Songklanakarin J. Sci. Technol.
41 (2), 299-304, Mar. – Apr. 2019

http://www.sjst.psu.ac.th

Original Article

A-ideals and fuzzy A-ideals of ternary semigroups

Sudaporn Suebsung[1], Khwancheewa Wattanatripop[2], and Ronnason Chinram[1,3*]

[1] *Algebra and Applications Research Unit, Department of Mathematics and Statistics, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla, 90110 Thailand*

[2] *Department of Mathematics, Faculty of Science, Khon Kaen University, Mueang, Khon Kaen, 40002 Thailand*

[3] *Centre of Excellence in Mathematics, Commission on Higher Education (CHE), Ratchathewi, Bangkok, 10400 Thailand*

Received: 19 July 2017; Revised: 2 October 2017; Accepted: 9 November 2017

Abstract

A ternary semigroup is a nonempty set together with a ternary multiplication that is associative. Every semigroup can be reduced to a ternary semigroup, but a ternary semigroup does not necessarily reduce to a semigroup. The notion of A-ideals in semigroups was introduced by Grosek and Satko in 1980. Fuzzy sets were introduced by Zadeh in 1965. Applications of the fuzzy set theory have been found in various fields. The theory of fuzzy sets has been studied in various kinds of algebraic systems.

In this paper, we define and study some properties of A-ideals and fuzzy A-ideals of ternary semigroups. Moreover, we introduce the notion of minimal fuzzy A-ideals of ternary semigroups and study properties of them.

Keywords: A-ideals, fuzzy A-ideals, minimal A-ideals, ternary semigroups.

1. Introduction

A ternary semigroup is a nonempty set together with a ternary multiplication that is associative. Every semigroup can be reduced to a ternary semigroup, but a ternary semigroup need not necessarily reduce to a semigroup: the ternary semigroup $\mathbb{Z}^-$ under the usual multiplication serves as an example. Lehmer (1932) investigated certain triple systems called triplexes, which turn out to be commutative ternary groups. Moreover, he defined regular ternary semigroups. Los (1955) showed that every ternary semigroup can be embedded in a semigroup. Sioson (1965) studied ternary semigroups with special reference to ideals and radicals. Moreover, he has extended to ternary semigroups various well-known concepts concerning ideals, such as primality, semiprimality etc. Santiago and Sri Bala (2010) studied regularity conditions in a ternary semigroup. The concept of a fuzzy set was introduced by Zadeh (1965). Fuzzy subsets have been developed in many fields. Rosenfeld (1971) applied the concepts of Zadeh to define fuzzy subgroups and fuzzy ideals. Fuzzy semigroups were first studied by Kuroki (1981, 1979). He defined also fuzzy ideals in semigroups (Kuroki, 1991). Grosek (1979) introduced the notion of an almost-ideal (A-ideal) in a semilattice. Grosek and Satko (1980) introduced the notion of an A-ideal of semigroups. Minimal A-ideals, maximal A-ideals and smallest A-ideals were studied in Grosek and Satko (1981).

In this paper, we define and study some properties of A-ideals and fuzzy A-ideals of ternary semigroups. Moreover, we introduce the notion of minimal fuzzy A-ideals of ternary semigroups and study properties of them.

*Corresponding author
Email address: ronnason.c@psu.ac.th

2. Preliminaries

A nonempty set T is called a *ternary semigroup* if there exists a ternary operation $: T \times T \times T \to T$ written as $(a,b,c) \mapsto abc$ satisfying the following identity

$$(abc)de = a(bcd)e = ab(cde), \text{for all } a,b,c,d,e \in T.$$

Let A, B and C be nonempty subsets of a ternary semigroup T. A product ABC is defined by

$$ABC = \{abc : a \in A, b \in B \text{ and } c \in C\}.$$

Definition 2.1. Let T be a ternary semigroup and A be a nonempty subset of T.

(1) A is called a *ternary subsemigroup* of T if $TTT \subseteq T$.

(2) A is called a *left ideal* of T if $TTA \subseteq A$.

(3) A is called a *right ideal* of T if $ATT \subseteq A$.

(4) A is called a *middle ideal* of T if $TAT \subseteq A$.

If A is a left, right and middle ideal of T, then it is called an *ideal* of T.

A function f from T to the unit interval [0,1] is called a *fuzzy subset* of T. Let f and g be any two fuzzy subsets of T,

(1) $f \cap g$ is a fuzzy subset of T defined by $(f \cap g)(x) = \min\{f(x), g(x)\}$ for all $x \in T$.

(2) $f \cup g$ is a fuzzy subset of T defined by $(f \cup g)(x) = \max\{f(x), g(x)\}$ for all $x \in T$.

(3) $f \subseteq g$ if $f(x) \leq g(x)$ for all $x \in T$.

Let f be a fuzzy subset of T , the support of f is defined by

$$supp\ f := \{x \in T | f(x) \neq 0\}.$$

Let A be a nonempty subset of T, the characteristic mapping of A is a fuzzy subset of T defined by

$$C_A(x) = \begin{cases} 1 & x \in A, \\ 0 & x \notin A. \end{cases}$$

We note that the ternary semigroup T can be considered a fuzzy subset of itself and write $T = C_T$, i.e., $T(x) = 1$ for all $t \in T$.

Let t be any element in T, the characteristic mapping of t is a fuzzy subset of T defined by

$$C_t(x) = \begin{cases} 1 & x = t, \\ 0 & x \neq t. \end{cases}$$

Definition 2.2. Let T be a ternary semigroup and f be a fuzzy subset of T.

(1) f is called a *fuzzy left ideal* of T if $f(xyz) \geq f(z)$, for all $x, y, z \in T$.

(2) f is called a *fuzzy right ideal* of T if $f(xyz) \geq f(x)$, for all $x, y, z \in T$.

(3) f is called a *fuzzy middle ideal* of T if $f(xyz) \geq f(y)$, for all $x, y, z \in T$.

Then f is called a *fuzzy ideal* of T if it is a fuzzy left, fuzzy right and fuzzy middle ideal of T, i.e.

$$f(xyz) \geq \max\{f(x), f(y), f(z)\}, \qquad \text{for all } x, y, z \in T$$

Let f, g and h be fuzzy subsets of T. A *product* $f \circ g \circ h$ is defined by

$$(f \circ g \circ h)(x) = \begin{cases} \sup\limits_{x=pqr} \min\{f(p), g(q), h(r)\}, & x = pqr \ \ \exists p,q,r \in T, \\ 0, & \text{otherwise}. \end{cases}$$

Theorem 2.1. Let f be a fuzzy subset of a ternary semigroup T. Then

(1) f is a fuzzy subsemigroup of T if and only if $f \circ f \circ f \subseteq f$.

(2) f is a fuzzy left ideal of T if and only if $T \circ T \circ f \subseteq f$.

(3) f is a fuzzy right ideal of T if and only if $f \circ T \circ T \subseteq f$.

(4) f is a fuzzy middle ideal of T if and only if $T \circ f \circ T \subseteq f$.

(5) f is a fuzzy ideal of T if and only if $T \circ T \circ f \subseteq f, f \circ T \circ T \subseteq f$ and $T \circ f \circ T \subseteq f$.

3. A-ideals of ternary semigroups

Definition 3.1. Let T be a ternary semigroup. A nonempty subset L of T is called a *left A-ideal* of T if $ttL \cap L \neq \emptyset$, $\forall t \in T$. A nonempty subset R of T is called a *right A-ideal* of T if $Rtt \cap R \neq \emptyset$, $\forall t \in T$. A nonempty subset M of T is called a *middle A-ideal* of T if $tMt \cap M \neq \emptyset, \forall t \in T$. If I is a left, right and middle A-ideal of T, I is called an *A-ideal* of T.

Example 3.1. Every left ideal of$T$ is a left A-ideal of T.

Example 3.2. Consider the ternary semigroup $T = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ under the usual addition and $A = \{\bar{1}, \bar{3}, \bar{4}\}$. Then A is a left A-ideal of T because

If $t = \bar{0}$, we have $(t + t + A) \cap A = \{\bar{1}, \bar{3}, \bar{4}\}$.
If $t = \bar{1}$, we have $(t + t + A) \cap A = \{\bar{3}\}$.
If $t = \bar{2}$, we have $(t + t + A) \cap A = \{\bar{1}\}$.
If $t = \bar{3}$, we have $(t + t + A) \cap A = \{\bar{1}, \bar{3}, \bar{4}\}$.
If $t = \bar{4}$, we have $(t + t + A) \cap A = \{\bar{3}\}$.
If $t = \bar{5}$, we have $(t + t + A) \cap A = \{\bar{1}\}$.

However, A is not a left ideal of T because

$\bar{2} + \bar{2} + \bar{4} \in \bar{2} + \bar{2} + A$ but $\bar{2} + \bar{2} + \bar{4} = \bar{8} = \bar{2} \notin A$.

Example 3.2 shows that in general, left A-ideals need not be left ideals.

Theorem 3.1. Let T be a ternary semigroup.

(1) Let L be a left A-ideal of T. If A is a subset of T such that $L \subseteq A$, then A is a left A-ideal of T.
(2) Let R be a right A-ideal of T. If A is a subset of T such that $R \subseteq A$, then A is a right A-ideal of T.
(3) Let M be a middle A-ideal of T. If A is a subset of T such that $L \subseteq A$, then A is a middle A-ideal of T.
(4) Let I be an A-ideal of T. If A is a subset of T such that $I \subseteq A$, then A is an A-ideal of T.

Proof. (1) Let L be a left A-ideal of T and $t \in T$. Let A be a left A-ideal of T such that $L \subseteq A$. Then $\emptyset \neq ttL \cap L \subseteq ttA \cap A$. Thus A is a left A-ideal of T.

(2) and (3) can be proved similarly.
(4) This follows from (1), (2) and (3).

Corollary 3.2. Let T be a ternary semigroup.

(1) If L_1 and L_2 are two left A-ideals of T, then $L_1 \cup L_2$ is a left A-ideal of T.
(2) If R_1 and R_2 are two right A-ideals of T, then $R_1 \cup R_2$ is a right A-ideal of T.
(3) If M_1 and M_2 are two middle A-ideals of T, then $M_1 \cup M_2$ is a middle A-ideal of T.
(4) If I_1 and I_2 are two A-ideals of T, then $I_1 \cup I_2$ is an A-ideal of T.

Proof. (1) Let L_1 and L_2 be two left A-ideals of T. Since $L_1 \subseteq L_1 \cup L_2$, by Theorem 3.1 (1), $L_1 \cup L_2$ is a left A-ideal of T. (2), (3) and (4) can be proved similarly.

Example 3.3. Consider the ternary semigroup $T = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ under the usual addition. Let $L_1 = \{\bar{1}, \bar{3}, \bar{4}\}$ and $L_2 = \{\bar{2}, \bar{3}, \bar{5}\}$. Then L_1 and L_2 are left A-ideals of T but $L_1 \cap L_2 = \{\bar{3}\}$ is not a left A-ideal of T.

Example 3.3 shows that in general, if L_1 and L_2 are two left A-ideals of T, $L_1 \cap L_2$ need not be a left A-ideal of T.

Definition 3.2. The element $a \in T$ is called a *ternary idempotent* if $a^3 = a$.

Lemma 3.3. Let T be a ternary semigroup.

(1) T has no proper left A-ideals if and only if for all $a \in T$, there exists an element $t_a \in T$ such that $t_a t_a (T - \{a\}) = \{a\}$.

(2) T has no proper right A-ideals if and only if for all $a \in T$, there exists an element $t_a \in T$ such that $(T - \{a\}) t_a t_a = \{a\}$.

(3) T has no proper middle A-ideals if and only if for all $a \in T$, there exists an element $t_a \in T$ such that $t_a (T - \{a\}) t_a = \{a\}$.

Proof. (1) Assume T has no proper left A-ideals and let $a \in T$. Then $T - \{a\}$ is not a left A-ideal of T, this implies that there exists $t_a \in T$ such that $t_a t_a (T - \{a\}) \cap (T - \{a\}) = \emptyset$.

Then $t_a t_a (T - \{a\}) = \{a\}$. Conversely, assume that for all $a \in T$, there exists an element $t_a \in T$ such that $t_a t_a (T - \{a\}) = \{a\}$. Then for all $a \in T$, there exists an element $t_a \in T$ such that $t_a t_a (T - \{a\}) \cap \{a\} = \emptyset$. Then for all $a \in T$, $T - \{a\}$ is not a left A-ideal of T. Therefore T has no proper left A-ideal.

(2) and (3) can be proved similarly.

Theorem 3.4. Let T be a ternary semigroup such that $|T| > 1$ and $a \in T$.

(1) If $T - \{a\}$ is not a left A-ideal of T, then either a is a ternary idempotent or $a^3 = a^7$.

(2) If $T - \{a\}$ is not a right A-ideal of T, then either a is a ternary idempotent or $a^3 = a^7$.

(3) If $T - \{a\}$ is not a middle A-ideal of T, then either a is a ternary idempotent or $a^3 = a^7$.

(4) If $T - \{a\}$ is not an A-ideal of T, then either a is a ternary idempotent or $a^3 = a^7$.

Proof. (1) By Lemma 3.3(1), there exists an element $t_a \in T$ s.t. $t_a t_a (T - \{a\}) = \{a\}$. Suppose that a is not a ternary idempotent. Thus $a^3 \in T - \{a\}$, hence $t_a t_a a^3 = a$.

Case 1: $t_a = a$. So $a = a^5$, therefore $a^3 = a^7$.

Case 2: $t_a \neq a$. Since $t_a \in T - \{a\}$, $t_a t_a t_a = a$. If $t_a t_a a = a$, then $t_a t_a a^3 = a^3 \neq a$, a contradiction. Then $t_a t_a a \neq a$, we have $t_a t_a (t_a t_a a) = a$. So $t_a a a = a$, therefore $a^3 = a^7$.

(2) and (3) can be proved similarly.

(4) This follows from (1), (2) and (3).

Corollary 3.5. Let T be a ternary semigroup such that $|T| > 1$ and $a \in T$. If $a^3 \neq a^7$, then $T - \{a\}$ is an A-ideal of T.

Proof. This follows by Theorem 3.4.

4. Fuzzy A-ideals of ternary semigroups

Definition 4.1 Let T be a ternary semigroup.

(1) A fuzzy subset f of T is called a *fuzzy left A-ideal* of T if $(C_t \circ C_t \circ f) \cap f \neq 0$ for all $t \in T$.

(2) A fuzzy subset f of T is called a *fuzzy right A-ideal* of T if $(f \circ C_t \circ C_t) \cap f \neq 0$ for all $t \in T$.

(3) A fuzzy subset f of T is called a *fuzzy middle A-ideal* of T if $(C_t \circ f \circ C_t) \cap f \neq 0$ for all $t \in T$.

If f is a fuzzy left, right and middle A-ideal of T, f is called a *fuzzy A-ideal* of T.

Example 4.1 Every fuzzy left ideal of T is a fuzzy left A-ideal of T.

Example 4.2 Consider the ternary semigroup $T = \mathbb{Z}_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ under the usual addition and $f: T \to [0,1]$ defined by $f(\bar{0}) = 0, f(\bar{1}) = 1, f(\bar{2}) = 0, f(\bar{3}) = 0.2, f(\bar{4}) = 0.3$ and $f(\bar{5}) = 0$. We have

If $t = \bar{0}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{1}) = 1$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

If $t = \bar{1}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{3}) = 0.2$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

If $t = \bar{2}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{1}) = 0.2$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

If $t = \bar{3}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{1}) = 1$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

If $t = \bar{4}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{3}) = 0.2$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

If $t = \bar{5}$, we have $[(C_t \circ C_t \circ f) \cap f](\bar{1}) = 0.2$. So $(C_t \circ C_t \circ f) \cap f \neq 0$.

Then $(C_t \circ C_t \circ f) \cap f \neq 0$ for all $t \in T$. Therefore f is a fuzzy left A-ideal of T.

However, A is not a left ideal of T because let $t = \bar{2}$, we have $(C_t \circ C_t \circ f)(\bar{2}) = 0.3$ but $f(\bar{2}) = 0$, this implies $C_t \circ C_t \circ f \nsubseteq f$.

Example 4.2 shows that in general, fuzzy left A-ideals need not be fuzzy left ideals.

Theorem 4.1. Let S be a nonempty subset of a ternary semigroup T. Then

(1) S is a left A-ideal of T if and only if C_S is a fuzzy left A-ideal of T.

(2) S is a right A-ideal of T if and only if C_S is a fuzzy right A-ideal of T.

(3) S is a middle A-ideal of T if and only if C_S is a fuzzy middle A-ideal of T.

(4) S is an A-ideal of T if and only if C_S is a fuzzy A-ideal of T.

Proof. (1) Assume S is a left A-ideal of T. Then $ttS \cap S \neq \emptyset$ for all $t \in T$. Thus there exists $x \in ttS \cap S$. So $[(C_t \circ C_t \circ C_S) \cap C_S](x) \neq 0$. Hence C_S is a fuzzy left A-ideal of T. Conversely, assume C_S is a fuzzy left A-ideal of T. Thus $(C_t \circ C_t \circ C_S) \cap C_S \neq 0$ for all $t \in T$, this implies there exists $x \in T$ such that $[(C_t \circ C_t \circ C_S) \cap C_S](x) \neq 0$. Hence $x \in ttS \cap S$. Therefore $ttS \cap S \neq \emptyset$. That is S is a left A-ideal of T.

(2) and (3) can be proved similarly.
(4) This follows from (1), (2) and (3).

Theorem 4.2. Let f be a nonzero fuzzy subset of a ternary semigroup T. Then

(1) f is a fuzzy left A-ideal of T if and only if $supp\ f$ is a left A-ideal of T.

(2) f is a fuzzy right A-ideal of T if and only if $supp\ f$ is a right A-ideal of T.

(3) f is a fuzzy middle A-ideal of T if and only if $supp\ f$ is a middle A-ideal of T.

(4) f is a fuzzy A-ideal of T if and only if $supp\ f$ is an A-ideal of T.

Proof. (1) Assume f is a fuzzy left A-ideal of T. Then $(C_t \circ C_t \circ f) \cap f \neq 0$ for all $t \in T$. Then for each $t \in T$ there exist $x, y \in T$ such that $x = tty, f(x) \neq 0$ and $f(y) \neq 0$. So $x, y \in supp\ f$. This implies $(C_t \circ C_t \circ C_{supp\ f}) \cap C_{supp\ f}(x) \neq 0$. Hence $(C_t \circ C_t \circ C_{supp\ f}) \cap C_{supp\ f} \neq 0$ for all $t \in T$.

Therefore $C_{supp\ f}$ is a fuzzy left A-ideal of T. By Theorem 4.1 (1), we have $supp\ f$ is a left A-ideal of T. Conversely, assume $supp\ f$ is a left A-ideal of T. By Theorem 4.1 (1), $C_{supp\ f}$ is a fuzzy left A-ideal of T. This implies

$(C_t \circ C_t \circ C_{supp\ f}) \cap C_{supp\ f} \neq 0$ for all $t \in T$.

Then there exist $x, y \in T$ such that $x = tty$ and $x, y \in supp\ f$, so $f(x), f(y) \neq 0$. So $[(C_t \circ C_t \circ f) \cap f](x) \neq 0$. Thus $(C_t \circ C_t \circ f) \cap f \neq 0$ for all $t \in T$. Hence f is a fuzzy left A-ideal of T.

(2) and (3) can be proved similarly.
(4) This follows from (1), (2) and (3).

A fuzzy left A-ideal f is *minimal* if for all a fuzzy left A-ideal g of T such that $g \subseteq f$, we have $supp\ g = supp\ f$.

Theorem 4.3. Let S be a nonempty subset of a ternary semigroup T. Then

(1) S is a minimal left A-ideal of T if and only if C_S is a minimal fuzzy left A-ideal of T.

(2) S is a minimal right A-ideal of T if and only if C_S is a minimal fuzzy right A-ideal of T.

(3) S is a minimal middle A-ideal of T if and only if C_S is a minimal fuzzy middle A-ideal of T.

(4) S is a minimal A-ideal of T if and only if C_S is a minimal fuzzy A-ideal of T.

Proof. (1) Assume S is a minimal left A-ideal of T. By Theorem 4.1 (1), C_S is a fuzzy left A-ideal of T. Let g be a fuzzy left A-ideal of T such that $g \subseteq C_S$. By Theorem 4.2 (1), $supp\ g$ is a left A-ideal of T. Then $supp\ g \subseteq supp\ C_S = S$. Since $g \subseteq C_{supp\ g}$, we have

$$(C_s \circ C_s \circ g) \cap g \subseteq (C_s \circ C_s \circ C_{supp\ g}) \cap C_{supp\ g}.$$

Hence $C_{supp\ g}$ is a fuzzy left A-ideal of T. By Theorem 4.2 (1), $supp\ g$ is a left A-ideal of T. Since S is minimal, $supp\ g = S = supp\ C_S$. Therefore C_S is minimal.

Conversely, assume C_S is a minimal fuzzy left A-ideal of T. By Theorem 4.1 (1), S is a left A-ideal of T. Let L be a left A-ideal of T such that $L \subseteq S$. By Theorem 4.1 (1), C_L is a fuzzy left A-ideal of T such that $C_L \subseteq C_S$. Hence $L = supp\ C_L = supp\ C_S = S$. Therefore S is minimal.

(2), (3) and (4) can be proved similarly.

Corollary 4.4. Let S be a nonempty subset of a ternary semigroup T. Then

(1) S has no proper left A-ideal of T if and only if for all a fuzzy left A-ideal f of T, $supp\ f = S$.

(2) S has no proper right A-ideal of T if and only if for all a fuzzy right A-ideal f of T, $supp\ f = S$.

(3) S has no proper middle A-ideal of T if and only if for all a fuzzy middle A-ideal f of T, $supp\ f = S$.

(4) S has no proper A-ideal of T if and only if for all a fuzzy A-ideal f of T, $supp\ f = S$.

Proof. (1) Let f be a fuzzy left A-ideal of T. By Theorem 4.2 (1), $supp\ f$ is a left A-ideal of T. Since S has no proper left A-ideal, $supp\ f = S$.

Conversely, let f is a fuzzy left ideal of T and $supp\ f = S$. Suppose that L is proper left A-ideal of T. By Theorem 4.1 (1), C_L is a fuzzy left A-ideal of T. Thus $supp\ C_L = L \neq S$, a contradiction. Hence S has no proper left A-ideal of T.

(2), (3) and (4) can be proved similarly.

5. Conclusions

In the structural theory of fuzzy algebraic systems, fuzzy ideals with special properties always play an important role. In this paper, we applied the fuzzy set theory to A-ideals of ternary semigroups to characterize fuzzy A-ideals.

Acknowledgements

This paper was supported by Algebra and Applications Research Unit, Prince of Songkla University. The first author is grateful to Development and Promotion of Science and Technology (DPST) for financial support. The authors would like to thank the referees for their valuable comments and suggestions.

References

Grosek, O. (1979). Entropy on algebraic structures (Russian). *Mathematica Slovaca*, *29*, 411-424.

Grosek, O., & Satko, L. (1980). A new notion in the theory of semigroup. *Semigroup Forum*, *20*, 233-240.

Grosek, O., & Satko, L. (1981). Smallest A-ideals in semigroups. *Semigroup Forum*, *23*, 297-309.

Kuroki, N. (1981). On fuzzy ideals and bi-ideals in semigroups. *Fuzzy Sets and System*, *5*(2), 203-215.

Kuroki, N. (1979). Fuzzy bi-ideals in semigroups. *Commentarii Mathematici Universitatis Santi Pauli, 28*, 17-21.

Kuroki, N. (1991). On fuzzy semigroups. *Information Sciences*, *53*(3), 203-236.

Lehmer, D. H. (1932). A ternary analogue of abelian groups. *American Journal of Mathematics*, *54*(2), 329-338.

Los, J. (1955). On the extending of models I. *Fundamenta Mathematicae*, *42*, 38-54.

Rosenfeld, A. (1971). Fuzzy groups. *Journal of Mathematical Analysis Applications*, *35*, 512-517.

Santiago, M. L., & Sri Bala, S. (2010). Ternary semigroups. *Semigroup Forum*, *81*, 380-388.

Satko, L., & Grosek, O. (1981). On minimal A-ideals of semigroups. *Semigroup Forum*, *23*, 283-295.

Satko, L., & Grosek, O. (1981). On maximal A-ideals in semigroups. *Colloquia Mathematica Societatis Janos Bolyai*, *39*, 389-395.

Sioson, F. M. (1965). Ideal theory in ternary semigroups. *Japanese Journal of Mathematics*, *10*, 63-84.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338-353.