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Original Article

Optimum cost analysis of batch service retrial queuing system with server failure, threshold and multiple vacations

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Abstract

The aim of this paper is to analyze the queuing model entitled to cost optimization in bulk arrival and a batch service retrial queuing system with threshold, server failure, non-disruptive service, and multiple vacations. When bulk arrival of customers find the server is busy, then all customers will join in the orbit. On the other hand, if the server is free, then batch service will be provided according to the general bulk service rule. Batch size varies from a minimum of one and a maximum of 'b' number of customers. Customers in the orbit seek service one by one through constant retrial policy whenever the server is in idle state. The server may encounter failure during service. If the server fails, then 'renewal of service station' will be considered with probability δ. If there is no server failure with probability $1-\delta$ in the service completion or after the renewal process and if the orbit is empty, the server then leaves for multiple vacations. The server stays on vacation until the orbit size reaches the value N. For this proposed queuing model, a probability generating function of the orbit size will be obtained by using the supplementary variable technique and various performance measures will be presented with suitable numerical illustrations. A real time application is also discussed for this system. Additionally, a cost effective model is developed for this queuing model.

Keywords: bulk arrival, batch service, constant retrial policy, server failure, threshold, renewal time

1. Introduction

Mathematical modeling of a retrial queuing system with vacations is very useful in dealing with real life congestion problems like local area networks (LAN), communication networks, and media access protocols. In modern technology, communication networks play a vital role in transmitting and accessing data from anywhere at any time. A retrial queuing system is characterized by the arrival of customers that find a busy server and leave the service area but after some random delay they request service again. If the customer finds the server is busy then he joins an orbit which is defined as a virtual queue formed by the customers after finding that the server is busy.

Performance analysis of a LAN executing under transmission protocol CSMA-CD (Carrier Sense Multiple Access with Collision Detection) is one of the applications of our proposed queuing model. In order to remit data, any moderator on the segment of CSMA-CD is used to investigate whether the transmission channel (a bus) is free or not, to avoid collisions between the data. Moderator A transmits messages to another moderator through the transmission medium (server in our model). The messages are split into different packets (batch) in order to transmit to the destination station. First, Moderator A checks whether the bus is free or not. If the transmission medium is free, then a group of packets is picked for transmission and the surplus is stored in a buffer (retrial group). On the contrary, if the bus is busy, then all of the packets are stored in the buffer and Moderator A will retry the transmission later on. Sometimes while transmitting data, the server may be infected with a virus (server failure) which results in slow performance of the server. Though the server fails, service will not be interrupted, but will continue for the current batch of packets by including

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antivirus software. When the transmission medium fails, antivirus software will get stimulated immediately and helps in transmitting the data. The virus will be removed after the data transmission (renewal period). When the server is idle, maintenance activities (multiple vacations) such as temporary files can be cleaned to keep the server functioning well. This type of maintenance can be programmed to perform on a regular basis. This can be designed as a bulk arrival and batch service queuing model with server failure, non-disruptive service, and multiple vacations.

Analytical treatment of different models of retrial queues has been extensively given by Falin and Templeton (1997). A brief survey and an overview of retrial queues were explained by Artalejo (1999). Retrial queuing systems with vacations and breakdown were analyzed by many researchers which included a study on retrial queue with constant retrial rate and server breakdown by Li and Zhao (2005). Atencia, Bouza, and Moreno (2008) derived generating functions of system and orbit state of the bulk arrival retrial queue with server breakdown. Also they considered constant failure rate of the server. Chang and Ke (2009) used a supplementary variable technique to derive some important results in a batch arrival retrial queue with modified vacations. The M/G/1 retrial queue with a breakdown period and delay period were analyzed by Choudhury and Ke (2014). They used Bernoulli schedule vacation and derived system size at a departure time epoch. Choudhury, Tadj, and Deka (2010) analysed the $M^X/G/1$ retrial queuing system with optional two phases of service and breakdown. In this paper, delay time was also introduced. Yang and Wu (2015) studied a working vacation queuing model with threshold and server failure. In their work cost minimization was carried out.

In all of the above queuing models, customers were served one by one. But in many real-time applications it is essential to provide batch service too. Extensive review on a classical bulk arrival and batch service queuing model was given by Niranjan and Indhira (2016). Bulk arrival and batch service retrial queueing systems have been analyzed by Haridass, Arumuganathan, and Senthilkumar (2012). They used supplementary variable techniques, derived some important performance measures, and developed cost effective models for their proposed system.

In the literature of bulk arrival retrial queuing models, only a few authors studied bulk arrival and batch service retrial queuing models. The bulk arrival and batch service retrial queuing models with multiple vacations were not considered. Once the server breaks down, the service stops in all of the bulk arrival retrial queuing models with breakdown under consideration. But in this proposed model, though the server encounters failure, service will not stop but will continue for the current batch through some precaution in technical arrangements. The server will be repaired after completion of the service which is called the 'renewal period of the server'. The model under consideration is peculiar because multiple vacations with threshold and server failure with non-disruptive service are used to model the proposed bulk arrival and batch service retrial queuing system.

2. Model Description

This paper analyses a bulk arrival and batch service queuing model with threshold, server failure, multiple vacations, and constant retrial policy. Customers enter into the system in bulk according to the Poisson process with rate λ. Upon arrival, if the server is busy then all customers choose to join the virtual queue called orbit. Customers in the orbit request service again after some time. On the contrary, if the customers find that the server is free then batch service will be provided with a minimum of '1' and a maximum of 'b' number of customers. Let ξ be the queue length. If $1 \leq \xi \leq b$, then the entire batch will be served immediately. Additionally if $\xi > b$, then service will be provided for only 'b' customers. The remaining $\xi - b$ customers will join the orbit. Since this proposed system follows constant retrial policy, customers in the orbit explore service one by one with constant retrial rate 'γ'. The server may encounter failure while serving customers. This paper proposes a concept called server failure without service interruption. Though the server encounters failure the service will not be stopped, but will continue for the current batch by doing some technical precaution arrangements.

Proper maintenance of the server or repair of the server is defined as renewal of service station. When the server encounters failure with probability δ then the renewal of service station will be considered. After completing a renewal of service station or when there is no server failure with probability $1-\delta$ and the orbit size is zero, then the server leaves for vacation. If the orbit size is less than 'N' upon return from a vacation, then the server leaves for another vacation. Likewise, the server continuously goes for vacation (multiple vacations) until the orbit size reaches the threshold value 'N' (N > b). At a vacation completion time, if the orbit size reaches the threshold value 'N', then the server becomes idle in the system to provide service for customers from the primary source or orbit. The model under consideration is schematically represented in Figure 1.

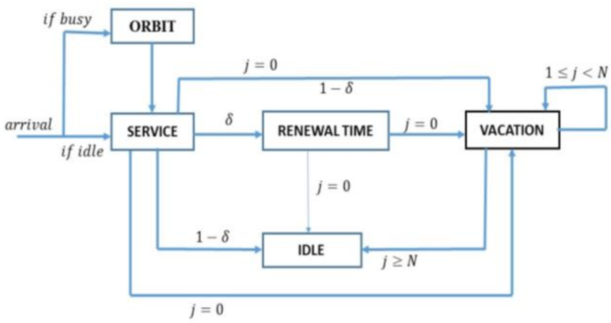

Figure 1. Schematic representation of the queuing model: J-orbit size

2.1. Notations

Let λ be the Poisson arrival rate, X be the group size random variable of the arrival, g_k be the probability that 'k' customers arrive in a batch, X (z) be the probability generating function of X, N_q(t) be the number of customers waiting for service at time t, N_s(t) be the number of customers under the service at time t, and $N(t)$ be the number of customers in the orbit at time t.

Let 'γ' be the retrial rate of the customer from the orbit and 'δ' be the probability of server failure. Let S(x) (s(x)) {$\tilde{s}(\theta)$} [$s^0(x)$] be the cumulative distribution function (probability density function) {Laplace-Stieltjes transform} [remaining service time] of service. Let V(x) (v(x) {($\tilde{v}(\theta)$}[$V^0(x)$] be the cumulative distribution function (probability density function){Laplace-Stieltjes transform} [remaining vacation time] of vacation. Let B(x)(b(x)){$\tilde{B}$ (θ)} [$B^0(x)$] be the cumulative distribution function (probability density function){Laplace-Stieltjes transform} [remaining renewal time] of renewal.

Let G (t) denotes different states of the server at time t, and define

$$G(t)=\begin{cases}0, \text{ if the server is busy with service}\\1, \text{ if the server is on vacation}\\2, \text{ if the server is on renewal}\\3, \text{ if the server is idle}\end{cases}$$

Let $C(t) = m$ be the server is on mth vacation.

The state probabilities are defined to obtain governing equations:

$$A_{ij}(x,t)dt = Pr\{N_s(t) = i, N_q(t) = j, x \le S^0(t) \le x + dt, G(t) = 0\}; 1 \le i \le b, j \ge 0$$
$$Q_{jn}(t)dt = Pr\{N(t) = n, x \le V^0(t) \le x + dt, G(t) = 1, C(t) = j\}, \quad n \ge 0$$
$$B_n(t)dt = Pr\{N(t) = n, x \le B^0(t) \le x + dt, G(t) = 2\}, n \ge 0$$
$$D_n(t)dt = Pr\{N(t) = n, G(t) = 3\},$$

3. Steady State Orbit Size Distribution

By using supplementary variable technique and using remaining service time as a supplementary variable the following equations are obtained.

$$0 = -(\lambda+\gamma)D_j + B_n(0) + (1-\delta)\sum_{m=1}^{b} A_{mj}(0) \qquad 1 \le j \le N-1 \qquad (1)$$

$$0 = -(\lambda+\gamma)D_j + B_n(0) + (1-\delta)\sum_{m=1}^{b} A_{mj}(0) + \sum_{l=1}^{\infty} Q_{lj}(0) \qquad j \ge N \qquad (2)$$

$$-\frac{d}{dx}A_{1j}(x) = -\lambda A_{1j}(x) + \gamma D_{j+1}s(x) + D_j\lambda g_1 s(x) \qquad j \ge 0 \qquad (3)$$

$$-\frac{d}{dx}A_{i0}(x) = -\lambda A_{1j}(x) + D_0\lambda g_i s(x) \qquad 2 \le i \le b \qquad (4)$$

$$-\frac{d}{dx}A_{ij}(x) = -\lambda A_{ij}(x) + D_j\lambda g_i s(x) + \sum_{k=1}^{j} A_{i\,j-k}(x)\lambda g_k s(x) \qquad 2 \le i \le b-1,\ j \ge 1 \qquad (5)$$

$$-\frac{d}{dx}A_{bj}(x) = -\lambda A_{bj}(x) + \sum_{k=1}^{j} A_{b\,j-k}(x)\lambda g_k + \sum_{k=0}^{j} D_{j-k}\,\lambda g_{b+k}s(x) \qquad j \ge 1 \qquad (6)$$

$$-\frac{d}{dx}Q_{10}(x) = -\lambda Q_{10}(x) + (1-\delta)A_{i0}(0)v(x) + B_0(0)v(x) \qquad 1 \le i \le b \qquad (7)$$

$$-\frac{d}{dx}Q_{1j}(x) = -\lambda Q_{1j}(x) + \sum_{k=1}^{j} Q_{1\,j-k}(x)\lambda g_k \qquad j \ge 1 \qquad (8)$$

$$-\frac{d}{dx}Q_{l0}(x) = -\lambda Q_{l0}(x) + Q_{l-1\,0}(0)v(x) \qquad l \ge 2 \qquad (9)$$

$$-\frac{d}{dx}Q_{lj}(x) = -\lambda Q_{lj}(x) + \sum_{k=1}^{j} Q_{l\,j-k}(x)\lambda g_k + Q_{l-1\,j}(0)v(x) \qquad j = 1,2,\dots N-1 \qquad (10)$$

$$-\frac{d}{dx}Q_{lj}(x) = -\lambda Q_{lj}(x) + \sum_{k=1}^{j} Q_{l\,j-k}(x)\lambda g_k \qquad j \ge N \quad l \ge 2 \qquad (11)$$

$$-\frac{d}{dx}B_0(x) = -\lambda B_0(x) + \delta\sum_{m=1}^{b} A_{m0}(0)b(x) \qquad (12)$$

$$-\frac{d}{dx}B_n(x) = -\lambda B_n(x) + \delta\sum_{m=1}^{b} A_{mn}(0)b(x) + \sum_{k=1}^{n} B_{n-k}(x)\lambda g_k \qquad n \ge 1 \qquad (13)$$

The Laplace-Stieltjes transform of $\mathbf{A_{in}}$(x), $\mathbf{Q_{jn}}$(x) and $\mathbf{B_n}$(x) are defined as

$$\tilde{A}_{in}(\theta) = \int_0^\infty e^{-\theta x} A_{in}(x)dx \qquad \tilde{Q}_{ln}(\theta) = \int_0^\infty e^{-\theta x} Q_{ln}(x)dx \qquad \tilde{B}_n(\theta) = \int_0^\infty e^{-\theta x} B_n(x)dx \tag{14}$$

Taking the Laplace-Stieltjes transform on both sides of the equations from equations (1) to (13), we get

$$(\theta-\lambda)\tilde{A}_{1j}(\theta) = A_{1j}(0) - \gamma D_{j+1}\tilde{S}(\theta) - D_j\lambda g_1\tilde{S}(\theta) \qquad j \geq 0 \tag{15}$$

$$(\theta-\lambda)\tilde{A}_{i0}(\theta) = A_{i0}(0) - D_0\lambda g_i\tilde{S}(\theta) \qquad 2 \leq i \leq b \tag{16}$$

$$(\theta-\lambda)\tilde{A}_{ij}(\theta) = A_{ij}(0) - D_j\lambda g_i\tilde{S}(\theta) - \sum_{k=1}^{j}\tilde{A}_{i\,j-k}(\theta)\,\lambda g_k \qquad 2 \leq i \leq b-1,\ j \geq 1 \tag{17}$$

$$(\theta-\lambda)\tilde{A}_{bj}(\theta) = A_{bj}(0) - \sum_{k=1}^{j}\tilde{A}_{b\,j-k}(\theta)\,\lambda g_k - \sum_{k=0}^{j} D_{j-k}\,\lambda g_{b+k}\tilde{S}(\theta) \qquad j \geq 1 \tag{18}$$

$$(\theta-\lambda)\tilde{Q}_{10}(\theta) = Q_{10}(0) - (1-\delta)A_{i0}(0)\tilde{V}(\theta) - B_0(0)\tilde{V}(\theta) \qquad 1 \leq i \leq b \tag{19}$$

$$(\theta-\lambda)\tilde{Q}_{1j}(\theta) = Q_{1j}(0) - \sum_{k=1}^{j}\tilde{Q}_{1\,j-k}(\theta)\,\lambda g_k \qquad j \geq 1 \tag{20}$$

$$(\theta-\lambda)\tilde{Q}_{l0}(\theta) = Q_{l0}(0) - Q_{l-1\,0}(0)\tilde{V}(\theta) \qquad l \geq 2 \tag{21}$$

$$(\theta-\lambda)\tilde{Q}_{lj}(\theta) = Q_{lj}(0) - \sum_{k=1}^{j}\tilde{Q}_{l\,j-k}(\theta)\,\lambda g_k + Q_{l-1\,j}(0)\tilde{V}(\theta) \qquad j = 1,2,\ldots N-1 \tag{22}$$

$$(\theta-\lambda)\tilde{Q}_{lj}(\theta) = Q_{lj}(0) - \sum_{k=1}^{j}\tilde{Q}_{l\,j-k}(\theta)\,\lambda g_k \qquad j \geq N \quad l \geq 2 \tag{23}$$

$$(\theta-\lambda)\tilde{B}_0(\theta) = B_0(0) - \delta\sum_{m=1}^{b} A_{m0}(0)\tilde{B}(\theta) \tag{24}$$

$$(\theta-\lambda)\tilde{B}_n(\theta) = B_n(0) - \delta\sum_{m=1}^{b} A_{mn}(0)\tilde{B}(\theta) - \sum_{k=1}^{j}\tilde{B}_{n-k}(\theta)\,\lambda g_k \qquad j \geq 1 \tag{25}$$

4. Probability Generating Function (PGF)

To obtain the PGF of an orbit size distribution at an arbitrary time epoch, the following generating functions are defined.

$$\tilde{A}_i(z,\theta) = \sum_{j=0}^{\infty}\tilde{A}_{ij}(\theta)z^j \qquad A_i(z,0) = \sum_{j=0}^{\infty} A_{ij}(0)z^j \qquad 2 \leq i \leq b$$
$$\tilde{Q}_j(z,\theta) = \sum_{l=1}^{\infty}\tilde{Q}_{1j}(0)z^j \qquad Q_j(z,0) = \sum_{l=1}^{\infty} Q_{lj}(0)z^j \qquad j \geq 1 \tag{26}$$

$$\tilde{B}(z,\theta) = \sum_{n=0}^{\infty}\tilde{B}_n(\theta)\,z^n \qquad B(z,0) = \sum_{n=0}^{\infty} B_n(0)z^n \qquad D(z) = \sum_{j=0}^{\infty} T_j\,z^j$$
$$(\lambda+\gamma)D(z) = (1-\delta)\sum_{m=1}^{b} A_m(z,0) - (1-\delta)A_{m0}(0) + B(z,0) - B_0(0) + \sum_{j=N}^{\infty}\sum_{l=1}^{\infty} Q_{lj}(z,0) - \gamma D_0 \tag{27}$$

$$(\theta-\lambda)\tilde{A}_1(z,\theta) = A_1(z,0) - \gamma\tilde{S}(\theta)\frac{1}{z}(D(z)-D_0) - \lambda g_1 D(z)\tilde{S}(\theta) \tag{28}$$

$$(\theta-\lambda+\lambda x(z))\tilde{A}_i(z,\theta) = A_i(z,0) - \lambda g_i D(z)\tilde{S}(\theta) \qquad 2 \leq i \leq b-1 \tag{29}$$

$$(\theta-\lambda+\lambda x(z))\tilde{A}_b(z,\theta) = A_b(z,0) - \lambda g_b D_0\tilde{S}(\theta) - \sum_{k=0}^{\infty}\lambda g_{b+k}\,z^k\,D(z)\tilde{S}(\theta) \tag{30}$$

$$(\theta-\lambda+\lambda x(z))\tilde{Q}_1(z,\theta) = Q_1(z,0) - \tilde{V}(\theta)\big((1-\delta)A_{i0}(0) + B_0(0)\big) \tag{31}$$

$$(\theta-\lambda+\lambda x(z))\tilde{Q}_l(z,\theta) = Q_l(z,0) - \tilde{V}(\theta)\sum_{j=0}^{N-1} Q_{l-1\,j}(0)\,z^j \tag{32}$$

$$(\theta-\lambda+\lambda x(z))\tilde{B}(z,\theta) = B(z,0) - \delta\sum_{m=1}^{b} A_m(z,0)\tilde{B}(\theta) \tag{33}$$

Substituting $\theta = \lambda$ in equation (28) implies

$$A_1(z,0) = \tilde{S}(\lambda) D(z)\left(\frac{\gamma}{z} + \lambda g_1\right) - \frac{\gamma}{z} D_0 \tilde{S}(\lambda) \tag{34}$$

Substituting $\theta = \lambda - \lambda x(z)$ from equations (29) to (33) implies

$$A_i(z,0) = \lambda g_i D(z) \tilde{S}(\lambda - \lambda x(z)) \qquad 2 \le i \le b-1 \tag{35}$$

$$A_b(z,0) = \lambda g_b D_0 \tilde{S}(\lambda - \lambda x(z)) + \sum_{k=0}^{\infty} \lambda g_{b+k} z^k D(z) \tilde{S}(\lambda - \lambda x(z)) \tag{36}$$

$$Q_1(z,0) = \tilde{V}\big(\lambda - \lambda x(z)\big)\big((1-\delta)A_{i0}(0) + B_0(0)\big) \tag{37}$$

$$Q_l(z,0) = \tilde{V}\big(\lambda - \lambda x(z)\big) \sum_{j=0}^{N-1} Q_{l-1\,j}(0) z^j \tag{38}$$

$$B(z,0) = \delta \sum_{m=1}^{b} A_m(z,0) \tilde{B}\big(\lambda - \lambda x(z)\big) \tag{39}$$

By using (28) and (34), we get

$$\tilde{A}_1(z,\theta) = \frac{(\tilde{S}(\lambda) - \tilde{S}(\theta))\left(D(z)\left(\frac{\gamma}{z} + \lambda g_1\right) - \frac{\gamma}{z} D_0\right)}{\theta - \lambda} \tag{40}$$

By using (29) and (35), we get

$$\tilde{A}_i(z,\theta) = \frac{(\tilde{S}(\lambda - \lambda x(z)) - \tilde{S}(\theta))\, \lambda g_i D(z)}{\theta - \lambda + \lambda x(z)} \qquad 2 \le i \le b-1 \tag{41}$$

By using (30) and (36), we get

$$\tilde{A}_b(z,\theta) = \frac{(\tilde{S}(\lambda - \lambda x(z)) - \tilde{S}(\theta))(\lambda g_b D_0 + \sum_{k=0}^{\infty} \lambda g_{b+k} z^k D(z))}{\theta - \lambda + \lambda x(z)} \tag{42}$$

By using (31), (32), (37) and (38), we get

$$\sum_{l=1}^{\infty} \tilde{Q}_l(z,\theta) = \frac{(\tilde{V}(\lambda - \lambda x(z)) - \tilde{V}(\theta))\left((1-\delta)A_{i0}(0) + B_0(0) + \sum_{j=0}^{N-1} Q_{l-1\,j}(0) z^j\right)}{\theta - \lambda + \lambda x(z)} \tag{43}$$

By using (33) and (39), we get

$$\tilde{B}(z,\theta) = \frac{(\tilde{B}(\lambda - \lambda x(z)) - \tilde{B}(\theta))\delta \sum_{m=1}^{b} A_m(z,0)}{\theta - \lambda + \lambda x(z)} \tag{44}$$

Substituting equations through (40) to (44), (38) and (39) in equation (27), we get

$$D(z) = \frac{D_0 z^b \left\{ \begin{matrix} \gamma z + \left((1-\delta) + \delta \tilde{B}\big(\lambda - \lambda x(z)\big)\right) \\ \times \left(\lambda z g_b \tilde{S}\big(\lambda - \lambda x(z)\big) - \gamma \tilde{S}(\lambda)\right) \end{matrix} \right\} + z^{b+1}\left(\tilde{V}\big(\lambda - \lambda x(z)\big) \sum_{j=0}^{N-1} Q_{l-1\,j}(0) z^j - \big((1-\delta)A_{i0}(0) + B_0(0)\big)\right)}{z^{b+1}(\lambda + \gamma) - \left((1-\delta) + \delta \tilde{B}\big(\lambda - \lambda x(z)\big)\right) \times \left(\begin{matrix} \tilde{S}(\lambda)(\gamma + \lambda z g_1) z^b \\ + \lambda \tilde{S}(\lambda - \lambda x(z))\big(z^{b+1} \sum_{i=2}^{b-1} g_i - z\big(X(z) - \sum_{j=1}^{b-1} g_j z^j\big)\big) \end{matrix} \right)} \tag{45}$$

Probability generating function of the orbit size at an arbitrary time is given by

$$P(z) = D(z) + \tilde{A}_1(z,0) + \sum_{i=2}^{b-1} \tilde{A}_i(z,0) + \tilde{A}_b(z,0) + \sum_{l=1}^{\infty} \tilde{Q}_l(z,0) + \tilde{B}(z,0) \tag{46}$$

Substituting $\theta = 0$ from equation (54) to (58) and using (45), the equation (46) is simplified as

$$P(z) = D_0\left[\frac{W_2(W_3+W_4+W_5)}{W_1(-\lambda+\lambda x(z))\lambda} + \frac{W_6+W_7}{(-\lambda+\lambda x(z))\lambda z}\right] + \frac{W_9}{(-\lambda+\lambda x(z))W_1} \quad (47)$$

where

$$W_1 = z^{b+1}(\lambda+\gamma) - \left((1-\delta)+\delta\tilde{B}(\lambda-\lambda x(z))\right) \times \begin{pmatrix} \tilde{S}(\lambda)(\gamma+\lambda z g_1)z^b + \lambda\tilde{S}(\lambda-\lambda x(z)) \\ +\lambda\tilde{S}(\lambda-\lambda x(z))M_1 \end{pmatrix}$$

$$W_2 = z^b\left(\gamma z + \left((1-\delta)+\delta\tilde{B}(\lambda-\lambda x(z))\right) \times \left(\lambda z g_b \tilde{S}(\lambda-\lambda x(z)) - \gamma\tilde{S}(\lambda)\right)\right)$$

$$W_3 = (-\lambda+\lambda x(z))\left(1+\left(1-\tilde{S}(\lambda)\right)(z+\lambda g_1)\right)$$

$$M_1 = z^{b+1}\sum_{i=2}^{b-1} g_i - z\left(X(z) - \sum_{i=1}^{b-1} g_i z^i\right)$$

$$W_4 = \left(\tilde{S}(\lambda-\lambda x(z))-1\right)\lambda\left(\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty}\lambda g_{b+k} z^k\right) \quad c_0 = \left((1-\delta)A_{i0}(0)+B_0(0)\right)$$

$$W_5 = \lambda\left(\tilde{B}(\lambda-\lambda x(z))-1\right)\delta\left(\tilde{S}(\lambda)\left(\frac{\gamma}{z}+\lambda g_1\right) + \tilde{S}(\lambda-\lambda x(z))\left(\sum_{k=0}^{\infty}\lambda g_{b+k} z^k + \sum_{i=2}^{b-1} g_i\right)\right)$$

$$W_6 = \lambda\left(\left(\tilde{S}(\lambda)-1\right)\gamma(-\lambda+\lambda x(z)) + \left(\tilde{S}(\lambda-\lambda x(z))-1\right)\lambda z g_b\right)$$

$$W_7 = \left(\tilde{B}(\lambda-\lambda x(z))-1\right)\delta g_b\tilde{S}(\lambda-\lambda x(z)) \quad W_8 = z^{b+1}\left(\tilde{V}(\lambda-\lambda x(z))\right)\left(\sum_{j=0}^{N-1} q_j z^j - c_0\right)$$

$$W_9 = W_1\left[\left(\tilde{V}(\lambda-\lambda x(z))-1\right)\left(\sum_{j=0}^{N-1} q_j z^j + c_0\right)\right] + W_8(-\lambda+\lambda x(z))$$

4.1 Steady state condition

The probability generating function given in equation (47) has to satisfy the condition P (1) = 1. The steady state condition for the proposed model under consideration is simplified as

$\rho = \lambda E(X)\left(E(S)+\delta E(B)\right) < 1$ and the unknown constant D_0 is obtained as

$$D_0 = \frac{\lambda E(X)\left(1-E(V)\left(\sum_{j=0}^{N-1} q_j - c_0\right)\right)\left((\lambda+\gamma)+\lambda M_1 - \tilde{S}(\lambda)(\gamma+\lambda g_1)+\lambda\right)}{\left(\gamma+\left(\lambda g_b - \gamma\tilde{S}(\lambda)\right)\right) + S1 + S2\left(\sum_{k=0}^{\infty}\lambda g_{b+k} + \sum_{i=2}^{b-1} g_i\right)}$$

where

$$S1 = \lambda E(X)\left(1+\left(1-\tilde{S}(\lambda)\right)(1+\lambda g_1)\right) + E(S)\lambda^3 E(X)\left(\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty} g_{b+k}\right)$$
$$+E(B)\lambda^2 E(X)\varphi\left(\tilde{S}(\lambda)(\gamma+\lambda g_1) + \lambda\left(\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty} g_{b+k}\right)\right)$$

$$S2 = \left(\gamma + E(B)\lambda E(X)\left(\lambda g_b - \gamma\tilde{S}(\lambda)\right) + \left(\lambda g_b + E(S)\lambda E(X)\right)\right) + b\left(\gamma+\lambda g_b - \gamma\tilde{S}(\lambda)\right)$$

4.2 Results

An unknown constant c_n is expressed in terms of known term D_n. Let β_n be the probability that 'n' customers arrive into the system during an idle period then.

$$c_0 = \frac{\beta_0 D_0}{(1-\beta_0)}$$

$$c_n = \frac{\beta_n D_0 + \sum_{j=0}^{n-1} c_j\beta_{n-j}}{(1-\beta_0)}, \quad n=1, 2, \ldots, N-1$$

5. Performance Measures

In this section some important performance measures for the given queuing system are derived.

5.1 Expected orbit length (E(Q))

The mean orbit length can be obtained by differentiating P(z) with respect to z at 1

$$E(Q) = \lim_{z\to 1}(P'(z))$$
$$E(Q) = D_0\left[\frac{\lambda(u_1'u_3'' - u_3'u_1'')}{2(u_1')^2\lambda^2} + \frac{u_2'u_4'' - u_4'u_2''}{2(u_2')^2}\right] + \frac{u_1'W_9'' - W_9'u_1''}{2(u_1')^2}$$

$F_1 = \lambda E(S)E(X)$ $G_1 = \lambda E(S)X''(1)$ $H_1 = \lambda^2 E(S^2)(E(X))^2$

$F_2 = \lambda E(B)E(X)$ $G_2 = \lambda E(B)X''(1)$ $H_2 = \lambda^2 E(B^2)(E(X))^2$

$F_3 = \lambda E(V)E(X)$ $G_3 = \lambda E(V)X''(1)$ $H_1 = \lambda^2 E(V^2)(E(X))^2$

$u_1 = W_1(-\lambda + \lambda x(z))$ $u_2 = (-\lambda + \lambda x(z))\lambda z$ $u_1' = c_1\,\lambda E(X)$

$u_3 = W_2(W_3 + W_4 + W_5)$ $u_4 = W_6 + W_7$

$c_1 = \tilde{S}(\lambda)(\gamma + \lambda g_1) + \lambda + \lambda M_1$ $c_1' = \tilde{S}(\lambda)(\gamma + \lambda g_1)b + \tilde{S}(\lambda)(\gamma + \lambda g_1) + \lambda F_1 M_1 + \lambda M_1'$

$$c_1'' = \tilde{S}(\lambda)(\gamma + \lambda g_1)b(b-1) + 2\tilde{S}(\lambda)(\gamma + \lambda g_1)b + \lambda(G_1 + H_1) + \lambda(G_1 + H_1)M_1 + 2\lambda F_1 M_1' + \lambda M_1''$$

$W_1' = (b+1)(\gamma + \lambda) - c_1' - \delta c_1 F_2$

$W_1'' = b(b+1)(\gamma + \lambda) - c_1'' - 2\,\delta c_1' F_2 - c_1\delta(G_2 + H_2)$

$W_2' = \gamma + E_2' + E_2E_1' + b(\gamma + E_2)$ $E_1' = \delta F_2$ $E_1'' = \delta(G_2 + H_2)$

$W_2'' = E_2'' + 2E_1'E_2' + E_2E_1'' + b(\gamma + E_2' + E_2E_1') + b(b-1)(\gamma + E_2)$

$E_2' = \lambda g_b(F_1 + 1)$ $E_2 = \lambda g_b - \tilde{S}(\lambda)\,\gamma$

$E_2'' = \lambda g_b(G_2 + H_2 + 2F_1)$ $W_3' = \lambda E(X)(1 + (1 - \tilde{S}(\lambda))(1 + \lambda g_1))$

$W_3'' = \lambda X''(1) + (1 - \tilde{S}(\lambda))(2\lambda^2 g_1 E(X) + (1 + \lambda g_1)(\lambda X''(1))$

$M_2 = \left(\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty} \lambda g_{b+k}\right)$

$W_4' = F_1\lambda M_3$ $W_4'' = \lambda(F_1M_2 + (G_1 + H_1)M_2)$

$M_3 = \sum_{k=0}^{\infty} k\lambda g_{b+k}$ $M_5 = \sum_{j=0}^{N-1} q_j + c_0$

$W_5' = F_2\lambda\,\delta(\tilde{S}(\lambda)(\lambda g_1 + \gamma) + M_2)$ $M_4 = \sum_{j=0}^{N-1} q_j - c_0$

$W_5'' = 2F_2\lambda\delta(\tilde{S}(\lambda)(\lambda g_1 - \gamma) + M_3 + F_1 M_2) + \lambda\delta(G_2 + H_2)(\tilde{S}(\lambda)(\lambda g_1 + \gamma) + M_2)$

$W_6' = \lambda\left((\tilde{S}(\lambda) - 1)\gamma\lambda E(X) + F_1\lambda g_b\right)$

$W_6'' = \lambda\{(\tilde{S}(\lambda) - 1)\gamma\lambda X''(1) + 2F_1\lambda g_b + (G_1 + H_1)\lambda g_b\}$

$W_7'' = F_2\lambda^2\delta E(X)\left(\lambda g_b - \gamma\tilde{S}(\lambda)\right)$ $W_8' = \sum_{j=0}^{N-1} jq_j + (F_3 + (b+1))\,M_4$

$W_8'' = \sum_{j=0}^{N-1} j(j-1)q_j + 2(F_3 + (b+1))\sum_{j=0}^{N-1} jq_j + ((G_3 + H_3) + 2(b+1)F_3 + b(b+1))M_4$

$W_9' = W_1F_3M_5 + W_8\lambda E(X)$

$W_9'' = 3W_1'F_3\sum_{j=0}^{N-1} jq_j + 2W_8'\,\lambda E(X) + (W_1(G_3 + H_3) + W_1'F_3)M_5 + W_8\,\lambda X''(1)$

5.2. Probability that the server is busy

$$P(B) = \frac{(\tilde{S}(\lambda)-1)(\lambda g_1 + \gamma(1 - D_0))}{-\lambda} + E(s)D(1)\left(\sum_{i=2}^{b-1} g_i + \lambda g_b D_0 + \sum_{k=0}^{\infty} \lambda g_{b+k}\right)$$

where

$$D(1) = \frac{D_0\left(\gamma + \lambda g_b - \gamma\tilde{S}(\lambda)\right) + \left(\sum_{j=0}^{N-1} Q_{i-1\,j}(0) - ((1-\delta)A_{i0}(0) + B_0(0))\right)}{(\lambda + \gamma) - \tilde{S}(\lambda)(\gamma + \lambda g_1) \times \lambda\left(\sum_{i=2}^{b-1} g_i - \left(1 - \sum_{j=1}^{b-1} g_j z^j\right)\right)}$$

5.3. Probability that the server is in renewal period

$$P(R) = \lim_{z\to 1} \tilde{B}(z, 0)$$
$$P(R) = E(B)\left(\frac{(\tilde{S}(\lambda) - 1)(\lambda g_1 + \gamma(1 - D_0))}{-\lambda}\right)$$

5.4. Probability that the server is idle

$$\mathrm{P(I)} = \lim_{z\to 1} D(z)$$

$$\mathrm{P(I)} = \frac{D_0\left(\gamma + \lambda g_b - \gamma \tilde{S}(\lambda)\right) + \left(\sum_{j=0}^{N-1} Q_{i-1\,j}(0) - \left((1-\delta)A_{i0}(0) + B_0(0)\right)\right)}{(\lambda+\gamma) - \tilde{S}(\lambda)(\gamma + \lambda g_1) \times \lambda\left(\sum_{i=2}^{b-1} g_i - \left(1 - \sum_{j=1}^{b-1} g_j z^j\right)\right)}$$

5.5. Mean waiting time in retrial queue

$$E(R) = \frac{E(Q)}{\lambda E(X)}$$

5.6. Mean number of customers in the system

$$L_s = E(Q) + \rho$$

5.7. Mean waiting time in the system

$$W_s = \frac{L_s}{\lambda E(X)}$$

5.8. Expected length of busy period

By the theory of alternating renewal process, the expected length of busy period is derived as

$$E(B) = \frac{1}{\lambda E(X)}\left(\frac{1}{D_0} - 1\right)$$

5.9. Expected length of busy cycle

The expression for expected length of busy cycle is obtained by the theory of alternating renewal process

$$E(B_c) = \frac{1}{\lambda E(X)}\left(\frac{1}{D_0}\right)$$

6. Special cases

The proposed model is developed with the assumption that the service time is arbitrary. However, to analyze real time systems, suitable distribution is required. This section presents some special cases of the system by indicating bulk service time as exponential distribution, hyper-exponential distribution and Erlangian distribution.

Case 1 Exponential bulk service time

The probability density function of exponential service time is $s(x) = e^{-\mu x}$, here μ is parameter, therefore, $\tilde{S}(\lambda - \lambda x(z)) = \left(\frac{\mu}{\mu + (\lambda - \lambda x(z))}\right)$

The PGF of the orbit size for exponential service time is derived by substituting the expression for $\tilde{S}(\lambda - \lambda x(z))$ in equation (61)

$$P(z) = D_0\left[\frac{W_2(W_3 + W_4 + W_5)}{W_1(-\lambda + \lambda x(z))\lambda} + \frac{W_6 + W_7}{(-\lambda + \lambda x(z))\lambda z}\right] + \frac{W_9}{(-\lambda + \lambda x(z))W_1}$$

where

$$W_1 = z^{b+1}(\lambda+\gamma) - \left((1-\delta) + \delta \tilde{B}(\lambda - \lambda x(z))\right) \times \begin{pmatrix} \left(\frac{\mu}{\mu+(\lambda-\lambda x(z))}\right)(\gamma + \lambda z g_1) z^b + \lambda\left(\frac{\mu}{\mu+\lambda}\right) \\ +\lambda\left(\frac{\mu}{\mu+(\lambda-\lambda x(z))}\right) M_1 \end{pmatrix}$$

$$W_2 = z^b\left(\gamma z + \left((1-\delta) + \delta\tilde{B}\big(\lambda - \lambda x(z)\big)\right) \times \left(\lambda z g_b\left(\frac{\mu}{\mu + (\lambda - \lambda x(z))}\right) - \gamma\left(\frac{\mu}{\mu+\lambda}\right)\right)\right)$$

$$W_3 = \big(-\lambda + \lambda x(z)\big)\left(1 + \left(1 - \left(\frac{\mu}{\mu+\lambda}\right)\right)(z + \lambda g_1)\right)$$

$$M_1 = z^{b+1}\textstyle\sum_{i=2}^{b-1} g_i - z\big(X(z) - \sum_{i=1}^{b-1} g_i z^i\big)$$

$$W_4 = \left(\frac{\mu}{\mu+(\lambda-\lambda x(z))} - 1\right)\lambda\big(\textstyle\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty}\lambda g_{b+k} z^k\big)\; c_0 = \big((1-\delta)A_{i0}(0) + B_0(0)\big)$$

$$W_5 = \lambda\big(\tilde{B}\big(\lambda - \lambda x(z)\big) - 1\big)\,\delta\left(\left(\frac{\mu}{\mu+\lambda}\right)\left(\frac{\gamma}{z} + \lambda g_1\right) + \left(\frac{\mu}{\mu+(\lambda-\lambda x(z))}\right)\big(\textstyle\sum_{k=0}^{\infty}\lambda g_{b+k} z^k + \sum_{i=2}^{b-1} g_i\big)\right)$$

$$W_6 = \lambda\left(\left(\left(\frac{\mu}{\mu+\lambda}\right) - 1\right)\gamma(-\lambda + \lambda x(z)) + \left(\left(\frac{\mu}{\mu+(\lambda-\lambda x(z))}\right) - 1\right)\lambda z g_b\right)$$

$$W_7 = \big(\tilde{B}\big(\lambda - \lambda x(z)\big) - 1\big)\delta g_b\left(\frac{\mu}{\mu+(\lambda-\lambda x(z))}\right)$$

$$W_8 = z^{b+1}\left(\tilde{V}\big(\lambda - \lambda x(z)\big)\right)\big(\textstyle\sum_{j=0}^{N-1} q_j z^j - c_0\big)$$

$$W_9 = W_1\big[\big(\tilde{V}\big(\lambda - \lambda x(z)\big) - 1\big)\big(\textstyle\sum_{j=0}^{N-1} q_j z^j + c_0\big)\big] + W_8\big(-\lambda + \lambda x(z)\big)$$

Case 2 Hyper-exponential bulk service time

When the service time follows hyper-exponential distribution with probability density function, then $s(x) = cde^{-dx} + (1-c)\,fe^{-fx}$, where d and f are parameters, then,

$$\tilde{S}\big(\lambda - \lambda x(z)\big) = \left(\frac{dc}{d + \big(\lambda - \lambda x(z)\big)}\right) + \left(\frac{f(1-c)}{f + \big(\lambda - \lambda x(z)\big)}\right)$$

The PGF of the orbit size for hyper-exponential service time is derived by substituting the expression for $\tilde{S}\big(\lambda - \lambda x(z)\big)$ in equation (61).

Case 3 K-Erlangian bulk service time

Let us consider that service time follows K-Erlang distribution with probability density function

$s(x) = \frac{(k\mu)^k x^{k-1} e^{-(k\mu x)}}{(k-1)!}$, k > 0; where μ is the parameter, then

$$\tilde{S}\big(\lambda - \lambda x(z)\big) = \left(\frac{k\mu}{k\mu + \big(\lambda - \lambda x(z)\big)}\right)^k$$

The PGF of the orbit size K-Erlangian bulk service time can be derived by substituting the expression for $\tilde{S}\big(\lambda - \lambda x(z)\big)$ in equation (61).

Case 4 When there is no server failure, multiple vacations and threshold

i. e ($\delta = 0$ *and* $\tilde{B}\big(\lambda - \lambda x(z)\big) = \tilde{V}\big(\lambda - \lambda x(z)\big) = 1$)

The PGF given in (61) is reduced to

$$P(z) = D_0\left[\frac{W_2(W_3 + W_4)}{W_1(-\lambda + \lambda x(z))\lambda} + \frac{W_6}{(-\lambda + \lambda x(z))\lambda z}\right]$$

where

$$W_1 = z^{b+1}(\lambda + \gamma) - \begin{pmatrix}\tilde{S}(\lambda)(\gamma + \lambda z g_1)z^b + \lambda\tilde{S}\big(\lambda - \lambda x(z)\big)\\ +\lambda\tilde{S}\big(\lambda - \lambda x(z)\big)M_1\end{pmatrix}$$

$$W_2 = z^b\left(\gamma z + \times\left(\lambda z g_b \tilde{S}\big(\lambda - \lambda x(z)\big) - \gamma\tilde{S}(\lambda)\right)\right)$$

$$W_3 = \big(-\lambda + \lambda x(z)\big)\left(1 + \left(1 - \tilde{S}(\lambda)\right)(z + \lambda g_1)\right)$$

$$M_1 = z^{b+1}\textstyle\sum_{i=2}^{b-1} g_i - z\big(X(z) - \sum_{i=1}^{b-1} g_i z^i\big)$$

$$W_4 = \left(\tilde{S}\left(\lambda - \lambda x(z)\right) - 1\right)\lambda\left(\sum_{i=2}^{b-1} g_i + \sum_{k=0}^{\infty} \lambda g_{b+k}\, z^k\right)$$

$$W_6 = \lambda\left(\left(\tilde{S}(\lambda) - 1\right)\gamma\left(-\lambda + \lambda x(z)\right) + \left(\tilde{S}\left(\lambda - \lambda x(z)\right) - 1\right)\lambda z g_b\right)$$

The above equation is coincides with Haridass *et al.* (2012).

7. Cost Effective Model

Optimization techniques take part in minimizing total average cost of the queuing system in many practical situations. Constraints in cost analysis are start-up cost, operating cost, holding cost, set-up cost and reward cost (if any). It is obvious that management of the system aims to minimize the total average cost. In this part, the cost analysis of the proposed queuing system is developed to obtain total average cost of the system with the following assumptions:

A_h : holding cost per customer	A_0 : operating cost per unit time
A_s : startup cost per cycle	A_r : reward cost per cycle due to vacation

Total average cost= Holding cost of customers per unit time in the queue
+ Start-up cost per cycle +Operating cost +Reward cost

$$\text{Total average cost} = A_s \frac{1}{E(B_c)} + A_h E(Q) + A_0 \frac{E(B)}{E(B_c)}$$

Therefore, the TAC is obtained as

$$\text{TAC} = A_s \lambda E(X) D_0 + A_h E(Q) + A_0 (1 - D_0)$$

The simple value direct search method is used to find an optimal policy for a maximum batch size *b** to minimize the total average cost.

Step 1: Fix the value of threshold 'N' (N>b)
Step 2: Select the value 'b' which will satisfy the following relation
$\text{TAC}(b^*) \leq \text{TAC(b)}, \quad b < N$
Step 3: The value *b** is optimum, since it gives minimum total average cost.

The above procedure gives the optimum value of 'b' which minimizes total average cost function. A numerical illustration is given in the next section to justify the above solution.

8. Numerical Illustration

In the performance evaluation of LAN executing under transmission protocol CSMA-CD, data are entered into the system according to Poisson arrival rate λ . This section presents a numerical example of the proposed queuing system, which is used by the moderator of a CSMA/CD protocol to make a decision in utilizing idle time effectively. All of the numerical results are obtained with the following assumptions:

Service time follows exponential distribution with parameter	μ
Batch size distribution of the arrival is geometric with mean	3
Retrial rate	γ
Vacation time follows exponential distribution with parameter	$\alpha = 5$
Renewal time follows exponential distribution with parameter	$\beta = 6$
Maximum service capacity	b
Threshold	N=10
Start-up cost	Rs. 1.40
Holding cost per customer	Rs. 0.50
Operating cost per unit time	Rs. 2.00
Reward per unit time due to vacation	Rs. 1.00
Renewal cost per unit time	Rs. 0.40

8.1. Effects of different parameters on the performance measures

The effects of retrial rate and service rate with respect to mean orbit size are given in Table 1. It is observed that if the retrial rate increases, then mean orbit size decreases. Also, when the service rate increases, the mean orbit size decreases.

Table 1. Retrial rate vs. mean orbit size (arrival rate λ=2).

Retrial rate	Expected orbit length (E(Q))							
	Service rate							
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	3.4592	3.0624	2.5317	1.9316	1.5296	1.1762	0.7915	0.2178
2	3.1972	2.8219	2..3125	1.7161	1.3261	0.9361	0.5715	0.0961
3	2.9635	2.6314	2.1362	1.5422	1.1965	0.5369	0.3191	0.0511
4	2.6192	2.3724	1.9352	1.3256	0.9561	0.2850	0.1061	0.0319
5	2.3207	2.0251	1.6541	1.1509	0.7169	0.0965	0.0951	0.0193
6	2.1579	1.8512	1.2981	0.9618	0.4196	0.0711	0.0621	0.00743
7	1.8271	1.6379	1.0263	0.6315	0.2193	0.0562	0.0417	0.00419
8	1.5324	1.5162	0.9572	0.3259	0.0961	0.0379	0.01884	0.00165
9	1.2193	1.3292	0.5192	0.1538	0.0541	0.0099	0.00092	0.000863

8.2. Effects of different parameters on the total average cost

Cost estimation is essential for the management of the system because there is a chance to change the maximum capacity value ‘b’ for service and service rate when the arrival rate is large. The management can reduce the total average cost by increasing either the service rate or the batch size of the service. Various comparisons with respect to total average cost are given in Table 2 and Table 3. It is clear that when the retrial rate increases, the total average cost decreases. Also, when the service rate increases, the total average cost decreases.

Table 2. Retrial rate vs. mean orbit size and total average cost (Arrival rate λ=3, Service rate μ=2).

Retrial rate	Threshold value ‘b’					
	b=3		b=4		b=5	
	E(Q)	TAC	E(Q)	TAC	E(Q)	TAC
2	2.7913	4.5623	2.4031	4.3028	2.3765	4.2193
3	2.2357	4.3261	2.2568	4.2968	2.2061	4.1562
4	1.8291	4.2392	2.1982	4.1192	2.0591	3.9843
5	1.7032	4.0569	1.9768	3.8369	1.8561	3.7521
6	1.5639	3.7894	1.6893	3.6528	1.7961	3.5361
7	1.4091	3.4063	1.4331	3.2391	1.5369	3.3549
8	1.1938	3.2569	1.2941	3.1965	1.3291	3.0391

Table 3. Retrial rate vs. total average cost (Arrival rate λ=3 and b=5)

Retrial rate	Total Average Cost		
	Arrival rate=2.0	Arrival rate=2.5	Arrival rate=3.0
2	5.5639	5.1861	4.8932
4	5.2962	4.8182	4.5362
6	4.8296	4.6365	4.3293
8	4.5192	4.3192	4.1368
10	4.3265	3.9370	4.8964
12	3.9493	3.7568	3.5256
14	3.6591	3.4293	3.2128

8.3. Optimum cost

This section presents a numerical example to explain how the moderator of the CSMA/CD protocol can effectively use the results obtained in Sections 4 and 5 to fix the maximum capacity (b), which minimizes the total average cost. An optimal policy regarding the maximum capacity ‘*b*’ which will minimize the total average cost is presented in Table 4 and Figure 7. From the observations it is clear that the optimum value which reduces the total average cost is b=5.

Table 4. Maximum capacity (b) vs. total average cost (λ=2, γ=3, N=10)

Maximum capacity(b)	Total average cost
3	4.5962
4	4.3249
5	4.2964
6	4.4565
7	4.6974
8	4.7421
9	4.8462
10	4.9523

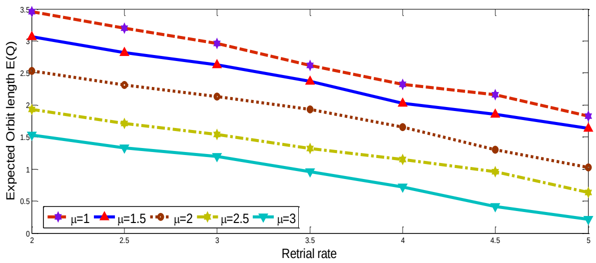

Figure 2. Retrial rate vs. E (Q) for different service rates.

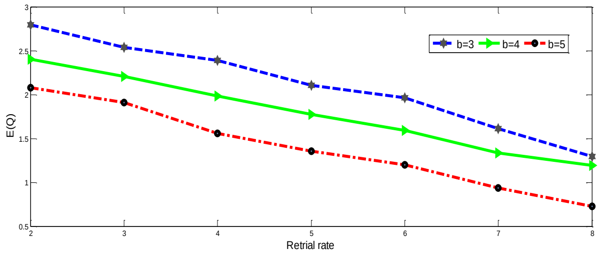

Figure 3. Retrial rate vs. E (Q) for different values of ‘b’.

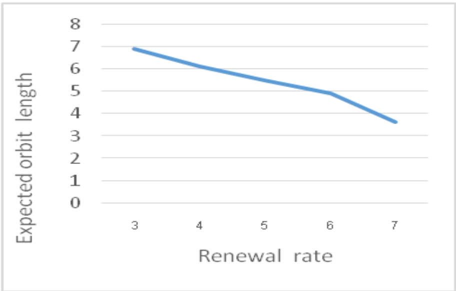

Figure 4. Renewal rate vs. E(Q).

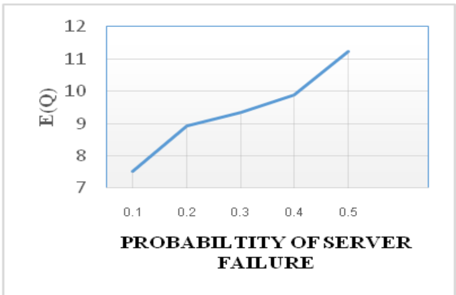

Figure 5. Probability of server breakdown vs. (E (Q)).

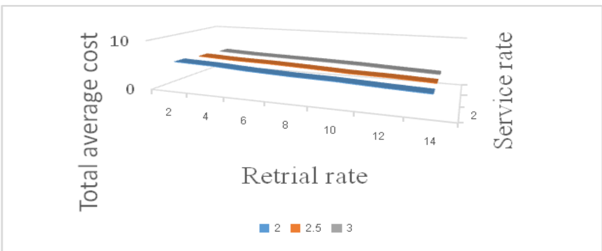

Figure 6. Retrial rate vs. total average cost for different service rates.

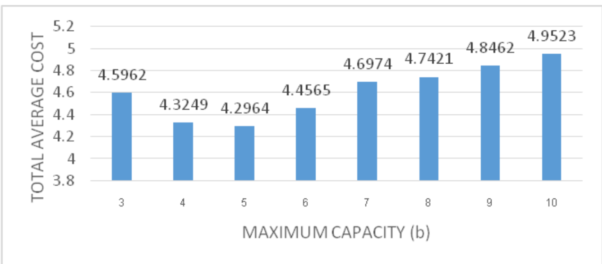

Figure 7. Maximum capacity vs. Total average cost.

9. Conclusions

This paper analyzed a bulk arrival and batch service retrial queuing system with server failure, threshold, and multiple vacations. A probability generating function of the orbit size at an arbitrary time epoch was obtained by using a supplementary variable technique. Various performance measures, a particular case, and special cases were also discussed. A cost estimation analysis was also carried out with numerical example. All the obtained results will be useful in making decisions to estimate overall cost and search for the best operating policy in a queuing system. In the future, this model can be improved by including vacation interruption concept. It is also possible to extend this model to a fuzzy environment.

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