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Original Article

More on generalized *b*-closed sets in double fuzzy topological spaces

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Abstract

The purpose of this paper is to introduce and study a new class of sets called (*r, s*)-generalized fuzzy *b*-regular weakly closed sets which is lies between the new class of (*r, s*)-fuzzy regular weakly closed sets and (*r, s*)-generalized fuzzy *b*-closed sets in double fuzzy topological spaces. Several fundamental properties are introduced and discussed. Furthermore, the relationships between the new concepts are introduced and established with some interesting counter examples.

Keywords: double fuzzy topology, (*r, s*)-fuzzy regular weakly closed sets, (*r, s*)-generalized fuzzy *b*-regular weakly closed sets

1. Introduction

After the development of Zadeh's theory of fuzzy sets Zadeh (1965), the idea of intuitionistic fuzzy set was first introduced by Atanassov (1993), then Çoker (1997) introduced the notion of intuitionistic fuzzy topological space. After that Samanta and Mondal (2002), introduced the notion of intuitionistic gradation of openness of fuzzy sets and gave the definition of intuitionistic fuzzy topological space as a generalization of smooth topology and the topology of intuitionistic fuzzy sets. Working under the name "intuitionistic" did not continue and ended in 2005 by Gutierrez Garcia and Rodabaugh (2005) when they proved that this term is unsuitable in mathematics and applications and they concluded that they work under the name "double".

In 2009, Omari and Noorani (2009) introduced the class of generalized *b*-closed sets (briefly, *gb*-closed) in topological spaces. As a generalization of this work, we will apply the notions of (*r, s*)-generalized $\psi\rho$-closed sets (see Mohammed *et al.* (2013, 2014)) to introduce and study the (*r, s*)-generalized fuzzy *b*-regular weakly closed sets in double fuzzy topological spaces. The new notion lies between the class of (*r, s*)-fuzzy regular weakly closed sets and the class of (*r, s*)-generalized fuzzy *b*-closed sets. Finally, some inter-relations between the new concepts are introduced and established with some interesting counter examples.

2. Preliminaries

Throughout this paper, X will be a non-empty set, I is the closed unit interval $[0,1]$, $I_0 = (0,1]$ and $I_1 = [0,1)$. A fuzzy set λ is quasi-coincident with a fuzzy set μ denoted by $\lambda q \mu$ iff there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$ and they are not quasi-coincident otherwise which denoted by $\lambda \overline{q} \mu$ (see Pao-Ming and Ying-Ming (1980)). The family of all fuzzy sets on X is denoted by I^X. By $\underline{0}$ and $\underline{1}$, we denote the smallest and the largest fuzzy sets on X. For a fuzzy set $\lambda \in I^X$, $\underline{1} - \lambda$ denotes its complement. All other notations are standard notations of fuzzy set theory.

Now, we recall the following definitions which are useful in the sequel.

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Definition 2.1 (see Samanta and Mondal (2002)) A double fuzzy topology (τ,τ^*) on X is a pair of maps $\tau,\ \tau^*: I^X \to I$, which satisfies the following properties:

(1) $\tau(\lambda) \le \underline{1} - \tau^*(\lambda)$ for each $\lambda \in I^X$.

(2) $\tau(\lambda_1 \wedge \lambda_2) \ge \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^*(\lambda_1 \wedge \lambda_2) \le \tau^*(\lambda_1) \vee \tau^*(\lambda_2)$ for each $\lambda_1,\ \lambda_2 \in I^X$.

(3) $\tau(\vee_{i\in\Gamma}\lambda_i) \ge \wedge_{i\in\Gamma}\tau(\lambda_i)$ and $\tau^*(\vee_{i\in\Gamma}\lambda_i) \le \vee_{i\in\Gamma}\tau^*(\lambda_i)$ for each $\lambda_i \in I^X$, $i \in \Gamma$.

The triplet (X,τ,τ^*) is called a double fuzzy topological space(dfts, for short). A fuzzy set λ is called an (r,s)-fuzzy open ((r,s)-fo, for short) if $\tau(\lambda) \ge r$ and $\tau^*(\lambda) \le s$. A fuzzy set λ is called an (r,s)-fuzzy closed ((r,s)-fc, for short) set iff $\underline{1} - \lambda$ is an (r,s)-fo set.

Theorem 2.1 (see Lee and Im (2001)) Let (X,τ,τ^*) be a dfts. Then double fuzzy closure operator and double fuzzy interior operator of $\lambda \in I^X$ are defined by

$$C_{\tau,\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X | \lambda \le \mu, \tau(\underline{1}-\mu) \ge r, \tau^*(\underline{1}-\mu) \le s\},$$

$$I_{\tau,\tau^*}(\lambda,r,s) = \vee\{\mu \in I^X | \mu \le \lambda, \tau(\mu) \ge r, \tau^*(\mu) \le s\}.$$

Where $r \in I_0$ and $s \in I_1$ such that $r + s \le 1$.

Definition 2.2 Let (X,τ,τ^*) be a dfts. For each $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$. A fuzzy set λ is called:

(1) An (r,s)-regular fuzzy open (see Ramadan *et al.* (2005)) (briefly, (r,s)-rfo) if $\lambda = (I_{\tau,\tau^*}(C_{\tau,\tau^*}(\lambda,r,s),r,s))$. λ is called an (r,s)-regular fuzzy closed (briefly, (r,s)-rfc) iff $\underline{1}-\lambda$ is (r,s)-rfo set.

(2) An (r,s)-fuzzy semi open (see Kim and Abbas (2004)) (briefly, (r,s)-fso) if $\lambda \le C_{\tau,\tau^*}(I_{\tau,\tau^*}(\lambda,r,s),r,s)$. λ is called an (r,s)-fuzzy semi closed (briefly, (r,s)-fsc) iff $\underline{1}-\lambda$ is an (r,s)-fso set.

(3) An (r,s)-regular generalized fuzzy closed (see Ghareeb (2011)) (briefly, (r,s)-rgfc) if $C_{\tau,\tau^*}(\lambda,r,s) \le \mu$, $\lambda \le \mu$, μ is (r,s)-rfo. λ is called an (r,s)-regular generalized fuzzy open (briefly, (r,s)-rgfo) iff $\underline{1}-\lambda$ is (r,s)-rgfc set.

(4) An (r,s)-fuzzy regular semiopen (see El-Saady and Ghareeb (2012)) (briefly, (r,s)-frso) if there exists an (r,s)-fuzzy regular open set μ such that $\mu \le \lambda \le C_{\tau,\tau^*}(\mu,r,s)$.

(5) An (r,s)-generalized fuzzy closed (see Abbas and El-Sanousy (2012)) (briefly, (r,s)-gfc) if $C_{\tau,\tau^*}(\lambda,r,s) \le \mu$, $\lambda \le \mu$, $\tau(\mu) \ge r$ and $\tau^*(\mu) \le s$. λ is called an -generalized fuzzy open (briefly, (r,s)-gfo) iff $\underline{1}-\lambda$ is (r,s)-gfc set.

(6) An (r,s)-fuzzy b-closed (see Mohammed *et al.* (2015)) (briefly, (r,s)-fbc) if

$$\lambda \ge \left(I_{\tau,\tau^*}\left(C_{\tau,\tau^*}(\lambda,r,s),r,s\right)\right) \wedge \left(C_{\tau,\tau^*}\left(I_{\tau,\tau^*}(\lambda,r,s),r,s\right)\right).$$

λ is called an (r,s)-fuzzy b-open (briefly, (r,s)-fbo) iff $\underline{1}-\lambda$ is (r,s)-fbc set.

(7) An (r,s)-generalized fuzzy b-closed (see Mohammed *et al.* (2015)) (briefly, (r,s)-gfbc) if $bC_{\tau,\tau^*}(\lambda,r,s) \le \mu$, $\lambda \le \mu$, $\tau(\mu) \ge r$ and $\tau^*(\mu) \le s$. λ is called an (r,s)-generalized fuzzy b-open (briefly, (r,s)-gfbo) iff $\underline{1}-\lambda$ is (r,s)-gfbc set.

3. An (r,s)-Generalized Fuzzy b-Regular Weakly Closed Sets

In this section, we introduce and study some basic properties of a new class of sets called (r,s)-generalized fuzzy b-regular weakly closed sets.

Definition 3.1 Let (X,τ,τ^*) be a dfts. Then double fuzzy b-closure operator and double fuzzy b-interior operator of $\lambda \in I^X$ are defined by

$$bC_{\tau,\tau^*}(\lambda,r,s) = \wedge\{\mu \in I^X | \lambda \le \mu \text{ and } \mu \text{ is } (r,s)-fbc\},$$

$$bI_{\tau,\tau^*}(\lambda,r,s) = \vee\{\mu \in I^X | \mu \le \lambda \text{ and } \mu \text{ is } (r,s)-fbo\},$$

where $r \in I_0$ and $s \in I_1$ such that $r + s \le 1$.

Definition 3.2 Let (X,τ,τ^*) be a dfts. For each $\lambda,\ \mu \in I^X$, $r \in I_0$ and $s \in I_1$. A fuzzy set λ is called:

(1) An (r,s)-fuzzy b-clopen (briefly, (r,s)-fb-clopen) if λ is an (r,s)-fbo and an (r,s)-fbc set.

(2) An (r,s)-fuzzy regular weakly closed (briefly, (r,s)-frwc) sets if $C_{\tau,\tau^*}(\lambda,r,s) \le \mu$ whenever $\lambda \le \mu$ and μ is (r,s)-frso. λ is called an (r,s)-fuzzy regular weakly open (briefly, (r,s)-frwo)sets iff $\underline{1}-\lambda$ is (r,s)-frwc set

(3) An (r,s)-generalized fuzzy b-regular weakly closed (briefly, (r,s)-gfbrwc)sets if $bC_{\tau,\tau^*}(\lambda,r,s) \le \mu$ whenever $\lambda \le \mu$ and μ is (r,s)-fuzzy regular semiopen. λ is called an (r,s)-generalized fuzzy b-regular weakly open (briefly, (r,s)-gfbrwo)sets iff $\underline{1}-\lambda$ is (r,s)-gfbrwc set.

Theorem 3.1 Let (X,τ,τ^*) be a dfts. A fuzzy set $\lambda \in I^X$ is (r,s)-gfbrwo set, $r \in I_0$ and $s \in I_1$ if and only if $\mu \le bI_{\tau,\tau^*}(\lambda,r,s)$ whenever μ is (r,s)-frsc.

Proof. Suppose that λ is an (r,s)-gfbrwo set in I^X, $r \in I_0$ and $s \in I_1$ and let μ is (r,s)-frsc such that $\mu \le \lambda$, so by the definition, $\underline{1}-\lambda$ is an (r,s)-gfbrwc set in I^X. Hence $bC_{\tau,\tau^*}(\underline{1}-\lambda,r,s) \le \underline{1}-\mu$ and $\underline{1}-bI_{\tau,\tau^*}(\lambda,r,s) \le \underline{1}-\mu$. So, $\mu \le bI_{\tau,\tau^*}(\lambda,r,s)$.

Conversely, suppose that $\mu \le \lambda$, μ is (r,s)-frsc, $r \in I_0$ and $s \in I_1$ such that $\mu \le bI_{\tau,\tau^*}(\lambda,r,s)$. Now $\underline{1}-bI_{\tau,\tau^*}(\lambda,r,s) \le \underline{1}-\mu$, Thus $bC_{\tau,\tau^*}(\underline{1}-\lambda,r,s) \le \underline{1}-\mu$. So, $\underline{1}-\lambda$ is an (r,s)-gfbrwc set and λ is an (r,s)-gfbrwo set.

Example 3.1 Let $X = \{a,b\}$. Define μ and α follows:

$$\mu(a) = 0.3, \mu(b) = 0.4,$$

$$\alpha(a) = 0.7, \alpha(b) = 0.6,$$

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda \in \{\underline{0},\underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 0, & \text{otherwise.} \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & \text{if } \lambda \in \{\underline{0},\underline{1}\}, \\ \frac{1}{2}, & \text{if } \lambda = \mu, \\ 1, & \text{otherwise.} \end{cases}$$

Then α is an $(\frac{1}{2},\frac{1}{2})$-gfbrwc set.

Theorem 3.2 Let (X,τ,τ^*) be a dfts, $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$. If λ is an (r, s)-gfbrwc set, then

(1) $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ does not contain any non-zero (r, s)-frso sets.

(2) λ is an (r, s)-fbrwc iff $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ is (r, s)-rfc.

(3) μ is (r, s)-gfbrwc set for each set $\mu \in I^X$ such that $\lambda \le \mu \le bC_{\tau,\tau^*}(\lambda,r,s)$.

(4) For each (r, s)-rfo set $\mu \in I^X$ such that $\mu \le \lambda$, μ is an (r, s)-gfbrwc relative to λ, μ is an (r, s)-gfbrwc in X.

(5) $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ does not contain any non-zero (r, s)-rfo sets.

Proof. (1) Suppose that μ is a non-zero (r, s)-frso set of I^X such that $\mu \le bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ whenever $\lambda \in I^X$ is an (r, s)-gfbrwc set, $r \in I_0$ and $s \in I_1$. Hence, $\mu \le \underline{1}-\lambda$ or $\lambda \le \underline{1}-\mu$. Since $\underline{1}-\mu$ is an (r, s)-frso set. But, λ is an (r, s)-gfbrwc set, hence

$$\lambda \le (\underline{1}-\mu) \Rightarrow bC_{\tau,\tau^*}(\lambda,r,s) \le (\underline{1}-\mu) \Rightarrow \mu \le \left(\underline{1}-bC_{\tau,\tau^*}(\lambda,r,s)\right)$$

$$\Rightarrow \mu \le \left(\underline{1}-bC_{\tau,\tau^*}(\lambda,r,s)\right) \wedge \left(bC_{\tau,\tau^*}(\lambda,r,s)-\lambda\right) = \underline{0}$$

and hence $\mu = \underline{0}$ which is a contradiction. Then $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ does not contain any non-zero (r, s)-frso sets.

(2) Suppose λ is an (r, s)-gfbrwc set. So, for each $r \in I_0$ and $s \in I_1$ if λ is an (r, s)-fbrwc set then, $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda = \underline{0}$ which is an (r, s)-rfc set.

Conversely, suppose $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ is an (r, s)-frso set. Then by (1), $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ does not contain any non-zero an (r, s)-frso set. But $bC_{\tau,\tau^*}(\lambda,r,s)-\lambda$ is an (r, s)-rfo set, then

$$bC_{\tau,\tau^*}(\lambda,r,s)-\lambda = \underline{0} \Rightarrow \lambda = bC_{\tau,\tau^*}(\lambda,r,s).$$

So, λ is an (r, s)-rfc set.

(3) Suppose α is an (r, s)-frso set such that $\mu \le \alpha$ and let λ be an (r, s)-gfbrwc set such that $\lambda \le \alpha$, $r \in I_0$ and $s \in I_1$. Then

$$bC_{\tau,\tau^*}(\lambda,r,s) \le \alpha,$$

hence,

$$bC_{\tau,\tau^*}(\mu,r,s) \le bC_{\tau,\tau^*}\left(bC_{\tau,\tau^*}(\lambda,r,s),r,s\right) = bC_{\tau,\tau^*}(\lambda,r,s) \le \mu.$$

Therefore μ is an (r, s)-gfbrwc set.

(4) Suppose that μ is an (r, s)-gfbrwc and ν is an (r, s)-frso in I^X such that $\mu \le \nu$, $r \in I_0$, $s \in I_1$. But $\mu \le \lambda \le \underline{1}$, therefore $\mu \le \lambda$ and $\mu \le \nu$. So

$$\mu \le \lambda \wedge \nu.$$

But μ is an (r, s)-gfbrwc relative to λ,

$$\lambda \wedge bC_{\tau,\tau^*}(\mu,r,s) \le \lambda \wedge \nu \Rightarrow \lambda \wedge bC_{\tau,\tau^*}(\mu,r,s) \le \nu.$$

Thus

$$\left(\lambda \wedge bC_{\tau,\tau^*}(\mu,r,s)\right) \vee \left(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s)\right) \le \nu \vee \left(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s)\right).$$

$$\Rightarrow \lambda \vee \left(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s)\right) \le \nu \vee \left(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s)\right).$$

Since λ is an (r, s)-fro and (r, s)-gfbrwc, then

$$bC_{\tau,\tau^*}(\lambda,r,s) \le \nu \vee (\underline{1}-\mu).$$

Also,

$$\mu \le \lambda \Rightarrow bC_{\tau,\tau^*}(\mu,r,s) \le bC_{\tau,\tau^*}(\lambda,r,s).$$

Thus

$$bC_{\tau,\tau^*}(\mu,r,s) \le bC_{\tau,\tau^*}(\lambda,r,s) \le \nu \vee \left(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s)\right).$$

Therefore, $bC_{\tau,\tau^*}(\mu,r,s) \le \nu$ but, $bC_{\tau,\tau^*}(\mu,r,s)$ is not contained in $(\underline{1}-bC_{\tau,\tau^*}(\mu,r,s))$. Therefore, μ is an (r, s)-gfbrwc relative to X.

(5) Directly from the fact that every (r, s)-rfo set is (r, s)-frso set.

Theorem 3.3 Let (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) be dfts's. If $\lambda \le \underline{1}_Y \le \underline{1}_X$ such that λ is an (r, s)-gfbrwc in $\underline{1}_X$, $r \in I_0$ and $s \in I_1$, then λ is an (r, s)-gfbrwc relative to $\underline{1}_Y$.

Proof. Suppose that (X,τ_1,τ_1^*) and (Y,τ_2,τ_2^*) are dfts's such that $\lambda \le \underline{1}_Y \le \underline{1}_X$, $r \in I_0$, $s \in I_1$ and λ is an (r, s)-gfbrwc in I^X. Now, let $\lambda \le \underline{1}_Y \wedge \mu$ such that μ is an (r, s)-rfo set in I^X. Since λ is an (r, s)-gfbrwc in I^X,

$$\lambda \le \mu \Rightarrow bC_{\tau,\tau^*}(\lambda,r,s) \le \mu.$$

So that

$$\underline{1}_Y \wedge bC_{\tau,\tau^*}(\lambda,r,s) \le \underline{1}_Y \wedge \mu.$$

Hence λ is an (r, s)-gfbrwc relative to Y.

Proposition 3.1 Let (X,τ_1,τ_1^*) be dfts's. For each λ and $\mu \in I^X$, $r \in I_0$, $s \in I_1$,

(1) If λ and μ are (r, s)-gfbrwc, then $\lambda \wedge \mu$ is an (r, s)-gfbrwc.

(2) If λ is an (r, s)-gfbrwc and $\tau(\underline{1}-\mu) \ge r$, $\tau^*(\underline{1}-\mu) \le s$, then $\lambda \wedge \mu$ is an (r, s)-gfbrwc.

Proof. 1-Suppose that λ and μ are (r, s)-gfbrwc sets in I^X such that $\lambda \wedge \mu \le \nu$ for each an (r, s)-frso set $\nu \in I^X$, $r \in I_0$ and $s \in I_1$. Since λ is an (r, s)-gfbrwc,

$$bC_{\tau,\tau^*}(\lambda,r,s) \le \nu$$

for each an (r, s)-frso set $\nu \in I^X$ and $\lambda \le \nu$. Also, μ is an (r, s)-gfbrwc,

$$bC_{\tau,\tau^*}(\mu,r,s) \le \nu$$

for each an (r, s)-frso set $\nu \in I^X$ and $\mu \le \nu$. So,

$$bC_{\tau,\tau^*}(\lambda,r,s) \wedge bC_{\tau,\tau^*}(\mu,r,s) \le \nu,$$

whenever $\lambda \wedge \mu \le \nu$ i.e, $\lambda \wedge \mu$ is an (r, s)-gfbrwc.

(2) Since every an (r, s)-fc set is an (r, s)-gfbrwc and from (1) we get the proof.

Proposition 3.2 Let (X,τ,τ^*) be dfts's. For each λ and $\mu \in I^X$, $r \in I_0$ and $s \in I_1$,

(1) If $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$ such that λ is an (r, s)-gfbc set. Then λ is an (r, s)-gfbrw-closed set.

(2) If λ is both an (r, s)-fro and an (r, s)-gfbrwc set, then λ is an (r, s)-fb-clopen.

(3) If λ is both an (r, s)-fro and an (r, s)-rgfc, then λ is an (r, s)-gfbrwc set.

(4) If λ is both an (r, s)-frso and an (r, s)-gfbrwc set, then λ is an (r, s)-fbc.

(5) If λ is both an (r, s)-frso and an (r, s)-gfbrwc set, such that $\tau(\underline{1}-\mu) \geq r$ and $\tau^*(\underline{1}-\mu) \leq s$. Then $\lambda \wedge \mu$ is an (r, s)-gfbrwc set.

Proof. (1) Suppose that $\lambda \leq \mu$ and μ is an (r, s)-frso in I^X such that $r \in I_0$ and $s \in I_1$. Since $\tau(\lambda) \geq r$ and $\tau^*(\lambda) \leq s$ such that λ is an (r, s)-gfbc set and $\lambda \leq \lambda$, then

$$bC_{\tau,\tau^*}(\lambda,r,s) \leq \lambda \Rightarrow bC_{\tau,\tau^*}(\lambda,r,s) \leq \mu.$$

Hence, λ is an (r, s)-gfbrwc set.

(2) Suppose that λ is an (r, s)-fro and an (r, s)-gfbrwc set, $r \in I_0$ and $s \in I_1$. Since every (r, s)-fro set is (r, s)-frso set and $\lambda \leq \lambda$, $bC_{\tau,\tau^*}(\lambda,r,s) \leq \lambda$. Also, $\lambda \leq bC_{\tau,\tau^*}(\lambda,r,s)$. Therefore, $\lambda = bC_{\tau,\tau^*}(\lambda,r,s)$, that is λ is (r, s)-fbc set. But, λ is an (r, s)-fro set, hence, is (r, s)-fbo set. Therefore, λ is (r, s)-fb-clopen set.

(3) Suppose that μ is an (r, s)-frso in I^X such that $\lambda \leq \mu$, $r \in I_0$ and $s \in I_1$. But, λ is an (r, s)-fro and an (r, s)-rgfc. Then by (2), $bC_{\tau,\tau^*}(\lambda,r,s) \leq \lambda$. Hence, $bC_{\tau,\tau^*}(\lambda,r,s) \leq \mu$. So, λ is an (r, s)-gfbrwc set.

(4) Obvious.

(5) Suppose that λ is an (r, s)-frso and an (r, s)-gfbrwc set of I^X, $r \in I_0$ and $s \in I_1$. Then by (4), λ is an (r, s)-fbc. But, $\tau(\underline{1}-\mu) \geq r$ and $\tau^*(\underline{1}-\mu) \leq s$, so $\lambda \wedge \mu$ is an (r, s)-fbc set. Therefore, $\lambda \wedge \mu$ is an (r, s)-gfbrwc set.

4. Interrelations

The following implication illustrates the relationships between different fuzzy sets:

$$(r,s)-fc \rightarrow (r,s)-frwc \rightarrow (r,s)-gfbrwc \rightarrow (r,s)-gfbc$$

None of these implications is reversible where $A \rightarrow B$ represents A implies B, as shown by the following examples.

Example 4.1

(1) Let $X = \{a,b,c\}$ and let μ_1, μ_2 and μ_3 are fuzzy sets defined as follows:

$$\mu_1(a) = 0.7, \mu_1(b) = 0.7, \mu_1(c) = 0.7,$$

$$\mu_2(a) = 0.6, \mu_2(b) = 0.7, \mu_2(c) = 0.7,$$

$$\mu_3(a) = 0.4, \mu_3(b) = 0.6, \mu_3(c) = 0.7.$$

Defined (τ,τ^*) on X as follows:

$$\tau(\lambda) = \begin{cases} 1, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.3, & if \ \lambda = \mu_1, \\ 0, & otherwise. \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.6, & if \ \lambda = \mu_1, \\ 1, & otherwise. \end{cases}$$

Then μ_1 is an (0.3,0.6)-frwc set, but not an (0.3,0.6)-fc set.

(2) Let $X = \{a,b,c\}$ and let μ_1, μ_2 and μ_3 are fuzzy sets defined as follows:

$$\mu_1(a) = 0.3, \mu_1(b) = 0.3, \mu_1(c) = 0.3,$$

$$\mu_2(a) = 0.3, \mu_2(b) = 0.3, \mu_2(c) = 0.3,$$

$$\mu_3(a) = 0.4, \mu_3(b) = 0.3, \mu_3(c) = 0.3.$$

Defined (τ,τ^*) on X as follows:

$$\tau(\lambda) = \begin{cases} 1, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.6, & if \ \lambda = \mu_2, \\ 0, & otherwise. \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.3, & if \ \lambda = \mu_2, \\ 1, & otherwise. \end{cases}$$

Then μ_2 is an (0.6,0.3)-gfbrwc set, but not an (0.6,0.3)-frwc set.

(3) Take X, μ_1 and μ_2 in (2). Define (τ,τ^*) on X as follows:

$$\tau(\lambda) = \begin{cases} 1, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.3, & if \ \lambda = \mu_1, \\ 0, & otherwise. \end{cases} \qquad \tau^*(\lambda) = \begin{cases} 0, & if \ \lambda \in \{\underline{0},\underline{1}\}, \\ 0.6, & if \ \lambda = \mu_1, \\ 1, & otherwise. \end{cases}$$

Then μ_2 is an (0.3,0.6)-gfbc set, but not an (0.3,0.6)-gfbrwc set.

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