



Original Article

New types of bipolar fuzzy sets in Γ -semihypergroups

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Abstract

The notion of bipolar fuzzy set was initiated by Lee (2000) as a generalization of the notion fuzzy sets and intuitionistic fuzzy sets, which have drawn attention of many mathematicians and computer scientists. In this paper, we initiate a study on bipolar (λ, δ) -fuzzy sets in Γ -semihypergroups. By using the concept of bipolar (λ, δ) -fuzzy sets (Yaqoob and Ansari, 2013), we introduce the notion of bipolar (λ, δ) -fuzzy sub Γ -semihypergroups (Γ -hyperideals and bi- Γ -hyperideals) and discuss some basic results on bipolar (λ, δ) -fuzzy sets in Γ -semihypergroups. Furthermore, we define the bipolar fuzzy subset $\mathcal{B}_\lambda^\delta = \langle \mu_{\mathcal{B}_\lambda^\delta}^+, \mu_{\mathcal{B}_\lambda^\delta}^- \rangle$ and prove that if $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., Γ -hyperideal and bi- Γ -hyperideal) of H ; then $\mathcal{B}_\lambda^\delta = \langle \mu_{\mathcal{B}_\lambda^\delta}^+, \mu_{\mathcal{B}_\lambda^\delta}^- \rangle$ is also a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., Γ -hyperideal and bi- Γ -hyperideal) of H .

Keywords: Γ -semihypergroups, bipolar fuzzy sets, bipolar (λ, δ) -fuzzy Γ -hyperideals

1. Introduction

Uncertainty is an attribute of information and uncertain data are presented in various domains. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh (1965), where he introduced the notion of a fuzzy subset of a non-empty set X , as a function from X to $[0; 1]$. Several researchers have conducted research on the generalizations of the notions of fuzzy sets with huge applications in computer science, logics and many branches of pure and applied mathematics. Rosenfeld (1971) defined the concept of fuzzy group. Since then many papers

have been published in the field of fuzzy algebra; for instance, Kuroki (1979, 1981) applied fuzzy set theory to the ideal theory of semigroups. Shabir (2005) studied fully fuzzy prime semigroups. Kehayopulu and Tsingelis (2007) studied fuzzy ideals in ordered semigroups. Akram and Dar (2005) studied fuzzy d-algebras.

There are several kinds of fuzzy set extensions in the fuzzy set theory; for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, etc. Bipolar-valued fuzzy set is another extension of fuzzy set whose membership degree range is different from the above extensions. Lee (2000) introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0, 1]$ to $[-1, 1]$. In a bipolar-valued fuzzy set, the membership degree 0 indicate that elements are irrelevant to

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the corresponding property, the membership degrees on $(0; 1]$ assign that elements somewhat satisfy the property, and the membership degrees on $[-1, 0)$ assign that elements somewhat satisfy the implicit counter-property (Lee, 2004). Jun and park (2009) applied the notion of bipolar-valued fuzzy sets to BCH-algebras. They introduced the concept of bipolar fuzzy subalgebras and bipolar fuzzy ideals of a BCH-algebra. Lee (2009) applied the notion of bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras. Also some results on bipolar-valued fuzzy BCK/BCI-algebras were introduced by Saeid (2009). Akram *et al.* (2011, 2012a, 2012b) applied bipolar fuzzy sets to Lie algebras.

The theory of hyperstructure was born in 1934 when Marty (1934) defined hypergroups, began to analysis their properties and applied them to groups, rational algebraic functions. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science and they are studied in many countries of the world. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. A recent book on hyperstructures (Corsini and Leoreanu, 2003) points out their applications in rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs and hypergraphs.

Recently, Anvariye *et al.* (2010), Heidri *et al.* (2010), Mirvakili *et al.* (2013), Hila *et al.* (2012) and Abdullah *et al.* (2011, 2012) introduced the notion of Γ -semihypergroups as a generalization of semigroups, semihypergroups and of Γ -semigroups. They proved some results in this respect and presented many interesting examples of Γ -semihypergroups. Fuzzy set theory has been well developed in the context of hyperalgebraic structure theory (Davvaz, 2010). Several authors investigated the different aspects of fuzzy semihypergroups and fuzzy Γ -semihypergroups; for instance, Aslam *et al.* (2012), Corsini *et al.* (2011), Davvaz (2000, 2006, 2007), Ersoy *et al.* (2012, 2013a, 2013b), Shabir and Mahmood (2013, 2015) and Yaqoob *et al.* (2014).

Yao introduced a new type of fuzzy sets called (λ, θ) -fuzzy sets, and studied (λ, θ) -fuzzy normal subfields (2005). Several authors extended Yao's idea and continued their researches in applying (λ, θ) -fuzzy sets on different algebraic structures. Coumaressane (2010) characterized near-rings by their (λ, θ) -fuzzy quasi-ideals. Shabir *et al.* (2011) characterized semigroups by the properties of their fuzzy ideals with thresholds and Khan *et al.* (2012) characterized ordered semigroups by their (λ, θ) -fuzzy bi-ideals. Li and Feng (2013) extended the idea of (λ, θ) -fuzzy sets in intuitionistic fuzzy sets and studied intuitionistic fuzzy (λ, μ) -fuzzy sets in Γ -semigroups. Yaqoob and Ansari (2013) also extended the idea of (λ, θ) -fuzzy sets in bipolar fuzzy sets and studied bipolar (λ, δ) -fuzzy ideals in ternary semigroups.

In this paper, we study bipolar (λ, δ) -fuzzy sets in Γ -semihypergroups and introduce the notion of bipolar (λ, δ) -fuzzy sub Γ -semihypergroups (Γ -hyperideals and bi- Γ -hyperideals).

2. Preliminaries and Basic Definitions

In this section, we will recall the basic terms and definitions from the hyperstructure theory and bipolar fuzzy sets.

Definition 2.1 A map $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set H , where H is a non-empty set and $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all non-empty subsets of H .

A hypergroupoid is a set H with together a (binary) hyperoperation.

Definition 2.2 A hypergroupoid (H, \circ) , which is associative, that is $x \circ (y \circ z) = (x \circ y) \circ z, \forall x, y, z \in H$, is called a semi-hypergroup.

Definition 2.3 (Anvariye *et al.*, 2010) Let H and Γ be two non-empty sets. Then H is called a Γ -semihypergroup if every $\gamma \in \Gamma$ is a hyperoperation on H , i.e.: $x\gamma y \subseteq H$ for every $x, y \in H$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in H$ we have $x\alpha(y\beta z) = (x\alpha y)\beta z$.

If every $\gamma \in \Gamma$ is an operation, then H is a Γ -semigroup. If (H, γ) is a hypergroup for every $\gamma \in \Gamma$, then H is called a Γ -hypergroup. Let A and B be two non-empty subsets of H . Then we define

$$A\Gamma B = \bigcup_{\gamma \in \Gamma} A\gamma B = \bigcup \{a\gamma b \mid a \in A, b \in B \text{ and } \gamma \in \Gamma\}.$$

Let H be a Γ -semihypergroup and $\gamma \in \Gamma$. A non-empty subset A of H is called a sub Γ -semihypergroup of H if $x\gamma y \subseteq A$ for every $x, y \in A$. A Γ -semihypergroup H is called commutative if for all $x, y \in H$ and $\gamma \in \Gamma$, we have $x\gamma y = y\gamma x$. A non-empty subset A of a Γ -semihypergroup H is a right (left) Γ -hyperideal of H if $A\Gamma H \subseteq A$ ($H\Gamma A \subseteq A$), and is a Γ -hyperideal of H if it is both a right and a left Γ -hyperideal. A non-empty subset B of a Γ -semihypergroup H is called bi- Γ -hyperideal of H if (1) $B\Gamma B \subseteq B$; (2) $B\Gamma H\Gamma B \subseteq B$.

Lee (2000) introduced the concept of bipolar fuzzy set defined on a non-empty set X as objects having the form:

$$\mathcal{B} = \left\{ \left\langle x, \mu_B^+(x), \mu_B^-(x) \right\rangle : x \in X \right\}.$$

Where $\mu_B^+ : X \rightarrow [0, 1]$ and $\mu_B^- : X \rightarrow [-1, 0]$. The positive membership degree $\mu_B^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set \mathcal{B} , and the negative membership degree $\mu_B^-(x)$ denotes the satisfaction degree of x to some implicit counter property of \mathcal{B} . For the sake of simplicity, we shall use the symbol $\mathcal{B} = \left\langle \mu_B^+, \mu_B^- \right\rangle$ for the bipolar fuzzy set $\mathcal{B} = \left\{ \left\langle x, \mu_B^+(x), \mu_B^-(x) \right\rangle : x \in X \right\}$.

Definition 2.4 $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be two bipolar fuzzy subsets of a Γ -semihypergroup H . Then for all $x \in H$, their intersection $\mathcal{A} \cap \mathcal{B}$ is defined by

$$\mathcal{A} \cap \mathcal{B} = \left\{ \left\langle x, \mu_{\mathcal{A} \cap \mathcal{B}}^+(x), \mu_{\mathcal{A} \cap \mathcal{B}}^-(x) \right\rangle : x \in H \right\},$$

where

$$\mu_{\mathcal{A} \cap \mathcal{B}}^+(x) = \left(\mu_{\mathcal{A}}^+ \wedge \mu_{\mathcal{B}}^+ \right)(x) = \mu_{\mathcal{A}}^+(x) \wedge \mu_{\mathcal{B}}^+(x),$$

$$\mu_{\mathcal{A} \cap \mathcal{B}}^-(x) = \left(\mu_{\mathcal{A}}^- \vee \mu_{\mathcal{B}}^- \right)(x) = \mu_{\mathcal{A}}^-(x) \vee \mu_{\mathcal{B}}^-(x).$$

Their union $\mathcal{A} \cup \mathcal{B}$ is defined by

$$\mathcal{A} \cup \mathcal{B} = \left\{ \left\langle x, \mu_{\mathcal{A} \cup \mathcal{B}}^+(x), \mu_{\mathcal{A} \cup \mathcal{B}}^-(x) \right\rangle : x \in H \right\},$$

where

$$\mu_{\mathcal{A} \cup \mathcal{B}}^+(x) = \left(\mu_{\mathcal{A}}^+ \vee \mu_{\mathcal{B}}^+ \right)(x) = \mu_{\mathcal{A}}^+(x) \vee \mu_{\mathcal{B}}^+(x),$$

$$\mu_{\mathcal{A} \cup \mathcal{B}}^-(x) = \left(\mu_{\mathcal{A}}^- \wedge \mu_{\mathcal{B}}^- \right)(x) = \mu_{\mathcal{A}}^-(x) \wedge \mu_{\mathcal{B}}^-(x).$$

Definition 2.5 Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be two bipolar fuzzy subsets of a Γ -semihypergroup H . Then their product $\mathcal{A} \circ_{\Gamma} \mathcal{B}$ is defined by

$$\mathcal{A} \circ_{\Gamma} \mathcal{B} = \left\{ \left\langle x, \mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^+(x), \mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^-(x) \right\rangle : x \in H \right\},$$

where

$$\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^+(x) = \begin{cases} \sup_{x \in \gamma y z} \{ \mu_{\mathcal{A}}^+(y) \wedge \mu_{\mathcal{B}}^+(z) \} & \text{if } x \in \gamma y z, \forall \gamma \in \Gamma \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^-(x) = \begin{cases} \inf_{x \in \gamma y z} \{ \mu_{\mathcal{A}}^-(y) \vee \mu_{\mathcal{B}}^-(z) \} & \text{if } x \in \gamma y z, \forall \gamma \in \Gamma \\ 0 & \text{otherwise,} \end{cases}$$

for $x, y, z \in H$.

Definition 2.6 Let $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be a bipolar fuzzy set and $(s, t) \in [-1, 0] \times [0, 1]$. Define:

1) the sets $\mathcal{B}_t^+ = \{x \in X \mid \mu_{\mathcal{B}}^+(x) \geq t\}$ and $\mathcal{B}_s^- = \{x \in X \mid \mu_{\mathcal{B}}^-(x) \leq s\}$, which are called positive t -cut of $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ and the negative s -cut of $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$, respectively,

2) the sets ${}^>\mathcal{B}_t^+ = \{x \in X \mid \mu_{\mathcal{B}}^+(x) > t\}$ and ${}^<\mathcal{B}_s^- = \{x \in X \mid \mu_{\mathcal{B}}^-(x) < s\}$, which are called strong positive t -cut of $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ and the strong negative s -cut of $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$, respectively,

3) the set $X_{\mathcal{B}}^{(t,s)} = \{x \in X \mid \mu_{\mathcal{B}}^+(x) \geq t, \mu_{\mathcal{B}}^-(x) \leq s\}$ is called an (s, t) -level subset of \mathcal{B} ,

4) the set ${}^H X_{\mathcal{B}}^{(t,s)} = \{x \in X \mid \mu_{\mathcal{B}}^+(x) > t, \mu_{\mathcal{B}}^-(x) < s\}$ is called a strong (s, t) -level subset of \mathcal{B} .

3. Bipolar (λ, δ) -fuzzy Γ -hyperideals in Γ -semihypergroups

Yaqoob and Ansari (2013) introduced the notion bipolar (λ, δ) -fuzzy ideals in ternary semigroups. In this section, we introduce and study the notion bipolar (λ, δ) -fuzzy Γ -hyperideals (resp., interior Γ -hyperideals and bi- Γ -hyperideals) in Γ -semihypergroups and discuss some related properties.

In what follows, let $\lambda^+, \delta^+ \in [0, 1]$ be such that $0 \leq \lambda^+ < \delta^+ \leq 1$ and $\lambda^-, \delta^- \in [-1, 0]$ be such that $-1 \leq \delta^- < \lambda^- \leq 0$. Both $\lambda, \delta \in [0, 1]$ are arbitrary but fixed.

Definition 3.1 A bipolar fuzzy subset $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ of a Γ -semihypergroup H is called

(1) a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H if

$$\max \left\{ \inf_{x \in \gamma y z} \mu_{\mathcal{B}}^+(x), \lambda^+ \right\} \geq \min \{ \mu_{\mathcal{B}}^+(y), \mu_{\mathcal{B}}^+(z), \delta^+ \}$$

$$\text{and } \min \left\{ \sup_{x \in \gamma y z} \mu_{\mathcal{B}}^-(x), \lambda^- \right\} \leq \max \{ \mu_{\mathcal{B}}^-(y), \mu_{\mathcal{B}}^-(z), \delta^- \},$$

for all $x, y, z \in H$ and $\gamma \in \Gamma$.

(2) a bipolar (λ, δ) -fuzzy left Γ -hyperideal of H if

$$\max \left\{ \inf_{x \in \gamma y z} \mu_{\mathcal{B}}^+(x), \lambda^+ \right\} \geq \min \{ \mu_{\mathcal{B}}^+(z), \delta^+ \}$$

$$\text{and } \min \left\{ \sup_{x \in \gamma y z} \mu_{\mathcal{B}}^-(x), \lambda^- \right\} \leq \max \{ \mu_{\mathcal{B}}^-(z), \delta^- \},$$

for all $x, y, z \in H$ and $\gamma \in \Gamma$.

(3) a bipolar (λ, δ) -fuzzy right Γ -hyperideal of H if

$$\max \left\{ \inf_{x \in \gamma y z} \mu_{\mathcal{B}}^+(x), \lambda^+ \right\} \geq \min \{ \mu_{\mathcal{B}}^+(y), \delta^+ \}$$

$$\text{and } \min \left\{ \sup_{x \in \gamma y z} \mu_{\mathcal{B}}^-(x), \lambda^- \right\} \leq \max \{ \mu_{\mathcal{B}}^-(y), \delta^- \},$$

for all $x, y, z \in H$ and $\gamma \in \Gamma$.

Example 3.2 Let $M = (0, 1)$, $\Gamma = \{ \gamma_n \mid n \in \mathbb{N} \}$ and for every $n \in \mathbb{N}$ we define hyperoperation γ_n on M as follows

$$x \gamma_n y = \left\{ \frac{xy}{2^k} \mid 0 \leq k \leq n \right\}, \forall x, y \in M.$$

Then, $x \gamma_n y \subset M$ and for every $m, n \in \mathbb{N}$ and $x, y, z \in M$

$$(x \gamma_n y) \gamma_m z = \left\{ \frac{xyz}{2^k} \mid 0 \leq k \leq n + m \right\} = x \gamma_n (y \gamma_m z).$$

So M is a Γ -semihypergroup (Hila and Abdullah, 2014). Now we define a bipolar fuzzy set $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ on M as:

$$\mu_{\mathcal{B}}^+(x) = \begin{cases} 0.7 & \text{if } 0 < x < \frac{1}{2^k} \\ 0.6 & \text{if } \frac{1}{2^k} \leq x < 1 \end{cases} \text{ where } k \in \mathbb{N}$$

$$\mu_B^-(x) = \begin{cases} -0.9 & \text{if } 0 < x < \frac{1}{2^k} \\ -0.5 & \text{if } \frac{1}{2^k} \leq x < 1 \end{cases} \text{ where } k \in \mathbb{N}.$$

Then by routine calculations, $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (0.2, 0.35)-fuzzy Γ -hyperideal of M .

Definition 3.3 A bipolar fuzzy subset $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of H is called a bipolar (λ, δ) -fuzzy interior Γ -hyperideal of H if

$$\max \left\{ \inf_{a \in x\beta y\gamma z} \mu_B^+(a), \lambda^+ \right\} \geq \min \{ \mu_B^+(y), \delta^+ \}$$

and $\min \left\{ \sup_{a \in x\beta y\gamma z} \mu_B^-(a), \lambda^- \right\} \leq \max \{ \mu_B^-(y), \delta^- \},$

for all $x, y, z \in H$ and $\gamma \in \Gamma$.

Definition 3.4 A bipolar (λ, δ) -fuzzy sub Γ -semihypergroup $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of H is called a bipolar (λ, δ) -fuzzy bi- Γ -hyperideal of H if

$$\max \left\{ \inf_{a \in x\alpha\beta y\gamma z} \mu_B^+(a), \lambda^+ \right\} \geq \min \{ \mu_B^+(x), \mu_B^+(z), \delta^+ \}$$

and $\min \left\{ \sup_{a \in x\alpha\beta y\gamma z} \mu_B^-(a), \lambda^- \right\} \leq \max \{ \mu_B^-(x), \mu_B^-(z), \delta^- \},$

for all $x, y, z \in H$ and $\gamma \in \Gamma$.

Example 3.5 Let $H = \{x, y, z, w\}$ and $\Gamma = \{\beta, \gamma\}$ be the sets of binary hyperoperations defined below:

β	x	y	z	w
x	x	$\{x, y\}$	$\{z, w\}$	w
y	$\{x, y\}$	$\{x, y\}$	$\{z, w\}$	w
z	$\{z, w\}$	$\{z, w\}$	z	w
w	w	w	w	w
γ	x	y	z	w
x	$\{x, y\}$	$\{x, y\}$	$\{z, w\}$	w
y	$\{x, y\}$	y	$\{z, w\}$	w
z	$\{z, w\}$	$\{z, w\}$	z	w
w	w	w	w	w

Clearly H is a Γ -semihypergroup. Now we define a bipolar fuzzy set $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ on H as:

$$\mu_B^+(t) = \begin{cases} 0.5 & \text{if } t \in \{x, y\} \\ 0.6 & \text{if } t = z \\ 0.8 & \text{if } t = w \end{cases}$$

and $\mu_B^-(t) = \begin{cases} -0.6 & \text{if } t \in \{x, y\} \\ -0.8 & \text{if } t = z \\ -0.9 & \text{if } t = w \end{cases}$

Then by routine calculation, $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (0.3, 0.4)-fuzzy bi- Γ -hyperideal of H .

Definition 3.6 A bipolar (λ, δ) -fuzzy sub Γ -semihypergroup $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of H is called a bipolar (1,2)-fuzzy bi- Γ -hyperideal of H if

$$\max \left\{ \inf_{a \in x\alpha w\beta(y\gamma z)} \mu_B^+(a), \lambda^+ \right\} \geq \min \{ \mu_B^+(x), \mu_B^+(y), \mu_B^+(z), \delta^+ \}$$

and $\min \left\{ \sup_{a \in x\alpha w\beta(y\gamma z)} \mu_B^-(a), \lambda^- \right\} \leq \max \{ \mu_B^-(x), \mu_B^-(y), \mu_B^-(z), \delta^- \},$

or all $x, y, z \in H$ and $\gamma \in \Gamma$.

Theorem 3.7 A bipolar fuzzy subset $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of a Γ -semihypergroup H is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H if and only if $\emptyset \neq H_B^{(t,s)}$ is a sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H for all $(s, t) \in [\delta^-, \lambda^-] \times [\lambda^+, \delta^+]$.

Proof. Let $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ be a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . Let $x, y \in H, (s, t) \in [\delta^-, \lambda^-] \times [\lambda^+, \delta^+]$ and $x, y \in H_B^{(t,s)}$. Then $\mu_B^+(x) \geq t$ and $\mu_B^+(y) \geq t$, also $\mu_B^-(x) \leq s$ and $\mu_B^-(y) \leq s$. As $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . Therefore,

$$\max \left\{ \inf_{z \in xy\gamma} \mu_B^+(z), \lambda^+ \right\} \geq \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \}$$

$$\geq \min \{ t, t, \delta^+ \} = t,$$

and

$$\min \left\{ \sup_{z \in xy\gamma} \mu_B^-(z), \lambda^- \right\} \leq \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}$$

$$\leq \max \{ s, s, \delta^- \} = s.$$

This implies that $\inf_{z \in xy\gamma} \mu_B^+(z) \geq t$ and $\sup_{z \in xy\gamma} \mu_B^-(z) \leq s$.

Thus $x\gamma y \subseteq H_B^{(t,s)}$. Hence $H_B^{(t,s)}$ is a sub Γ -semihypergroup of H .

Conversely, suppose that $H_B^{(t,s)}$ is a sub Γ -semi-hypergroup of H . Let $x, y \in H$ such that

$$\max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} < \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \},$$

and

$$\min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} > \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}.$$

Then there exist $(s, t) \in [\delta^-, \lambda^-] \times (\lambda^+, \delta^+]$ such that

$$\max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} < t \leq \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \},$$

and

$$\min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} > s \geq \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}.$$

This shows that $\mu_B^+(x) \geq t, \mu_B^+(y) \geq t$ and $\inf_{z \in xy} \mu_B^+(z) < t$, also $\mu_B^-(x) \leq s, \mu_B^-(y) \leq s$ and $\sup_{z \in xy} \mu_B^-(z) > s$.

Thus $x, y \in H_B^{(t,s)}$, since $H_B^{(t,s)}$ is a sub Γ -semihypergroup of H . Therefore $x\gamma z \subseteq H_B^{(t,s)}$ for some $z \in xy$, but this is a contradiction to $\inf_{z \in xy} \mu_B^+(z) < t$ and $\sup_{z \in xy} \mu_B^-(z) > s$. Thus,

$$\max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} \geq \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \},$$

and

$$\min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} \leq \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}.$$

Hence $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semi-hypergroup of H . The other cases can be seen in a similar way. \square

Corollary 3.8 Every bipolar fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H with $\lambda^+ = 0, \delta^+ = 1$ and $\lambda^- = 0, \delta^- = -1$.

Theorem 3.9 If a bipolar fuzzy subset $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H . Then the set $\mathcal{B}_\lambda = \langle \mathcal{B}_{\lambda^+}^+, \mathcal{B}_{\lambda^-}^- \rangle$ is a sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H , where $\mathcal{B}_{\lambda^+}^+ = \{x \in H \mid \mu_B^+(x) > \lambda^+\}$ and $\mathcal{B}_{\lambda^-}^- = \{x \in H \mid \mu_B^-(x) < \lambda^-\}$.

Proof. Suppose that $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . Let $x, y \in H$ such that $x, y \in \mathcal{B}_\lambda$. Then $\mu_B^+(x) > \lambda^+, \mu_B^+(y) > \lambda^+$ and $\mu_B^-(x) < \lambda^-, \mu_B^-(y) < \lambda^-$.

Since $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup, therefore

$$\begin{aligned} \max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} &\geq \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \} \\ &> \min \{ \lambda^+, \lambda^+, \delta^+ \} = \lambda^+, \end{aligned}$$

and

$$\begin{aligned} \min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} &\leq \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \} \\ &< \max \{ \lambda^-, \lambda^-, \delta^- \} = \lambda^-. \end{aligned}$$

Hence $\inf_{z \in xy} \mu_B^+(z) > \lambda^+$ and $\sup_{z \in xy} \mu_B^-(z) < \lambda^-$. This shows that $x\gamma y \subseteq \mathcal{B}_\lambda$, for $z \in xy$ and $\gamma \in \Gamma$. Hence \mathcal{B}_λ is a sub Γ -semihypergroup of H . The other cases can be seen in a similar way. \square

Theorem 3.10 A non-empty subset A of H is a sub Γ -semi-hypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H if and only if the bipolar fuzzy subset $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of H defined as follows:

$$\mu_B^+(x) = \begin{cases} \geq \delta^+ & \text{if } x \in A \\ \lambda^+ & \text{if } x \notin A, \end{cases}$$

and

$$\mu_B^-(x) = \begin{cases} \leq \delta^- & \text{if } x \in A \\ \lambda^- & \text{if } x \notin A, \end{cases}$$

is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H .

Proof. Suppose that A is a sub Γ -semihypergroup of H . Let $x, y \in H$ be such that $x, y \in A$, then $x\gamma y \subseteq A$ for $\gamma \in \Gamma$. Hence $\inf_{z \in xy} \mu_B^+(z) \geq \delta^+$ and $\sup_{z \in xy} \mu_B^-(z) \leq \delta^-$. Therefore

$$\max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} \geq \delta^+ = \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \},$$

and

$$\min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} \leq \delta^- = \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}.$$

If $x \notin A$ or $y \notin A$, then $\min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \} = \lambda^+$ and $\max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \} = \lambda^-$. Thus

$$\max \left\{ \inf_{z \in xy} \mu_B^+(z), \lambda^+ \right\} \geq \lambda^+ = \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \},$$

and

$$\min \left\{ \sup_{z \in xy} \mu_B^-(z), \lambda^- \right\} \leq \lambda^- = \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}.$$

Consequently $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H .

Conversely: Let $x, y \in A$. Then $\mu_{\mathcal{B}}^+(x) \geq \delta^+, \mu_{\mathcal{B}}^+(y) \geq \delta^+$ and $\mu_{\mathcal{B}}^-(x) \leq \delta^-, \mu_{\mathcal{B}}^-(y) \leq \delta^-$. As $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H , therefore

$$\begin{aligned} \max \left\{ \inf_{z \in xy\gamma} \mu_{\mathcal{B}}^+(z), \lambda^+ \right\} &\geq \min \left\{ \mu_{\mathcal{B}}^+(x), \mu_{\mathcal{B}}^+(y), \delta^+ \right\} \\ &\geq \min \left\{ \delta^+, \delta^+, \delta^+ \right\} = \delta^+, \end{aligned}$$

and

$$\begin{aligned} \min \left\{ \sup_{z \in xy\gamma} \mu_{\mathcal{B}}^-(z), \lambda^- \right\} &\leq \max \left\{ \mu_{\mathcal{B}}^-(x), \mu_{\mathcal{B}}^-(y), \delta^- \right\} \\ &\leq \max \left\{ \delta^-, \delta^-, \delta^- \right\} = \delta^-. \end{aligned}$$

This implies that $x\gamma y \subseteq A$. Hence A is a sub Γ -semihypergroup of H . The other cases can be seen in a similar way. \square

Theorem 3.11 A non-empty subset A of H is a sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H if and only if $\mathcal{B}_A = \langle \mu_{\mathcal{B}_A}^+, \mu_{\mathcal{B}_A}^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H .

Proof. Let A be a sub Γ -semihypergroup of H . Then $\mathcal{B}_A = \langle \mu_{\mathcal{B}_A}^+, \mu_{\mathcal{B}_A}^- \rangle$ is a bipolar fuzzy sub Γ -semihypergroup of H and by Corollary 3.8, \mathcal{B}_A is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H .

Conversely, let $x, y \in H$ be such that $x, y \in A$. Then $\mu_{\mathcal{B}_A}^+(x) = \mu_{\mathcal{B}_A}^+(y) = 1$ and $\mu_{\mathcal{B}_A}^-(x) = \mu_{\mathcal{B}_A}^-(y) = -1$. Since \mathcal{B}_A is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . Therefore

$$\begin{aligned} \max \left\{ \inf_{z \in xy\gamma} \mu_{\mathcal{B}_A}^+(z), \lambda^+ \right\} &\geq \min \left\{ \mu_{\mathcal{B}_A}^+(x), \mu_{\mathcal{B}_A}^+(y), \delta^+ \right\} \\ &= \min \left\{ 1, 1, \delta^+ \right\} = \delta^+, \end{aligned}$$

and

$$\begin{aligned} \min \left\{ \sup_{z \in xy\gamma} \mu_{\mathcal{B}_A}^-(z), \lambda^- \right\} &\leq \max \left\{ \mu_{\mathcal{B}_A}^-(x), \mu_{\mathcal{B}_A}^-(y), \delta^- \right\} \\ &= \max \left\{ -1, -1, \delta^- \right\} = \delta^-. \end{aligned}$$

It implies that $\inf_{z \in xy\gamma} \mu_{\mathcal{B}_A}^+(z) \geq \delta^+$ and $\sup_{z \in xy\gamma} \mu_{\mathcal{B}_A}^-(z) \leq \delta^-$.

Thus $z \in x\gamma y \subseteq A$ for $z \in xy\gamma$ and $\gamma \in \Gamma$. Therefore $\mathcal{B}_A = \langle \mu_{\mathcal{B}_A}^+, \mu_{\mathcal{B}_A}^- \rangle$ is a sub Γ -semihypergroup of H . The other cases can be seen in a similar way. \square

Theorem 3.12 Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ be a bipolar (λ, δ) -fuzzy left Γ -hyperideal and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be a bipolar (λ, δ) -fuzzy right Γ -hyperideal of a Γ -semihypergroup H . Then $\mathcal{A} \circ_{\Gamma} \mathcal{B}$ is a bipolar (λ, δ) -fuzzy Γ -hyperideal of H .

Proof. The proof is straightforward. \square

Lemma 3.13 Intersection of any family of bipolar (λ, δ) -fuzzy sub Γ -semihypergroups (resp., left Γ -hyperideals, right Γ -hyperideals, interior Γ -hyperideals, bi- Γ -hyperideals, (1, 2)- Γ -hyperideals) of a Γ -semihypergroup H is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H .

Proof. The proof is straightforward. \square

Now we prove that if $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ is a bipolar (λ, δ) -fuzzy Γ -hyperideal of H then $\mathcal{B}_{\lambda}^{\delta} = \langle \mu_{\mathcal{B}_{\lambda}^{\delta}}^+, \mu_{\mathcal{B}_{\lambda}^{\delta}}^- \rangle$ is also a bipolar (λ, δ) -fuzzy Γ -hyperideal of H .

Definition 3.14 Let $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be a bipolar fuzzy subset of a Γ -semihypergroup $H, \lambda^+, \delta^+ \in (0, 1]$ such that $\lambda^+ < \delta^+$. We define the bipolar fuzzy subset $\mathcal{B}_{\lambda}^{\delta} = \langle \mu_{\mathcal{B}_{\lambda}^{\delta}}^+, \mu_{\mathcal{B}_{\lambda}^{\delta}}^- \rangle$ of H as follows,

$$\begin{aligned} \mu_{\mathcal{B}_{\lambda}^{\delta}}^+(x) &= (\mu_{\mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ \text{ and } \mu_{\mathcal{B}_{\lambda}^{\delta}}^-(x) = (\mu_{\mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^-, \\ \text{for all } x &\in H. \end{aligned}$$

Definition 3.15 Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be bipolar fuzzy subsets of a Γ -semihypergroup H . Then we define,

(1) the bipolar fuzzy subset $\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B} = \langle \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^+, \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^- \rangle$ as follows:

$$\begin{aligned} \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^+(x) &= (\mu_{\mathcal{A} \wedge \mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ \text{ and } \\ \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^-(x) &= (\mu_{\mathcal{A} \wedge \mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^- \end{aligned}$$

(2) the bipolar fuzzy subset $\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B} = \langle \mu_{\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B}}^+, \mu_{\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B}}^- \rangle$ as follows:

$$\begin{aligned} \mu_{\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B}}^+(x) &= (\mu_{\mathcal{A} \vee \mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ \text{ and } \\ \mu_{\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B}}^-(x) &= (\mu_{\mathcal{A} \vee \mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^- \end{aligned}$$

(3) the bipolar fuzzy subset $\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B} = \langle \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^+, \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^- \rangle$ as follows:

$$\begin{aligned} \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^+(x) &= (\mu_{\mathcal{A} \circ \mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ \text{ and } \\ \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^-(x) &= (\mu_{\mathcal{A} \circ \mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^- \end{aligned}$$

for all $x \in H$.

Lemma 3.16 Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be bipolar fuzzy subsets of a Γ -semihypergroup H . Then the following holds:

- (1) $(\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}) = (\mathcal{A}_{\lambda}^{\delta} \wedge \mathcal{B}_{\lambda}^{\delta})$,
- (2) $(\mathcal{A} \vee_{\lambda}^{\delta} \mathcal{B}) = (\mathcal{A}_{\lambda}^{\delta} \vee \mathcal{B}_{\lambda}^{\delta})$,
- (3) $(\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}) \geq (\mathcal{A}_{\lambda}^{\delta} \circ_{\Gamma} \mathcal{B}_{\lambda}^{\delta})$.

Proof. The proof is straightforward. \square

Theorem 3.17 A bipolar fuzzy subset $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ of a Γ -semihypergroup H is a bipolar (λ, δ) -fuzzy,

- (1) sub Γ -semihypergroup of H if and only if $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$,
- (2) left Γ -hyperideal of H if and only if $\mathcal{H}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$,
- (3) right Γ -hyperideal of H if and only if $\mathcal{B}_{\lambda}^{\circ\delta} H \subseteq \mathcal{B}_{\lambda}^{\delta}$,
- (4) bi- Γ -hyperideal of H if and only if $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$ and $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{H}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$,
- (5) interior Γ -hyperideal of H if and only if $\mathcal{H}_{\lambda}^{\circ\delta} \mathcal{B}_{\lambda}^{\circ\delta} \mathcal{H} \subseteq \mathcal{B}_{\lambda}^{\delta}$,
- (6) (1, 2)- Γ -hyperideal of H if and only if $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$ and $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{H}_{\lambda}^{\circ\delta} \mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$.

Proof. (1) Let $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ be a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H and $z \in H$. Let us suppose that $z \in x\gamma y$ for $x, y \in H$ and $\gamma \in \Gamma$. Then

$$\begin{aligned} \mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^+(z) &= (\mu_{\mathcal{B}_{\circ\Gamma} \mathcal{B}}^+(z) \wedge \delta^+) \vee \lambda^+ \\ &= \left(\left\{ \bigvee_{z \in x\gamma y} \{ \mu_B^+(x) \wedge \mu_B^+(y) \} \right\} \wedge \delta^+ \right) \vee \lambda^+ \\ &= \left(\left\{ \bigvee_{z \in x\gamma y} \{ \mu_B^+(x) \wedge \mu_B^+(y) \wedge \delta^+ \} \right\} \wedge \delta^+ \right) \vee \lambda^+ \\ &\leq \left(\left\{ \bigvee_{z \in x\gamma y} \left(\inf_{z \in x\gamma y} \mu_B^+(z) \vee \lambda^+ \right) \right\} \wedge \delta^+ \right) \vee \lambda^+ \\ &= \left(\left\{ \bigvee_{z \in x\gamma y} \left(\inf_{z \in x\gamma y} \mu_B^+(z) \wedge \delta^+ \right) \right\} \vee \lambda^+ \right) \vee \lambda^+ \\ &\leq \left(\bigvee_{z \in x\gamma y} (\mu_B^+(x\gamma y) \wedge \delta^+) \right) \vee \lambda^+ \vee \lambda^+ \\ &= (\mu_B^+(z) \wedge \delta^+) \vee \lambda^+ = \mu_{\mathcal{B}_{\lambda}^{\delta}}^+(z), \end{aligned}$$

and

$$\begin{aligned} \mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^-(z) &= (\mu_{\mathcal{B}_{\circ\Gamma} \mathcal{B}}^-(z) \vee \delta^-) \wedge \lambda^- \\ &= \left(\left\{ \bigwedge_{z \in x\gamma y} \{ \mu_B^-(x) \vee \mu_B^-(y) \} \right\} \vee \delta^- \right) \wedge \lambda^- \\ &= \left(\left\{ \bigwedge_{z \in x\gamma y} \{ \mu_B^-(x) \vee \mu_B^-(y) \vee \delta^- \} \right\} \vee \delta^- \right) \wedge \lambda^- \\ &\geq \left(\left\{ \bigwedge_{z \in x\gamma y} \left(\sup_{z \in x\gamma y} \mu_B^-(z) \wedge \lambda^- \right) \right\} \vee \delta^- \right) \wedge \lambda^- \\ &= \left(\left\{ \bigwedge_{z \in x\gamma y} \left(\sup_{z \in x\gamma y} \mu_B^-(z) \vee \delta^- \right) \right\} \wedge \lambda^- \right) \wedge \lambda^- \\ &\geq \left(\bigwedge_{z \in x\gamma y} (\mu_B^-(x\gamma y) \vee \delta^-) \right) \wedge \lambda^- \wedge \lambda^- \\ &= (\mu_B^-(z) \vee \delta^-) \wedge \lambda^- = \mu_{\mathcal{B}_{\lambda}^{\delta}}^-(z). \end{aligned}$$

If there do not exist any $x, y \in H$ and $\gamma \in \Gamma$ such that $z \in x\gamma y$, then

$$\mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^+(x) = \lambda^+ \leq \mu_{\mathcal{B}_{\lambda}^{\delta}}^+(x),$$

and

$$\mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^-(x) = \lambda^- \geq \mu_{\mathcal{B}_{\lambda}^{\delta}}^-(x).$$

Hence $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$.

Conversely, assume that $\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B} \subseteq \mathcal{B}_{\lambda}^{\delta}$. If there exist $m, n \in H$ and $\beta \in \Gamma$ such that $x\gamma y \subseteq m\beta n$, then

$$\begin{aligned} &\max \{ \mu_B^+(x\gamma y), \lambda^+ \} \\ &\geq (\mu_B^+(x\gamma y) \wedge \delta^+) \vee \lambda^+ = \mu_{\mathcal{B}_{\lambda}^{\delta}}^+(x\gamma y) \geq \mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^+(x\gamma y) \\ &= \left(\left\{ \bigvee_{x\gamma y \subseteq m\beta n} \{ \mu_B^+(m) \wedge \mu_B^+(n) \} \right\} \wedge \delta^+ \right) \vee \lambda^+ \\ &= x\gamma y \subseteq m\beta n \{ \mu_B^+(m) \wedge \mu_B^+(n) \} \wedge \delta^+ \vee \lambda^+ \\ &\geq ((\mu_B^+(x) \wedge \mu_B^+(y)) \wedge \delta^+) \vee \lambda^+ \\ &\geq \mu_B^+(x) \wedge \mu_B^+(y) \wedge \delta^+ \\ &\max \left\{ \inf_{z \in x\gamma y} \mu_B^+(z), \lambda^+ \right\} \\ &\geq \min \{ \mu_B^+(x), \mu_B^+(y), \delta^+ \}, \end{aligned}$$

and

$$\begin{aligned} &\min \{ \mu_B^-(x\gamma y), \lambda^- \} \\ &\leq (\mu_B^-(x\gamma y) \vee \delta^-) \wedge \lambda^- = \mu_{\mathcal{B}_{\lambda}^{\delta}}^-(x\gamma y) \leq \mu_{\mathcal{B}_{\lambda}^{\circ\delta} \mathcal{B}}^-(x\gamma y) \\ &= \left(\left\{ \bigwedge_{x\gamma y \subseteq m\beta n} \{ \mu_B^-(m) \vee \mu_B^-(n) \} \right\} \vee \delta^- \right) \wedge \lambda^- \\ &\leq ((\mu_B^-(x) \vee \mu_B^-(y)) \vee \delta^-) \wedge \lambda^- \\ &\leq \mu_B^-(x) \vee \mu_B^-(y) \vee \delta^- \\ &\min \left\{ \sup_{z \in x\gamma y} \mu_B^-(z), \lambda^- \right\} \\ &\leq \max \{ \mu_B^-(x), \mu_B^-(y), \delta^- \}. \end{aligned}$$

Hence $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . The proofs of (2), (3), (4), (5) and (6) are similar to the proof of (1). \square

Proposition 3.18 If $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of a Γ -semihypergroup H , then $\mathcal{B}_{\lambda}^{\delta} = \langle \mu_{\mathcal{B}_{\lambda}^{\delta}}^+, \mu_{\mathcal{B}_{\lambda}^{\delta}}^- \rangle$ is also a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup (resp., left Γ -hyperideal, right Γ -hyperideal, interior Γ -hyperideal, bi- Γ -hyperideal, (1, 2)- Γ -hyperideal) of H .

Proof. Suppose $\mathcal{B} = \langle \mu_B^+, \mu_B^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semihypergroup of H . Let $x, y \in H$ and $\gamma \in \Gamma$.

Then

$$\begin{aligned} &\max \left\{ \inf_{z \in x\gamma y} \mu_{\mathcal{B}_{\lambda}^{\delta}}^+(z), \lambda^+ \right\} \\ &= \left(\inf_{z \in x\gamma y} \{ (\mu_B^+(z) \wedge \delta^+) \vee \lambda^+ \} \right) \vee \lambda^+ \\ &= \left(\inf_{z \in x\gamma y} \mu_B^+(z) \wedge \delta^+ \right) \vee \lambda^+ \\ &= \left(\inf_{z \in x\gamma y} \mu_B^+(z) \vee \lambda^+ \right) \wedge (\delta^+ \vee \lambda^+) \end{aligned}$$

$$\begin{aligned}
 &= \left(\inf_{z \in x\gamma y} \mu_B^+(z) \vee \lambda^+ \right) \wedge \delta^+ \\
 &= \left(\left(\inf_{z \in x\gamma y} \mu_B^+(z) \vee \lambda^+ \right) \vee \lambda^+ \right) \wedge \delta^+ \\
 &\geq \left((\mu_B^+(x) \wedge \mu_B^+(y) \wedge \delta^+) \vee \lambda^+ \right) \wedge \delta^+ \\
 &= \left((\mu_B^+(x) \wedge \mu_B^+(y) \wedge \delta^+ \wedge \delta^+) \vee \lambda^+ \right) \wedge \delta^+ \\
 &= \left(((\mu_B^+(x) \wedge \delta^+) \wedge (\mu_B^+(y) \wedge \delta^+)) \vee \lambda^+ \right) \wedge \delta^+ \\
 &= \left(((\mu_B^+(x) \wedge \delta^+) \vee \lambda^+) \wedge ((\mu_B^+(y) \wedge \delta^+) \vee \lambda^+) \right) \wedge \delta^+ \\
 &= \mu_{B_\lambda^\delta}^+(x) \wedge \mu_{B_\lambda^\delta}^+(y) \wedge \delta^+ = \min \left\{ \mu_{B_\lambda^\delta}^+(x), \mu_{B_\lambda^\delta}^+(y), \delta^+ \right\},
 \end{aligned}$$

and

$$\begin{aligned}
 &\min \left\{ \sup_{z \in x\gamma y} \mu_{B_\lambda^\delta}^-(z), \lambda^- \right\} \\
 &= \left(\sup_{z \in x\gamma y} \left\{ (\mu_B^-(z) \vee \delta^-) \wedge \lambda^- \right\} \right) \wedge \lambda^- \\
 &= \left(\sup_{z \in x\gamma y} \mu_B^-(z) \vee \delta^- \right) \wedge \lambda^- \\
 &= \left(\sup_{z \in x\gamma y} \mu_B^-(z) \wedge \lambda^- \right) \vee (\delta^- \wedge \lambda^+) \\
 &= \left(\sup_{z \in x\gamma y} \mu_B^-(z) \wedge \lambda^- \right) \vee \delta^- \\
 &= \left(\left(\sup_{z \in x\gamma y} \mu_B^-(z) \wedge \lambda^- \right) \wedge \lambda^- \right) \vee \delta^- \\
 &\leq \left((\mu_B^-(x) \vee \mu_B^-(y) \vee \delta^-) \wedge \lambda^- \right) \vee \delta^- \\
 &= \left((\mu_B^-(x) \vee \mu_B^-(y) \vee \delta^- \vee \delta^-) \wedge \lambda^- \right) \vee \delta^- \\
 &= \left(((\mu_B^-(x) \vee \delta^-) \vee (\mu_B^-(y) \vee \delta^-)) \wedge \lambda^- \right) \vee \delta^- \\
 &= \left(((\mu_B^-(x) \vee \delta^-) \wedge \lambda^-) \vee ((\mu_B^-(y) \vee \delta^-) \wedge \lambda^-) \right) \vee \delta^- \\
 &= \mu_{B_\lambda^\delta}^-(x) \vee \mu_{B_\lambda^\delta}^-(y) \vee \delta^- = \max \left\{ \mu_{B_\lambda^\delta}^-(x), \mu_{B_\lambda^\delta}^-(y), \delta^- \right\}.
 \end{aligned}$$

Hence $B_\lambda^\delta = \langle \mu_{B_\lambda^\delta}^+, \mu_{B_\lambda^\delta}^- \rangle$ is a bipolar (λ, δ) -fuzzy sub Γ -semi-hypergroup of H . The other cases can be seen in a similar way. \square

Theorem 3.19 Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ be a bipolar (λ, δ) -fuzzy right Γ -hyperideal and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be a bipolar (λ, δ) -fuzzy left Γ -hyperideal of H . Then $\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B} \subseteq \mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}$.

Proof. Let $\mathcal{A} = \langle \mu_{\mathcal{A}}^+, \mu_{\mathcal{A}}^- \rangle$ be a bipolar (λ, δ) -fuzzy right Γ -hyperideal and $\mathcal{B} = \langle \mu_{\mathcal{B}}^+, \mu_{\mathcal{B}}^- \rangle$ be a bipolar (λ, δ) -fuzzy left Γ -hyperideal of H . Then for all $z \in H$, we have

$$\begin{aligned}
 &\mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^+(z) \\
 &= (\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^+(z) \wedge \delta^+) \vee \lambda^+
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\left\{ \bigvee_{z \in x\gamma y} \{ \mu_{\mathcal{A}}^+(x) \wedge \mu_{\mathcal{B}}^+(y) \} \wedge \delta^+ \right\} \vee \lambda^+ \right) \\
 &= \left(\left\{ \bigvee_{z \in x\gamma y} \{ \mu_{\mathcal{A}}^+(x) \wedge \mu_{\mathcal{B}}^+(y) \} \wedge \delta^+ \wedge \delta^+ \right\} \vee \lambda^+ \right) \\
 &= \left(\left\{ \bigvee_{z \in x\gamma y} \{ (\mu_{\mathcal{A}}^+(x) \wedge \delta^+) \wedge (\mu_{\mathcal{B}}^+(y) \wedge \delta^+) \} \wedge \delta^+ \right\} \vee \lambda^+ \right) \\
 &\leq \left(\left\{ \bigvee_{z \in x\gamma y} \left\{ \left(\inf_{z \in x\gamma y} \mu_{\mathcal{A}}^+(z) \vee \lambda^+ \right) \wedge \left(\inf_{z \in x\gamma y} \mu_{\mathcal{B}}^+(z) \vee \lambda^+ \right) \right\} \wedge \delta^+ \right\} \vee \lambda^+ \right) \\
 &= \left((\mu_{\mathcal{A}}^+(z) \vee \lambda^+) \wedge (\mu_{\mathcal{B}}^+(z) \vee \lambda^+) \wedge \delta^+ \right) \vee \lambda^+ \\
 &= \left(((\mu_{\mathcal{A}}^+(z) \wedge \mu_{\mathcal{B}}^+(z)) \vee \lambda^+) \wedge \delta^+ \right) \vee \lambda^+ \\
 &= \left\{ (\mu_{\mathcal{A}}^+ \wedge \mu_{\mathcal{B}}^+)(z) \wedge \delta^+ \right\} \vee \lambda^+ = \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^+(z),
 \end{aligned}$$

and

$$\begin{aligned}
 &\mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^-(z) \\
 &= (\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^-(z) \vee \delta^-) \wedge \lambda^- \\
 &= \left(\left\{ \bigwedge_{z \in x\gamma y} \{ \mu_{\mathcal{A}}^-(x) \vee \mu_{\mathcal{B}}^-(y) \} \vee \delta^- \right\} \wedge \lambda^- \right) \\
 &= \left(\left\{ \bigwedge_{z \in x\gamma y} \{ \mu_{\mathcal{A}}^-(x) \vee \mu_{\mathcal{B}}^-(y) \} \vee \delta^- \vee \delta^- \right\} \wedge \lambda^- \right) \\
 &= \left(\left\{ \bigwedge_{z \in x\gamma y} \{ (\mu_{\mathcal{A}}^-(x) \vee \delta^-) \vee (\mu_{\mathcal{B}}^-(y) \vee \delta^-) \} \vee \delta^- \right\} \wedge \lambda^- \right) \\
 &\geq \left(\left\{ \bigwedge_{z \in x\gamma y} \left\{ \left(\sup_{z \in x\gamma y} \mu_{\mathcal{A}}^-(z) \wedge \lambda^- \right) \vee \left(\sup_{z \in x\gamma y} \mu_{\mathcal{B}}^-(z) \wedge \lambda^- \right) \right\} \vee \delta^- \right\} \wedge \lambda^- \right) \\
 &= \left((\mu_{\mathcal{A}}^-(z) \wedge \lambda^-) \vee (\mu_{\mathcal{B}}^-(z) \wedge \lambda^-) \vee \delta^- \right) \wedge \lambda^- \\
 &= \left(((\mu_{\mathcal{A}}^-(z) \vee \mu_{\mathcal{B}}^-(z)) \wedge \lambda^-) \vee \delta^- \right) \wedge \lambda^- \\
 &= \left\{ (\mu_{\mathcal{A}}^- \vee \mu_{\mathcal{B}}^-)(z) \vee \delta^- \right\} \wedge \lambda^- = \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^-(z).
 \end{aligned}$$

If there do not exist any $x, y \in H$ and $\gamma \in \Gamma$ such that $z \in x\gamma y$, then

$$\begin{aligned}
 \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^+(x) &= (\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ = 0 \vee \lambda^+ \\
 &= \lambda^+ \leq (\mu_{\mathcal{A} \wedge \mathcal{B}}^+(x) \wedge \delta^+) \vee \lambda^+ = \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^+(x),
 \end{aligned}$$

and

$$\begin{aligned}
 \mu_{\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B}}^-(x) &= (\mu_{\mathcal{A} \circ_{\Gamma} \mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^- = 0 \wedge \lambda^- \\
 &= \lambda^- \geq (\mu_{\mathcal{A} \vee \mathcal{B}}^-(x) \vee \delta^-) \wedge \lambda^- = \mu_{\mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}}^-(x).
 \end{aligned}$$

Hence we get $\mathcal{A} \circ_{\lambda}^{\delta} \mathcal{B} \subseteq \mathcal{A} \wedge_{\lambda}^{\delta} \mathcal{B}$. \square

References

Abdullah, S., Aslam, M. and Anwar, T. 2011. A note on M-hypersystems and N-hypersystems in Γ -semihypergroups. Quasigroups and Related Systems. 19, 101-104.

Abdullah, S., Hila, K. and Aslam, M. 2012. On bi- Γ -hyperideals of Γ -semihypergroups. U.P.B. Scientific Bulletin, Series A. 74(4), 79-90.

Akram, M. 2011. Bipolar fuzzy L-Lie algebras. World Applied Sciences Journal. 14(12), 1908-1913.

- Akram, M., Chen, W. and Lin, Y. 2012a. Bipolar fuzzy Lie superalgebras. *Quasigroups and Related Systems*. 20, 139-156.
- Akram, M. and Al-Shehrie, N.O. 2012b. Bipolar fuzzy Lie ideals. *Utilitas Mathematica*. 87, 265-278.
- Anvariye, S.M., Mirvakili, S. and Davvaz, B. 2010. On Γ -hyperideals in Γ -semihypergroups. *Carpathian Journal of Mathematics*. 26(1), 11-23.
- Aslam, M., Abdullah, S., Davvaz, B. and Yaqoob, N. 2012. Rough M-hypersystems and fuzzy M-hypersystems in Γ -semihypergroups. *Neural Computing and Applications*. 21(1), 281-287.
- Corsini, P. and Leoreanu, V. 2003. *Applications of hyperstructure theory*. Kluwer Academic Publications, The Netherlands.
- Corsini, P., Shabir, M. and Mahmood, T. 2011. Semisimple semihypergroups in terms of hyperideals and fuzzy hyperideals. *Iranian Journal of Fuzzy Systems*. 8(1), 95-111.
- Coumaressane, S. 2010. Near-rings characterized by their (λ, δ) -fuzzy quasi-ideals. *International Journal of Computational Cognition*. 8, 5-11.
- Davvaz, B. 2010. A survey of fuzzy algebraic hyperstructures. *Algebra Groups and Geometries*. 27(1), 37-62.
- Davvaz, B. 2000. Fuzzy hyperideals in semihypergroups. *Italian Journal of Pure and Applied Mathematics*. 8, 67-74.
- Davvaz, B. 2006. Intuitionistic hyperideals of semihypergroups. *Bulletin of the Malaysian Mathematical Sciences Society*. 29(1), 203-207.
- Davvaz, B. 2007. Extensions of fuzzy hyperideals in H_v -semigroups. *Soft Computing*. 11(9), 829-837.
- Ersoy, B.A., Saricaoglu, Y., Yenigun, M. and Davvaz, B. 2012. On fuzzy interior Γ -hyperideals of Γ -semihypergroups. *Utilitas Mathematica*. 88, 157-170.
- Ersoy, B.A. and Davvaz, B. 2013a. Structure of intuitionistic fuzzy sets in Γ -semihypergroups. *Abstract and Applied Analysis*. 2013, 9 pages.
- Ersoy, B.A. and Davvaz, B. 2013b. Atanassov's intuitionistic fuzzy Γ -hyperideals of Γ -semihypergroups. *Journal of Intelligent and Fuzzy Systems*. 25(2), 463-470.
- Heidari, D., Dehkordi, S.O. and Davvaz, B. 2010. Γ -semihypergroups and their properties. *U.P.B. Scientific Bulletin, Series A*. 72, 197-210.
- Hila, K., Davvaz, B. and Dine, J. 2012. Study on the structure of \hat{U} -semihypergroups. *Communications in Algebra*. 40, 2932-2948.
- Hila, K. and Abdullah, S. 2014. A study on intuitionistic fuzzy sets in Γ -semihypergroups. *Journal of Intelligent and Fuzzy Systems*. 26(4), 1695-1710.
- Jun, Y.B. and Park, C.H. 2009. Filters of BCH-algebras based on bi-polar-valued fuzzy sets. *International Mathematical Forum*. 13, 631-643.
- Kehayopulu, N. and Tsingelis, M. 2007. Fuzzy ideals in ordered semigroups. *Quasigroups and Related Systems*. 15, 279-289.
- Khan, F.M., Sarmin, N.H. and Khan, A. 2012. Some new characterization of ordered semigroups in terms of (λ, θ) -fuzzy bi-ideals. *International Journal of Algebra and Statistics*. 1(1), 22-32.
- Kuroki, N. 1979. Fuzzy bi-ideals in semigroups. *Commentarii Mathematici Universitatis Sancti Pauli*. 28, 17-21.
- Kuroki, N. 1981. On fuzzy ideals and fuzzy bi-ideals in semigroups. *Fuzzy Sets and Systems*. 5(2), 203-215.
- Lee, K.M. 2000. Bi-polar-valued fuzzy sets and their operations. *Proceeding of the International Conference on Intelligent Technologies, Bangkok, Thailand*. 307-312.
- Lee, K.M. 2004. Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bi-polar-valued fuzzy sets. *Journal of Fuzzy Logic and Intelligent Systems*. 14, 125-129.
- Lee, K.J. 2009. Bi-polar fuzzy subalgebras and bi-polar fuzzy ideals of BCK/BCI-algebras. *Bulletin of the Malaysian Mathematical Sciences Society*. 32(3), 361-373.
- Li, B. and Feng, Y. 2013. Intuitionistic (λ, μ) -fuzzy sets in Γ -semigroups. *Journal of Inequalities and Applications*. 107, 9 pages.
- Marty, F. 1934. Sur une generalization de la notion de group. *Proceeding of the 8th Congress Mathematics Scandinaves, Stockholm*. 45-49.
- Mirvakili, S., Anvariye, S.M. and Davvaz, B. 2013. Γ -semihypergroups and regular relations. *Journal of Mathematics*. Article ID 915250, 9 pages.
- Rosenfeld, A. 1971. Fuzzy groups. *Journal of Mathematical Analysis and Applications*. 35, 512-517.
- Saeid, A.B. 2009. Bipolar-valued fuzzy BCK/BCI-algebras. *World Applied Sciences Journal*. 7(11), 1404-1411.
- Shabir, M. 2005. Fully fuzzy prime semigroups. *International Journal of Mathematics and Mathematical Sciences*. 1, 163-168.
- Shabir, M., Nawaz, Y. and Aslam, M. 2011. Semigroups characterized by the properties of their fuzzy ideals with thresholds. *World Applied Sciences Journal*. 14(12), 1851-1865.
- Shabir, M. and Mahmood, T. 2013. Semihypergroups Characterized by $(\epsilon, \in \vee qk)$ -fuzzy Hyperideals. *Information Sciences Letters*. 2(2), 101-121.
- Shabir, M. and Mahmood, T. 2015. Semihypergroups characterized by $(\in \gamma, \in \gamma \vee q\delta)$ -fuzzy hyperideals. *Journal of Intelligent and Fuzzy Systems*. 28(6), 2667-2678.
- Yao, B. 2005. (λ, θ) -fuzzy normal sub.elds and (λ, θ) -fuzzy quotient sub.elds. *The Journal of Fuzzy Mathematics*. 13(3), 695-705.
- Yaqoob, N., Aslam, M., Davvaz, B. and Ghareeb, A. 2014. Structures of bipolar fuzzy Γ -hyperideals in Γ -semihypergroups. *Journal of Intelligent and Fuzzy Systems*. 27(6), 3015-3032.
- Yaqoob, N. and Ansari, M.A. 2013. Bipolar (λ, δ) -fuzzy ideals in ternary semigroups. *International Journal of Mathematical Analysis*. 7(36), 1775-1782.
- Zadeh, L.A. 1965. Fuzzy sets. *Information and Control*. 8, 338-353.