resp., intuitionistic quasi-coincident) with intuitionistic fuzzy set $A=\{(x,\mu_A(x),\gamma_A(x)):\ x\in X\}$ written $x_{(\alpha,\beta)}\in A$ $\left(\text{resp., } x_{(\alpha,\beta)}qA\right)$ if $\mu_A(x)\ge\alpha$ and $\gamma_A(x)\le\beta$ (resp., $\mu_A(x)+\alpha>1$ and $\gamma_A(x)+\beta<1$) By the symbol $x_{(\alpha,\beta)}q_kA$ we mean $\mu_A(x)+\alpha+k>1$ and $\gamma_A(x)+\beta+k<1$, where $k\in(0,1)$.

We use the symbol $x_t\in\mu_A$ implies $\mu_A(x)\ge t$ and $\frac{t}{x}[\in]\gamma_A$ implies $\gamma_A(x)\le t$, in the whole paper.

Definition 3.3 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of X if $x_{(t_1,t_3)}\in A$,

$$y_{(t_2,t_4)}\in A\Rightarrow(x-y)_{(t_1\wedge t_2,t_3\vee t_4)}\in\vee q_kA,$$

for all $x,y\in X$, $t_1,t_2,t_3,t_4,k\in(0,1)$.

OR

An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of X if it satisfy the following conditions,

(i) $x_{t_1}\in\mu_A,\ y_{t_2}\in\mu_A\Rightarrow(x-y)_{t_1\wedge t_2}\in\vee q_k\mu_A$, for all $x,y\in X$, $t_1,t_2,k\in(0,1]$,

(ii) $\frac{t_3}{x}[\in]\gamma_A$, $\frac{t_4}{y}[\in]\gamma_A\Rightarrow\frac{t_3\vee t_4}{x-y}[\in]\wedge[q_k]\gamma_A$, for all $x,y\in X$, $t_3,t_4,k\in[0,1)$.

Example 3.4 Let $X=\{0,a,b\}$ be a subtraction algebra with the following Cayley table

$-$	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

Let us define the intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X as

X	0	a	b
$\mu_A(x)$	0.5	0.6	0.7
$\gamma_A(x)$	0.1	0.2	0.21

,

and

t_1	0.4
t_2	0.5
t_3	0.22
t_4	0.23
k	0.5

then $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_{0.5})$-intuitionistic fuzzy subalgebra of X.

Definition 3.5 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of X if it satisfy the following conditions,

(i) $x_{(t,t)}\in A,\ y\in X\Rightarrow(x-y)_t\in\vee q_kA,$

(ii) If there exist $x\vee y$, then $x_{(t_1,t_3)}\in A,\ y_{(t_2,t_4)}\in A\Rightarrow(x\vee y)_{(t_1\wedge t_2,t_3\vee t_4)}\in\vee q_kA$, for all $x,y\in X$, $t,t_1,t_2,t_3,t_4,k\in(0,1)$.

OR

An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of X if it satisfy the following conditions,

(i) $x_t\in\mu_A,\ y\in X\Rightarrow(x-y)_t\in\vee q_k\mu_A$,

(ii) If there exist $x\vee y$ in X then $x_{t_1}\in\mu_A,\ y_{t_2}\in\mu_A\Rightarrow(x\vee y)_{t_1\wedge t_2}\in\vee q_k\mu_A$, for all $x,y\in X$, $t,t_1,t_2,k\in(0,1]$,

(iii) $\frac{t}{x}[\in]\gamma_A,\ y\in X\Rightarrow\frac{t}{x-y}[\in]\wedge[q_k]\gamma_A$,

(iv) If there exist $x\vee y$ in X then $\frac{t_3}{x}[\in]\gamma_A$, $\frac{t_4}{y}[\in]\gamma_A\Rightarrow\frac{t_3\vee t_4}{x\vee y}[\in]\wedge[q_k]\gamma_A$, for all $x,y\in X$, $t,t_3,t_4,k\in[0,1)$.

Example 3.6 Let $X=\{0,a,b\}$ be a subtraction algebra with the Cayley table define in Example 3.4, and let us define an intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of X as

X	0	a	b
$\mu_A(x)$	0.5	0.6	0.7
$\gamma_A(x)$	0.1	0.2	0.3

,

and

t	0.4
t_1	0.4
t_2	0.45
t_3	0.42
t_4	0.47
k	0.2

then $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_{0.2})$-intuitionistic fuzzy ideal of *X*.

Theorem 3.7 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of *X* is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X* if and only if $\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$ and $\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$.

Proof. Let $A=(\mu_A,\gamma_A)$ be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X,* and assume on contrary that there exist some $t\in(0,1]$ and $r\in[0,1)$ such that

$$\mu_A(x-y)<t<\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$$

and

$$\gamma_A(x-y)>r>\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}.$$

This implies that

$$\mu_A(x)\geq t,\ \mu_A(y)\geq t,\ \mu_A(x-y)<t$$
$$\Rightarrow\mu_A(x-y)+t+k<\frac{1-k}{2}+\frac{1-k}{2}+k=1$$
$$\Rightarrow(x-y)_t\overline{\in\vee q_k}\mu_A,$$

which is a contradiction. Hence

$$\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}.$$

Also from

$$\gamma_A(x-y)>r>\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$$

we get

$$\gamma_A(x)\leq r,\ \gamma_A(y)\leq r,\ \gamma_A(x-y)>r$$
$$\Rightarrow\gamma_A(x-y)+r+k>\frac{1-k}{2}+\frac{1-k}{2}+k=1$$
$$\Rightarrow\frac{r}{x-y}\overline{[\in]\wedge[q_k]}\gamma_A(x),$$

which is a contradiction. Hence

$$\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}.$$

Conversely, let $\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$ and $\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$. Let $x_{t_1}\in\mu_A$, $y_{t_2}\in\mu_A$ for all $x,y\in X$, $t_1,t_2,k\in(0,1]$ and $\frac{t_3}{x}[\in]\gamma_A$, $\frac{t_4}{y}[\in]\gamma_A$ for all $x,y\in X$, $t_3,t_4,k\in[0,1)$. This implies that $\mu_A(x)\geq t_1$, $\mu_A(y)\geq t_2$ and $\gamma_A(x)\leq t_3$, $\gamma_A(y)\leq t_4$. Consider

$$\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\tfrac{1-k}{2}\}\geq\min\{t_1,t_2,\tfrac{1-k}{2}\}.$$

If $t_1\wedge t_2>\frac{1-k}{2}$, then $\mu_A(x-y)\geq\frac{1-k}{2}$. So $\mu_A(x-y)+(t_1\wedge t_2)+k>\frac{1-k}{2}+\frac{1-k}{2}+k=1$, which implies that $(x-y)_{t_1\wedge t_2}q_k\mu_A$. If $t_1\wedge t_2\leq\frac{1-k}{2}$, then $\mu_A(x-y)\geq t_1\wedge t_2$. So $(x-y)_{t_1\wedge t_2}\in\mu_A$. Thus $(x-y)_{t_1\wedge t_2}\in\vee q_k\mu_A$. Also consider

$$\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\tfrac{1-k}{2}\}\leq\max\{t_3,t_4,\tfrac{1-k}{2}\}.$$

If $t_3\vee t_4<\frac{1-k}{2}$, then $\gamma_A(x-y)\leq\frac{1-k}{2}$. So $\gamma_A(x-y)+(t_3\vee t_4)+k<\frac{1-k}{2}+\frac{1-k}{2}+k=1$, which implies that $\frac{t_3\vee t_4}{x-y}[q_k]\gamma_A$. If $t_3\vee t_4>\frac{1-k}{2}$, then $\gamma_A(x-y)\leq t_3\vee t_4$. So $\frac{t_3\vee t_4}{x-y}[\in]\gamma_A$. Thus $\frac{t_3\vee t_4}{x-y}[\in]\vee[q_k]\gamma_A$. Hence $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X.*

Theorem 3.8 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of *X* is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of *X* if and only if

(*i*) $\mu_A(x-y)\geq\min\{\mu_A(x),\frac{1-k}{2}\}$,

(*ii*) $\gamma_A(x-y)\leq\max\{\gamma_A(x),\frac{1-k}{2}\}$,

(*iii*) $\mu_A(x\vee y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$,

(*iv*) $\gamma_A(x\vee y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$.

Proof. The proof is similar to the proof of the Theorem 3.7.

Proposition Every $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of *X* is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X,* but converse is not true.

Example 3.9 Let $X=\{0,a,b\}$ be a subtraction algebra with the Cayley table define in Example 3.4, and let

X	0	a	b
$\mu_A(x)$	0.9	0.3	0.6
$\gamma_A(x)$	0.2	0.6	0.4

then by Theorem 3.7, $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X.* But by Theorem 3.8, we observe that $A=(\mu_A,\gamma_A)$ is not a $(\in,\in\vee q_{0.4})$-intuitionistic fuzzy ideal of *X.* As

$$\mu_A(a\vee b)=\mu_A(0-((0-b)-a)))=\mu_A(0)=0.9$$
$$\ngeq\min\{\mu_A(a),\mu_A(b),\tfrac{1-k}{2}\}=\min\{0.3,0.6,0.3\}=0.3.$$

Definition 3.10 Let $A=(\mu_A,\gamma_A)$ is an intuitionistic fuzzy set of *X*. Define the intuitionistic level set as $A_{(\alpha,\beta)}=\{x\in X \mid \mu_A(x)\geq\alpha,\ \gamma_A(x)\leq\beta,\text{ where } \alpha\in(0,\frac{1-k}{2}],\ \beta\in[\frac{1-k}{2},1)\}$.

Theorem 3.11 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of *X* is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X* if and only if $A_{(\alpha,\beta)}\neq\varphi$ is a subalgebra of *X*.

Proof. Let $A=(\mu_A,\gamma_A)$ be an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X*. Suppose that $x,y\in A_{(\alpha,\beta)}$ then $\mu_A(x)\geq\alpha$, $\mu_A(y)\geq\alpha$ and $\gamma_A(x)\leq\beta$, $\gamma_A(y)\leq\beta$. Since $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X*, so

$$\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$$
$$\geq\min\{\alpha,\alpha,\frac{1-k}{2}\}=\min\{\alpha,\frac{1-k}{2}\}=\alpha$$

and

$$\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$$
$$\leq\max\{\beta,\beta,\frac{1-k}{2}\}=\max\{\beta,\frac{1-k}{2}\}=\beta.$$

Thus $(x-y)\in A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}\neq\varphi$ is a subalgebra of *X*. Conversely, assume that $A_{(\alpha,\beta)}\neq\varphi$ is a subalgebra of *X*. Assume on contrary that there exist some $x,y\in X$ such that $\mu_A(x-y)<\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$ and $\gamma_A(x-y)>\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$. Choose $\alpha\in(0,\frac{1-k}{2}]$, $\beta\in[\frac{1-k}{2},1)$ such that $\mu_A(x-y)<\alpha<\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$ and $\gamma_A(x-y)>\beta>\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$. This implies that $(x-y)\notin A_{(\alpha,\beta)}$, which is a contradiction to the hypothesis. Hence $\mu_A(x-y)\geq\min\{\mu_A(x),\mu_A(y),\frac{1-k}{2}\}$ and $\gamma_A(x-y)\leq\max\{\gamma_A(x),\gamma_A(y),\frac{1-k}{2}\}$. Thus $A=(\mu_A,\gamma_A)$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X*.

Theorem 3.12 An intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ of *X* is said to be an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of *X* if and only if $A_{(\alpha,\beta)}\neq\varphi$ is an ideal of *X*.

Proof. The proof is similar to the proof of the Theorem 3.12.

Definition 3.13 Let *X* be a subtraction algebra and $A\subseteq X$, an $(\in,\in\vee q_k)$-intuitionistic characteristic function

$$\chi_A=\{\langle x,\mu_{\chi_A}(x),\gamma_{\chi_A}(x)\rangle \mid x\in S\},$$

where μ_{χ_A} and γ_{χ_A} are fuzzy sets respectively, defined as follows:

$$\mu_{\chi_A}\ :\ X\to[0,1]\mid x\to\mu_{\chi_A}(x):=\begin{cases}\frac{1-k}{2} & \text{if } x\in A\\ 0 & \text{if } x\notin A\end{cases}$$

and

$$\gamma_{\chi_A}\ :\ X\to[0,1]\mid x\to\gamma_{\chi_A}(x):=\begin{cases}1 & \text{if } x\in A\\ \frac{1-k}{2} & \text{if } x\notin A\end{cases}.$$

Lemma 3.14 For a non-empty subset *A* of a subtraction algebra *X*, we have

(*i*) *A* is a subalgebra of *X* if and only if the characteristic intuitionistic set $\chi_A=\langle\mu_{\chi_A},\gamma_{\chi_A}\rangle$ of *A* in *X* is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of *X*.

(*ii*) *A* is an ideal of *X* if and only if the characteristic intuitionistic set $\chi_A=\langle\mu_{\chi_A},\gamma_{\chi_A}\rangle$ of *A* in *X* is an $(\in,\in\vee q_k)$-intuitionistic fuzzy ideal of *X*.

Proof. The proof is straightforward.

4. $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideals

Molodtsov defined the notion of a soft set as follows.

Definition 4.1 (Molodtsov, 1999) A pair (F,A) is called a soft set over *U*, where *F* is a mapping given by $F\ :\ A\to P(U)$. In other words a soft set over *U* is a parametrized family of subsets of *U*.

The class of all intuitionistic fuzzy sets on *X* will be denoted by $IF(X)$.

Definition 4.2 (Maji *et al.*, 2001b) Let *U* be an initial universe and *E* be the set of parameters. Let $A\subseteq E$. A pair $(\tilde{F},A)$ is called an intuitionistic fuzzy soft set over *U*, where $\tilde{F}$ is a mapping given by $\tilde{F}\ :\ A\to IF(U)$.

In general, for every $\varepsilon\in A$, $\tilde{F}[\varepsilon]=\langle\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]}\rangle$ is an intuitionistic fuzzy set in *U* and it is called intuitionistic fuzzy value set of parameter *ε*.

Definition 4.3 Let *U* be an initial universe and *E* be a set of parameters. Suppose that $A,B\subseteq E$, $(\tilde{F},A)$ and $(\tilde{G},B)$ are two intuitionistic fuzzy soft sets, we say that $(\tilde{F},A)$ is an intuitionistic fuzzy soft subset of $(\tilde{G},B)$ if and only if

(1) $A\subseteq B$,

(2) for all $\varepsilon\in A$, $\tilde{F}[\varepsilon]$ is an intuitionistic fuzzy subset of $\tilde{G}[\varepsilon]$, that is, for all $x\in U$ and $\varepsilon\in A$, $\mu_{\tilde{F}[\varepsilon]}(x)\leq\mu_{\tilde{G}[\varepsilon]}(x)$, and $\gamma_{\tilde{F}[\varepsilon]}(x)\geq\gamma_{\tilde{G}[\varepsilon]}(x)$. This relationship is denoted by $(\tilde{F},A)\prec(\tilde{G},B)$.

Definition 4.4 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe *U*. Then

$(\tilde{F},A)\tilde{\wedge}(\tilde{G},B)$ is defined by $(\tilde{F},A)\tilde{\wedge}(\tilde{G},B)=(\tilde{\Theta},A\times B)$, where $\tilde{\Theta}[\varepsilon,\varepsilon]=\tilde{F}[\varepsilon]\cap\tilde{G}[\varepsilon]$ for all $(\varepsilon,\varepsilon)\in A\times B$, that is,

$$\tilde{\Theta}[\varepsilon,\varepsilon]=\left\langle \mu_{\tilde{F}[\varepsilon]}(x)\wedge\mu_{\tilde{G}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(x)\vee\gamma_{\tilde{G}[\varepsilon]}(x)\right\rangle,$$

for all $(\varepsilon,\varepsilon)\in A\times B,\ x\in U.$

Definition 4.5 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U. Then $(\tilde{F},A)\tilde{\vee}(\tilde{G},B)$ is defined by $(\tilde{F},A)\tilde{\vee}(\tilde{G},B)=(\tilde{\Omega},A\times B)$, where $\tilde{\Omega}[\varepsilon,\varepsilon]=\tilde{F}[\varepsilon]\cup\tilde{G}[\varepsilon]$ for all $(\varepsilon,\varepsilon)\in A\times B$, that is,

$$\tilde{\Omega}[\varepsilon,\varepsilon]=\left\langle \mu_{\tilde{F}[\varepsilon]}(x)\vee\mu_{\tilde{G}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(x)\wedge\gamma_{\tilde{G}[\varepsilon]}(x)\right\rangle,$$

for all $(\varepsilon,\varepsilon)\in A\times B,\ x\in U.$

Definition 4.6 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U. Then the intersection $(\tilde{\Theta},C)$, where $C=A\cup B$ and for all $\varepsilon\in C$ and $x\in U$,

$$\mu_{\tilde{\Theta}[\varepsilon]}(x)=\begin{cases}\mu_{\tilde{F}[\varepsilon]}(x) & \text{if } \varepsilon\in A-B\\ \mu_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in B-A\\ \mu_{\tilde{F}[\varepsilon]}(x)\wedge\mu_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in A\cap B,\end{cases}$$

$$\gamma_{\tilde{\Theta}[\varepsilon]}(x)=\begin{cases}\gamma_{\tilde{F}[\varepsilon]}(x) & \text{if } \varepsilon\in A-B\\ \gamma_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in B-A\\ \gamma_{\tilde{F}[\varepsilon]}(x)\vee\gamma_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in A\cap B.\end{cases}$$

We denote it by $(\tilde{F},A)\Cap(\tilde{G},B)=(\tilde{\Theta},C)$.

Definition 4.7 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U. Then the union $(\tilde{\Theta},C)$, where $C=A\cup B$ and for all $\varepsilon\in C$ and $x\in U$,

$$\mu_{\tilde{\Theta}[\varepsilon]}(x)=\begin{cases}\mu_{\tilde{F}[\varepsilon]}(x) & \text{if } \varepsilon\in A-B\\ \mu_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in B-A\\ \mu_{\tilde{F}[\varepsilon]}(x)\vee\mu_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in A\cap B,\end{cases}$$

$$\gamma_{\tilde{\Theta}[\varepsilon]}(x)=\begin{cases}\gamma_{\tilde{F}[\varepsilon]}(x) & \text{if } \varepsilon\in A-B\\ \gamma_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in B-A\\ \gamma_{\tilde{F}[\varepsilon]}(x)\wedge\gamma_{\tilde{G}[\varepsilon]}(x) & \text{if } \varepsilon\in A\cap B.\end{cases}$$

We denote it by $(\tilde{F},A)\Cup(\tilde{G},B)=(\tilde{\Theta},C)$.

In contrast with the above definitions of IF-soft set union and intersection, we may sometimes adopt different definitions of union and intersection given as follows.

Definition 4.8 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two IF-soft sets over a common universe U and $A\cap B\neq\varphi$. Then the bi-intersection of $(\tilde{F},A)$ and $(\tilde{G},B)$ is defined to be the fuzzy soft set $(\tilde{\Theta},C)$, where $C=A\cap B$ and $\tilde{\Theta}[\varepsilon]=\tilde{F}[\varepsilon]\cap\tilde{G}[\varepsilon]$ for all $\varepsilon\in C$. This is denoted by $(\tilde{\Theta},C)=(\tilde{F},A)\tilde{\Cap}(\tilde{G},B)$.

Definition 4.9 Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two IF-soft sets over a common universe U and $A\cap B\neq\varphi$. Then the bi-union of $(\tilde{F},A)$ and $(\tilde{G},B)$ is defined to be the fuzzy soft set $(\tilde{\Theta},C)$, where $C=A\cap B$ and $\tilde{\Theta}[\varepsilon]=\tilde{F}[\varepsilon]\cup\tilde{G}[\varepsilon]$ for all $\varepsilon\in C$. This is denoted by $(\tilde{\Theta},C)=(\tilde{F},A)\tilde{\Cup}(\tilde{G},B)$.

Definition 4.10 Let $(\tilde{F},A)$ be an IF-soft set over X. Then $(\tilde{F},A)$ is called an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft subtraction algebra if $\tilde{F}[\varepsilon]=\left\langle\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]}\right\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of X, for all $\varepsilon\in A$.

Example 4.11 Let $X=\{0,a,b\}$ be a subtraction algebra with the following Cayley table

$-$	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

and let U is the set of cellular brands of mobile companies in the market. Let Θ = {attractive, expensive, cheap} is a parameter space and A = {attractive, cheap}. Define

$$\tilde{F}([\text{attractive}])=\{\langle 0,0.9,0.1\rangle,\langle a,0.8,0.2\rangle,\langle b,0.7,0.3\rangle\},$$
$$\tilde{F}([\text{cheap}])=\{\langle 0,0.5,0.12\rangle,\langle a,0.6,0.15\rangle,\langle b,0.7,0.2\rangle\}.$$

Let $k=0.4$ then $\langle\tilde{F},A\rangle$ is an $(\in,\in\vee q_{0.4})$-intuitionistic fuzzy soft subalgebra of X.

Definition 4.12 An intuitionistic fuzzy soft set $\langle\tilde{F},A\rangle$ of X is called an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft subalgebra of X, if for all $\varepsilon\in A$, $\tilde{F}[\varepsilon]=\left\langle\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]}\right\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy subalgebra of X, if

$$(i)\ \mu_{\tilde{F}[\varepsilon]}(x-y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(y),\tfrac{1-k}{2}\},$$

$$(ii)\ \gamma_{\tilde{F}[\varepsilon]}(x-y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(y),\tfrac{1-k}{2}\},$$

for all $x,y\in X$.

Definition 4.13 An intuitionistic fuzzy soft set $\langle\tilde{F},A\rangle$ of X is called an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X, if for all $\varepsilon\in A$, $\tilde{F}[\varepsilon]=\left\langle\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]}\right\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X, if

$$(i)\ \mu_{\tilde{F}[\varepsilon]}(x-y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\tfrac{1-k}{2}\},$$

$$(ii)\ \gamma_{\tilde{F}[\varepsilon]}(x-y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\tfrac{1-k}{2}\},$$

(*iii*) $\mu_{\tilde{F}[\varepsilon]}(x \vee y) \geq \min\{\mu_{\tilde{F}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$,

(*iv*) $\gamma_{\tilde{F}[\varepsilon]}(x \vee y) \leq \max\{\gamma_{\tilde{F}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$,

for all $x, y \in X$.

Proposition 4.14 An $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideal of *X* is an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra of *X*, but converse is not true.

Example 4.15 From Example 4.11, $\langle \tilde{F}, A\rangle$ is not an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideal of *X*. As

$$\mu_{\tilde{F}([\text{attractive}])}(a \vee b) = \mu_{\tilde{F}([\text{attractive}])}(0-((0-b)-a))) = \mu_{\tilde{F}([\text{attractive}])}(0) = 0.9$$
$$\ngeq \min\{\mu_{\tilde{F}([\text{attractive}])}(a), \mu_{\tilde{F}([\text{attractive}])}(b), \frac{1-k}{2}\}$$
$$= 0.3 = \min\{0.8, 0.7, 0.3\}, \text{ for } k = 0.4.$$

Theorem 4.16 Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebras (resp., ideals) of *X*. Then $\langle \tilde{F}, A\rangle \tilde{\wedge} \langle \tilde{G}, B\rangle$ is also an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra (resp., ideal) of *X*.

Proof. Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideals of *X*. We know that $\langle \tilde{F}, A\rangle \tilde{\wedge} \langle \tilde{G}, B\rangle = \langle \tilde{\Theta}, A \times B\rangle$, where $\tilde{\Theta}[\varepsilon, \varepsilon] = \tilde{F}[\varepsilon] \cap \tilde{G}[\varepsilon]$ for all $(\varepsilon, \varepsilon) \in A \times B$, that is

$$\tilde{\Theta}[\varepsilon, \varepsilon](x) = \left\langle \mu_{\tilde{F}[\varepsilon]}(x) \wedge \mu_{\tilde{G}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(x) \vee \gamma_{\tilde{G}[\varepsilon]}(x) \right\rangle$$

for all $x \in X$. Let $x, y \in X$, we have

$$\left(\mu_{\tilde{F}[\varepsilon]} \wedge \mu_{\tilde{G}[\varepsilon]}\right)(x-y) = \mu_{\tilde{F}[\varepsilon]}(x-y) \wedge \mu_{\tilde{G}[\varepsilon]}(x-y)$$
$$\geq \left(\min\{\mu_{\tilde{F}[\varepsilon]}(x), \frac{1-k}{2}\}\right) \wedge \left(\min\{\mu_{\tilde{G}[\varepsilon]}(x), \frac{1-k}{2}\}\right)$$
$$= \left(\min\{\mu_{\tilde{F}[\varepsilon]}(x) \wedge \mu_{\tilde{G}[\varepsilon]}(x), \frac{1-k}{2}\}\right)$$
$$= \min\{\left(\mu_{\tilde{F}[\varepsilon]} \wedge \mu_{\tilde{G}[\varepsilon]}\right)(x), \frac{1-k}{2}\},$$

and

$$\left(\gamma_{\tilde{F}[\varepsilon]} \vee \gamma_{\tilde{G}[\varepsilon]}\right)(x-y) = \gamma_{\tilde{F}[\varepsilon]}(x-y) \vee \gamma_{\tilde{G}[\varepsilon]}(x-y)$$
$$\leq \left(\max\{\gamma_{\tilde{F}[\varepsilon]}(x), \frac{1-k}{2}\}\right) \wedge \left(\max\{\gamma_{\tilde{G}[\varepsilon]}(x), \frac{1-k}{2}\}\right)$$
$$= \left(\max\{\gamma_{\tilde{F}[\varepsilon]}(x) \vee \gamma_{\tilde{G}[\varepsilon]}(x), \frac{1-k}{2}\}\right)$$
$$= \max\{\left(\gamma_{\tilde{F}[\varepsilon]} \vee \gamma_{\tilde{G}[\varepsilon]}\right)(x), \frac{1-k}{2}\}.$$

Also we have

$$\left(\mu_{\tilde{F}[\varepsilon]} \wedge \mu_{\tilde{G}[\varepsilon]}\right)(x \vee y) = \mu_{\tilde{F}[\varepsilon]}(x \vee y) \wedge \mu_{\tilde{G}[\varepsilon]}(x \vee y)$$
$$\geq \left(\min\{\mu_{\tilde{F}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}\right) \wedge \left(\min\{\mu_{\tilde{G}[\varepsilon]}(x), \mu_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}\right)$$
$$= \left(\min\{\mu_{\tilde{F}[\varepsilon]}(x) \wedge \mu_{\tilde{G}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y) \wedge \mu_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}\right)$$
$$= \min\{\left(\mu_{\tilde{F}[\varepsilon]} \wedge \mu_{\tilde{G}[\varepsilon]}\right)(x), \left(\mu_{\tilde{F}[\varepsilon]} \wedge \mu_{\tilde{G}[\varepsilon]}\right)(y), \frac{1-k}{2}\},$$

and

$$\left(\gamma_{\tilde{F}[\varepsilon]} \vee \gamma_{\tilde{G}[\varepsilon]}\right)(x \vee y) = \gamma_{\tilde{F}[\varepsilon]}(x \vee y) \vee \gamma_{\tilde{G}[\varepsilon]}(x \vee y)$$
$$\leq \left(\max\{\gamma_{\tilde{F}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}\right) \wedge \left(\max\{\gamma_{\tilde{G}[\varepsilon]}(x), \gamma_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}\right)$$
$$= \left(\max\{\gamma_{\tilde{F}[\varepsilon]}(x) \vee \gamma_{\tilde{G}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y) \vee \gamma_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}\right)$$
$$= \max\{\left(\gamma_{\tilde{F}[\varepsilon]} \vee \gamma_{\tilde{G}[\varepsilon]}\right)(x), \left(\gamma_{\tilde{F}[\varepsilon]} \wedge \gamma_{\tilde{G}[\varepsilon]}\right)(y), \frac{1-k}{2}\}.$$

Hence $\langle \tilde{F}, A\rangle \tilde{\wedge} \langle \tilde{G}, B\rangle$ is also an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideal of *X*. The other case can be seen in a similar way.

Theorem 4.17 Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebras (resp., ideals) of *X*. Then $\langle \tilde{F}, A\rangle \tilde{\vee} \langle \tilde{G}, B\rangle$ is also an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra (resp., ideal) of *X*.

Proof. The proof is similar to the proof of the Theorem 4.16.

Theorem 4.18 Let $\langle \tilde{F}, A\rangle$ be an intuitionistic fuzzy soft set of *X*. Then $\langle \tilde{F}, A\rangle$ is an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra (resp., ideal) of *X* if and only if $\langle \tilde{F}, A\rangle^{(\alpha,\beta)} = \langle \tilde{F}^{(\alpha,\beta)}, A\rangle = \{x \in X \mid \mu_{\tilde{F}[\varepsilon]}(x) \geq \alpha, \ \gamma_{\tilde{F}[\varepsilon]}(x) \leq \beta$, where $\alpha \in (0, \frac{1-k}{2}]$, $\beta \in [\frac{1-k}{2}, 1)\}$ is a soft subalgebra (resp., ideal) of *X*.

Proof. Let $\langle \tilde{F}, A\rangle$ be an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra of *X*. Let $x, y \in \langle \tilde{F}, A\rangle^{(\alpha,\beta)}$ then $\mu_{\tilde{F}[\varepsilon]}(x) \geq \alpha$, $\mu_{\tilde{F}[\varepsilon]}(y) \geq \alpha$ and $\gamma_{\tilde{F}[\varepsilon]}(x) \leq \beta$, $\gamma_{\tilde{F}[\varepsilon]}(y) \leq \beta$, where $\alpha \in (0, \frac{1-k}{2}]$, $\beta \in [\frac{1-k}{2}, 1)$. Since $\langle \tilde{F}, A\rangle$ is an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra of *X*, so

$$\mu_{\tilde{F}[\varepsilon]}(x-y) \geq \min\{\mu_{\tilde{F}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \min\{\alpha, \alpha, \frac{1-k}{2}\} = \alpha,$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-y) \leq \max\{\gamma_{\tilde{F}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \max\{\beta, \beta, \frac{1-k}{2}\} = \beta.$$

Thus $(x-y) \in \langle \tilde{F}, A\rangle^{(\alpha,\beta)}$. Hence $\langle \tilde{F}, A\rangle^{(\alpha,\beta)}$ is a soft subalgebra of *X*.

Conversely assume that $\langle\tilde{F},A\rangle^{(\alpha,\beta)}$ is a soft sub-algebra of *X*. Let there exist some $r\in(0,1]$ and $s\in[0,1)$ such that

$$\mu_{\tilde{F}[\varepsilon]}(x-y)<r<\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-y)>s>\min\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}.$$

This implies that $(x-y)\notin\langle\tilde{F},A\rangle^{(\alpha,\beta)}$, which is contradiction. Thus

$$\mu_{\tilde{F}[\varepsilon]}(x-y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}.$$

Hence $\langle\tilde{F},A\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft sub-algebra of *X*.

Proposition 4.19 Every $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal $\langle\tilde{F},A\rangle$ of *X* satisfies the following,

(i) $\mu_{\tilde{F}[\varepsilon]}(0)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$,

(ii) $\gamma_{\tilde{F}[\varepsilon]}(0)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$, for all $x\in X$.

Proof: By letting $x=y$ in the Definition 4.13, we get the required proof.

Lemma 4.20 If an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft set $\langle\tilde{F},A\rangle$ of *X* satisfies the followings,

(i) $\mu_{\tilde{F}[\varepsilon]}(0)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$,

(ii) $\gamma_{\tilde{F}[\varepsilon]}(0)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$,

(iii) $\mu_{\tilde{F}[\varepsilon]}(x-z)\geq\min\{\mu_{\tilde{F}[\varepsilon]}((x-y)-z),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$,

(iv) $\gamma_{\tilde{F}[\varepsilon]}(x-z)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}((x-y)-z),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$

Then we have $x\leq a\Rightarrow\mu_{\tilde{F}[\varepsilon]}(x)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}$ and $\gamma_{\tilde{F}[\varepsilon]}(x)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}$, for all $a,x,y,z\in X$.

Proof. Let $a,x\in X$ and $x\leq a$. Consider

$$\begin{aligned}\mu_{\tilde{F}[\varepsilon]}(x)&=\mu_{\tilde{F}[\varepsilon]}(x-0)\text{ by }(a2),\\&\geq\min\{\mu_{\tilde{F}[\varepsilon]}((x-a)-0),\mu_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by (iii)},\\&=\min\{\mu_{\tilde{F}[\varepsilon]}(0),\mu_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by }x\leq a\text{ iff }x-a=0,\\&=\min\{\mu_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by (i).}\end{aligned}$$

Also consider

$$\begin{aligned}\gamma_{\tilde{F}[\varepsilon]}(x)&=\gamma_{\tilde{F}[\varepsilon]}(x-0)\text{ by }(a2),\\&\leq\max\{\gamma_{\tilde{F}[\varepsilon]}((x-a)-0),\gamma_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by (iv)},\\&=\max\{\gamma_{\tilde{F}[\varepsilon]}(0),\gamma_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by }x\leq a\text{ iff }x-a=0,\\&=\max\{\gamma_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}\text{ by (ii).}\end{aligned}$$

This complete the proof.

Theorem 4.21 An $(\in,\in\vee q_k)$-intuitionistic fuzzy soft set $\langle\tilde{F},A\rangle$ of *X* satisfies the conditions (i)-(iv) of the Lemma 4.20, if and only if $\langle\tilde{F},A\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of *X*.

Proof. Suppose that $\langle\tilde{F},A\rangle$, i.e, $\tilde{F}[\varepsilon]=\langle\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]}\rangle$ satisfies the conditions (i)-(iv) of the Lemma 4.20. Let $x,y\in X$, then by using (*a*3) we have $x-y\leq x.$ Now by the use of Lemma 4.20, $\mu_{\tilde{F}[\varepsilon]}(x-y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$ and $\gamma_{\tilde{F}[\varepsilon]}(x-y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$ for all $x,y\in X$.

Also, $\mu_{\tilde{F}[\varepsilon]}(x\vee y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$ and $\gamma_{\tilde{F}[\varepsilon]}(x\vee y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$ whenever $x\vee y$ exists in *X* and by using the Lemma 4.20, we have $\mu_{\tilde{F}[\varepsilon]}(x\vee y)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$ and $\gamma_{\tilde{F}[\varepsilon]}(x\vee y)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$ for all $x,y\in X$. Hence $\langle\tilde{F},A\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of *X*.

Conversely, assume that $\langle\tilde{F},A\rangle$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of *X*. Conditions (i) and (ii) directly follows from the Proposition 4.19, let $x,y,z\in X$. Put $x=y$ and $y=x-z$ in (*a*3) and (*a*4), We get from (*a*3), $(x-z)-y\leq x-z$ and from (*a*4), we have $y-(y-((x-z))\leq x-z.$ Which indicates that is an upper bound for $(x-z)-y$ and $y-(y-((x-z)).$ By using the Proposition 2.4, we have

$$\begin{aligned}&((x-z)-y)\vee(y-(y-((x-z)))\\&=(x-z)-(((x-z)-((x-z)-y))-(y-(y-(x-z))))\\&=x-z.\end{aligned}$$

Thus

$$\begin{aligned}\mu_{\tilde{F}[\varepsilon]}(x-z)&=\mu_{\tilde{F}[\varepsilon]}(((x-z)-y)\vee(y-(y-((x-z)))))\\&\geq\min\{\mu_{\tilde{F}[\varepsilon]}((x-z)-y),\mu_{\tilde{F}[\varepsilon]}(y-(y-(x-z)),\frac{1-k}{2}\}\\&=\min\{\mu_{\tilde{F}[\varepsilon]}((x-y)-z),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}\text{ by }(S_3)\text{ and }(a2)\text{ and }x=y.\end{aligned}$$

Which is (iii). And

$$\gamma_{\tilde{F}[\varepsilon]}(x-z)=\gamma_{\tilde{F}[\varepsilon]}(((x-z)-y)\vee(y-(y-((x-z)))))$$
$$\leq \max\{\gamma_{\tilde{F}[\varepsilon]}((x-z)-y),\gamma_{\tilde{F}[\varepsilon]}(y-(y-(x-z))),\frac{1-k}{2}\}$$
$$=\max\{\gamma_{\tilde{F}[\varepsilon]}((x-y)-z),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\} \text{ by } (S_3) \text{ and } (a2) \text{ and } x=y.$$

Which is (iv). Hence it completes the proof.

Theorem 4.22 An $(\in,\in\vee q_k)$-intuitionistic fuzzy soft set $\langle\tilde{F},A\rangle$ of X is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X if and only if it satisfies:

(i) $\mu_{\tilde{F}[\varepsilon]}(x-((x-a)-b))\geq\min\{\mu_{\tilde{F}[\varepsilon]}(a),\mu_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\}$,

(ii) $\gamma_{\tilde{F}[\varepsilon]}(x-((x-a)-b))\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(a),\gamma_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\}$

for all $x,a,b\in X$.

Proof. Let $(\in,\in\vee q_k)$-intuitionistic fuzzy set $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ of X satisfies (i) and (ii). Consider

$$\mu_{\tilde{F}[\varepsilon]}(x-y)=\mu_{\tilde{F}[\varepsilon]}((x-y)-(((x-y)-x)-x)) \text{ by (i)}$$
$$\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$$
$$=\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-y)=\gamma_{\tilde{F}[\varepsilon]}((x-y)-(((x-y)-x)-x)) \text{ by (ii)}$$
$$\leq\max c\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}$$
$$=\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}.$$

Also suppose that $x\vee y$ exists in X and by using the Proposition 2.4, we have $x\vee y=w-((w-x)-y)$. Now using the given conditions

$$\mu_{\tilde{F}[\varepsilon]}(x\vee y)=\mu_{\tilde{F}[\varepsilon]}(w-((w-x)-y))$$
$$\geq\min\{\mu_{\tilde{F}[\varepsilon]}(x),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\} \text{ by (i),}$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x\vee y)=\gamma_{\tilde{F}[\varepsilon]}(w-((w-x)-y))$$
$$\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\} \text{ by (ii).}$$

Hence $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X.

Conversely, let $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ be an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X. By Theorem 4.21,

$$\mu_{\tilde{F}[\varepsilon]}(x-z)\geq\min\{\mu_{\tilde{F}[\varepsilon]}((x-y)-z),\mu_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-z)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}((x-y)-z),\gamma_{\tilde{F}[\varepsilon]}(y),\frac{1-k}{2}\}.$$

Let $z=(x-a)-b$ and $y=b$, then

$$\mu_{\tilde{F}[\varepsilon]}(x-((x-a)-b))\geq\min\{\mu_{\tilde{F}[\varepsilon]}((x-y)-((x-a)-b)),\mu_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\}$$
$$=\min\{\mu_{\tilde{F}[\varepsilon]}((x-y)-((x-b)-a)),\mu_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\} \text{ by } (S_3)$$
$$=\min\{\mu_{\tilde{F}[\varepsilon]}(a),\mu_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\} \text{ by putting } x=x-b \text{ and } y=a \text{ in } (a4)$$

and

$$\gamma_{\tilde{F}[\varepsilon]}(x-((x-a)-b))\leq\max\{\gamma_{\tilde{F}[\varepsilon]}((x-y)-((x-a)-b)),\gamma_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\}$$
$$=\max\{\gamma_{\tilde{F}[\varepsilon]}((x-y)-((x-b)-a)),\gamma_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\} \text{ by } (S_3)$$
$$=\max\{\gamma_{\tilde{F}[\varepsilon]}(a),\gamma_{\tilde{F}[\varepsilon]}(b),\frac{1-k}{2}\} \text{ by putting } x=x-b \text{ and } y=a \text{ in } (a4).$$

Hence $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ satisfy condition (i) and (ii).

Lemma 4.23 An intuitionistic fuzzy set $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ of X is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X if and only if fuzzy sets $\mu_{\tilde{F}[\varepsilon]}$ and $\overline{\gamma}_{\tilde{F}[\varepsilon]}$ are $(\in,\in\vee q_k)$- fuzzy soft ideals of X.

Proof. The proof is straightforward.

Theorem 4.24 An intuitionistic fuzzy set $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ of X is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X if $\triangleright A=(\mu_{\tilde{F}[\varepsilon]},\overline{\mu}_{\tilde{F}[\varepsilon]})$ and only if and $\triangleleft A=(\gamma_{\tilde{F}[\varepsilon]},\overline{\gamma}_{\tilde{F}[\varepsilon]})$ are $(\in,\in\vee q_k)$- intuitionistic fuzzy soft ideal of X.

Proof. The proof is straightforward.

Theorem 4.25 If $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X then

$$X_{\mu_{\tilde{F}[\varepsilon]}}=\{x\in X\ :\ \mu_{\tilde{F}[\varepsilon]}(x)=\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}=\mu_{\tilde{F}[\varepsilon]}(0)\}$$

and

$$X_{\gamma_{\tilde{F}[\varepsilon]}}=\{x\in X\ :\ \gamma_{\tilde{F}[\varepsilon]}(x)=\max\{\gamma_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}=\gamma_{\tilde{F}[\varepsilon]}(0)\}$$

are soft ideals of X.

Proof. Let $a\in X_{\mu_{\tilde{F}[\varepsilon]}}$ and $x\in X$. Then by definition $\min\{\mu_{\tilde{F}[\varepsilon]}(x),\frac{1-k}{2}\}=\mu_{\tilde{F}[\varepsilon]}(0)$. Since $A=(\mu_{\tilde{F}[\varepsilon]},\gamma_{\tilde{F}[\varepsilon]})$ is an $(\in,\in\vee q_k)$-intuitionistic fuzzy soft ideal of X so we have

(i) $\mu_{\tilde{F}[\varepsilon]}(a-x)\geq\min\{\mu_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}=\mu_{\tilde{F}[\varepsilon]}(0)$, which implies that $(a-x)\in X_{\mu_{\tilde{F}[\varepsilon]}}$.

(ii) $\gamma_{\tilde{F}[\varepsilon]}(a-x)\leq\max\{\gamma_{\tilde{F}[\varepsilon]}(a),\frac{1-k}{2}\}=\gamma_{\tilde{F}[\varepsilon]}(0)$, which implies that $(a-x)\in X_{\gamma_{\tilde{F}[\varepsilon]}}$.

Let $x\vee y$ exists in X, we get

(iii) $\mu_{\tilde{F}[\varepsilon]}(x \vee y) \geq \min\{\mu_{\tilde{F}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\} = \mu_{\tilde{F}[\varepsilon]}(0)$ which implies that $(x \vee y) \in X_{\mu_{\tilde{F}[\varepsilon]}}$, $\gamma_{\tilde{F}[\varepsilon]}(x \vee y) \leq \max\{\gamma_{\tilde{F}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\} = \gamma_{\tilde{F}[\varepsilon]}(0)$, which implies that $(x \vee y) \in X_{\gamma_{\tilde{F}[\varepsilon]}}$. Hence

$$X_{\mu_{\tilde{F}[\varepsilon]}} = \{x \in X \ : \ \mu_{\tilde{F}[\varepsilon]}(x) = \min\{\mu_{\tilde{F}[\varepsilon]}(x), \frac{1-k}{2}\} = \mu_{\tilde{F}[\varepsilon]}(0)\}$$

and

$$X_{\gamma_{\tilde{F}[\varepsilon]}} = \{x \in X \ : \ \gamma_{\tilde{F}[\varepsilon]}(x) = \max\{\gamma_{\tilde{F}[\varepsilon]}(x), \frac{1-k}{2}\} = \gamma_{\tilde{F}[\varepsilon]}(0)\}$$

are soft ideals of X.

Theorem 4.26 Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebras (resp., ideals) of X. Then so is $\langle \tilde{F}, A\rangle \sqcap \langle \tilde{G}, B\rangle$.

Proof. Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebras of X. We know that $\langle \tilde{F}, A\rangle \sqcap \langle \tilde{G}, B\rangle = \langle \tilde{\Theta}, C\rangle$, where $C = A \cup B$. Now for any $\varepsilon \in C$ and $x, y \in X$, we consider the following cases

Case 1: For any $\varepsilon \in A - B$, we have

$$\mu_{\tilde{\Theta}[\varepsilon]}(x-y) = \mu_{\tilde{F}[\varepsilon]}(x-y) \geq \min\{\mu_{\tilde{F}[\varepsilon]}(x), \mu_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \min\{\mu_{\tilde{\Theta}[\varepsilon]}(x), \mu_{\tilde{\Theta}[\varepsilon]}(y), \frac{1-k}{2}\},$$

and

$$\gamma_{\tilde{\Theta}[\varepsilon]}(x-y) = \gamma_{\tilde{F}[\varepsilon]}(x-y) \leq \max\{\gamma_{\tilde{F}[\varepsilon]}(x), \gamma_{\tilde{F}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \max\{\gamma_{\tilde{\Theta}[\varepsilon]}(x), \gamma_{\tilde{\Theta}[\varepsilon]}(y), \frac{1-k}{2}\}.$$

Case 2: For any $\varepsilon \in B - A$, we have

$$\mu_{\tilde{\Theta}[\varepsilon]}(x-y) = \mu_{\tilde{G}[\varepsilon]}(x-y) \geq \min\{\mu_{\tilde{G}[\varepsilon]}(x), \mu_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \min\{\mu_{\tilde{\Theta}[\varepsilon]}(x), \mu_{\tilde{\Theta}[\varepsilon]}(y), \frac{1-k}{2}\},$$

and

$$\gamma_{\tilde{\Theta}[\varepsilon]}(x-y) = \gamma_{\tilde{G}[\varepsilon]}(x-y) \leq \max\{\gamma_{\tilde{G}[\varepsilon]}(x), \gamma_{\tilde{G}[\varepsilon]}(y), \frac{1-k}{2}\}$$
$$= \max\{\gamma_{\tilde{\Theta}[\varepsilon]}(x), \gamma_{\tilde{\Theta}[\varepsilon]}(y), \frac{1-k}{2}\}.$$

Case 3: For any $\varepsilon \in A \cap B$, we have $\mu_{\tilde{F}[\varepsilon]} \cap \mu_{\tilde{G}[\varepsilon]}$ and $\gamma_{\tilde{F}[\varepsilon]} \cup \gamma_{\tilde{G}[\varepsilon]}$. Analogous to the proof of Theorem 4.16. Hence $\langle \tilde{F}, A\rangle \sqcap \langle \tilde{G}, B\rangle$ is an $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra of X.

Theorem 4.27 Let $\langle \tilde{F}, A\rangle$ and $\langle \tilde{G}, B\rangle$ be two $(\in, \in \vee q_k)$-intuitionistic fuzzy soft subalgebra (resp., ideals) of X. Then so is $\langle \tilde{F}, A\rangle \sqcup \langle \tilde{G}, B\rangle$.

Proof. The proof is similar to the proof of the Theorem 4.26.

Acknowledgements

We highly appreciate the detailed valuable comments of the referees which greatly improve the quality of this paper.

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