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Original Article

Penetrate field behavior in superconducting shield by modified Beer-Lambert model: Applied to cylindrical MgB_2 superconductors

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Abstract

Beer-Lambert model is a relation concerning the absorption of radiant energy by an absorbing medium. In this paper, we modified the Beer-Lambert model to investigate the penetrating magnetic field behavior in a superconducting shield by taking into account the effect of a power-law term of previous penetrating magnetic fields internal materials. The penetrating fields were derived analytically and applied to cylindrical MgB_2 superconductors. We found that our calculation results of penetrating field fit well with experimental data of cylindrical MgB_2 superconductors in the range of 0.1-0.6 T.

Keywords: penetrating field, Beer-Lambert model, superconducting shield, MgB_2, power-law

1. Introduction

Superconductivity is a property displayed by certain materials at very low temperature. We call the material that can show the superconductivity phenomena as superconductor. One of the properties of superconductor is that it will exclude magnetic fields, this phenomenon called the Meissner effect (Ketterson & Song, 1999). Because of this effect, some can be made to float endlessly above a strong magnetic field. The magnetic shielding is one of the interesting applications of superconductors (Cavallin, Quarantiello, Matrone, & Giunchi, 2006). Isolation from external magnetic fields is a fundamental requirement that appears in various fields. Among them is the need to protect people and instruments from the high stray fields of magnetic sources or, alternatively, to avoid the ambient electromagnetic noise acting on very high-sensitive magnetic sensors, such as SQUIDs (Gozzelino *et al.*, 2013; Rabbers, Oomen, Bassani, Ripamonti, & Giunchi, 2010). In particular, for applications where static or slowly varying magnetic fields need to be screened, enclosures made of superconductors or high magnetic permeability metal alloys are typically utilized (Narayana & Sato, 2011). Denis *et al.* (2007) studied the magnetic shielding properties of a cylindrical shell of BiPbSrCaCuO subjected to low frequency AC axial magnetic fields. The magnetic response had been investigated as a function of the dimensions of the tube, the magnitude of the applied field and the frequency. Douine *et al.* (2014) improved YBCO superconductor magnetic shielding by using multiple bulks. Lousberg, Fagnard, Ausloos, Vanderbemden, and Vander bemden, (2010) investigated the magnetic shielding properties of hybrid ferromagnetic/superconductor structures consisting of two coaxial cylinders, with one of each material by axisymmetric finite-element model. Kulinov *et al.* (2014) showed the shielding opportunity of the magnetic field

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perpendicular component by a high-temperature superconductor (Y123) tape. Classsen, Namburi, and Vanderbemden (2015) evaluated the potential of a thin film technology to deposit YBCO film by the electrophoretic deposition on a silver substrate of arbitrary shape. Mutual inductance measurements could predict the maximum external field that could be excluded as well as the residual transmitted field at lower external fields. Posen, Transtrum, Catelani, Liepe, and Sethna (2015) evaluated the shielding potential of the superconducting-film-insulating-film-superconductor (SIS′) structure, a configuration that could provide benefits in screening large ac magnetic fields.

The magnetic shields made of low-temperature superconductor require elevated refrigeration costs while high-temperature superconductor cuprated need to overcome brittleness problems that make the system scalability difficult. MgB_2 is a good compromise between the need of working at intermediate temperature and reducing the manufacturing cost (Gozzelino *et al.*, 2013). Cavallin, Quarantiello, Matrone, & Giunchi (2006) investigated shielding efficiency of several tubular MgB2objects at 4.2 K in applied DC and AC field. Giunchi, Bassani, Cavallin, Bancone, and Pavese (2007) presented results indicate that the adopted geometry was suitable for the fabrication of a cryogenic current comparator (CCC) with high-temperature superconductor superconducting shield, using bulk MgB_2 produced by applying the reactive liquid Mg infiltration (RLI) technology. Rabbers *et al.* (Rabbers *et al.*, 2010) reported magnetic shielding capability of MgB_2 cylinders in DC magnetic field. A complete shielding capability had been verified up to the applied field of 2 T. Gozzelino *et al.* (Gozzelino *et al.*, 2013) studied the material magnetic shielding properties of disk-shaped MgB_2 bulk samples synthesized by the spark-plasma-sintering technique. Spatial distributions of the axial component of the shielding magnetic-induction field generated by the superconductor were evaluated in the temperature range 20-36 K and in applied magnetic field up to 1.5 T. Gozzelino *et al.* (2013) studied the magnetic shielding properties of a MgB_2/Fe hybrid structure consisting of two coaxial cups subjected to a magnetic field applied parallel to their axis. Arpaia, Ballarino, Giunchi and Montenero (2014) designed cylindrical hollow MgB_2 shields for cryogenic measurement devices operating in background fields of 1 T. A design case study for the shield of a cryogenic DC current transformer was reported.

The Beer-Lambert model is a relation concerning the absorption of radiant energy by an absorbing medium. Formulated by Johann Heinrich Lambert and August Beer. Lambert's law stated that absorbance of a material sample is directly proportional to its thickness (path length) in 1760. Much later, August Beer discovered another attenuation relation in 1852. Beer's law stated that absorbance is proportional to the concentrations of the attenuating species in the material sample. The Beer–Lambert model combines the two laws and correlates the absorbance to both the concentrations of the attenuating species as well as the thickness of the material sample (Ingle & Crouch, 1988). In analytical chemistry field, Beer–Lambert model used to measure the concentration of the solution by spectrophotometer. Moreover, Beer–Lambert model can be modified and applied in many fields that used an optical technique. Baker *et al.* (2014) developed and validated a Beer-Lambert model for blood flow based on diffuse correlation spectroscopy (DCS) measurements. Bhatt, Ayyalasomayajula and Yalavarthy (2016) modified Beer-Lambert model to described the attenuation of near-infrared (NIR) light intensity as it propagates in a turbid medium like biological tissue. Huong and Ngu (2014) presented the use of extended Modified Beer-Lambert model for accurate and continuous monitoring of percent blood carboxyhemoglobin (COHb) (SCO) and oxyhemoglobin (Oxy Hb) saturation (SO_2) via a fitting procedure. Changjan, Punchoo & Udomsamuthirun (2014) modified Beer-Lambert model to describe behaviour of magnetic field attenuation by superconducting cylinders. The penetrate field, London penetration depth and the attenuation of applied magnetic field are derived analytically and applied to cylindrical $Tl_2Ba_2Ca_2O_{10}$ superconductors.

In this paper, we developed Beer-Lambert model to describe behavior of penetrate magnetic field into superconducting cylinders by take into account the effect due to the power law term of previous penetrate magnetic fields internal materials. Finally, the penetrate field was investigated and applied to cylindrical MgB_2 superconductors (Arpaia *et al.*, 2014).

2. Materials and Methods

In optics, the Beer–Lambert model relates the absorption of light to the properties of the material through and path length which the light is travelling (Ingle & Crouch, 1988). From Figure 1(a), it is assumed that a beam of incident light Φ_0 enters a material sample. $\Phi(z)$ denotes the beam of penetrate light. Define z as an axis parallel to the direction of the beam. Divide the material sample into thin slices dz perpendicular to the beam of light. The radiant flux of the light that emerges from a slice is reduced, compared to that of the light that entered, by $d\Phi(z) = -\mu\Phi(z)dz.$, where μ is the attenuation coefficient. Beer-Lambert model can describe behavior of penetrate light as $\Phi(z) = \Phi_{\circ} e^{-\mu z}$. For describe behavior of magnetic field attenuation by HTS cylindrical magnetic shields, comparison between Figure 1(a) versus Figure 1(b), modified Beer-Lambert model relates the penetrate fields and distance that penetrate fields travels through the material (Changjan *et al.*, 2014) can be written as $\frac{1}{B(z)} \cdot \frac{dB(z)}{dz} = -\alpha.$ Here α is magnetic absorption coefficient of the substance. $B(z)$ and B_0 is the penetrate field and initial incident field respectively. From this case, Beer-Lambert model can describe behavior of penetrate field as $B(z) = B_0 e^{-\alpha z}$.

In this communication, we modified Beer-Lambert model for penetrate field by take into account the effect due to the power law term of previous penetrate magnetic fields internal materials as

$$\frac{1}{B(z)} \cdot \frac{dB(z)}{dz} = -\alpha - \beta[B(z)]^n, \tag{1}$$

where n is proportional constant. β is coupling magnetic absorption coefficient of the material. Let $m = [B(z)]^{-n}$, then

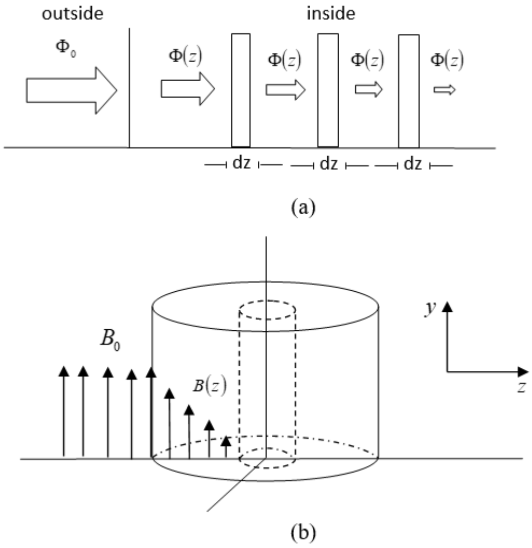

Figure 1. (a) The incident light penetrate into material in z direction, (b) The external magnetic fields penetrate into HTS cylindrical magnetic shields in z direction.

$$\frac{dB(z)}{dz} = \frac{1}{-n[B(z)]^{-(n+1)}} \cdot \frac{dm}{dz}. \tag{2}$$

Substitution Equation (2) into Equation (1), thus $\frac{dm}{dz} = n(\beta + \alpha m)$. We get $m = \frac{Ae^{\alpha nz} - \beta}{\alpha}$, here A is an arbitrary constant. Consequently, a general solution of penetrate field can be presented by $[B(z)]^n = \frac{\alpha}{Ae^{\alpha nz} - \beta}$. From boundary condition $B(0) = B_{\circ}$ the penetrate field at any position inside cylindrical superconductors is given by

$$B(z) = \left[\frac{\alpha}{\frac{\alpha e^{\alpha nz}}{B_{\circ}^{n}} + \beta\left(e^{\alpha nz} - 1\right)} \right]^{\frac{1}{n}}. \tag{3}$$

Our calculation can be reduced to ordinary modified Beer-Lambert formula (Changjan *et al.*, 2014) for penetrate magnetic field into superconducting material by setting n=1 and reduced to normal Beer-Lambert formula by setting $\beta = 0$.

3. Results and Discussion

The magnetic attenuation of cylindrical $Tl_2Ba_2Ca_2Cu_3O_{10}$ superconductors had been studied by Changjan *et al.* (Changjan *et al.*, 2014). The penetrate field and London penetration depth were derived analytically. The calculation of attenuation of applied magnetic field could be coincided with the experimental data of cylindrical Tl-based superconductors at low field scenery. In this paper, we compared the results of the penetrate field from our calculations with the experimental data of cylindrical MgB_2 superconductors (Arpaia *et al.*, 2014). The superconducting material was synthesized by the reactive Mg liquid infiltration process (Giunchi, 2003; Giunchi *et al.*, 2006). The tube that used in this experiment had 40 and 4.5 mm in the height and thickness respectively. The external magnetic field was generated by a superconducting solenoid capable of providing up to 11 T. High-sensitivity cryogenic Hall probe was mounted inside the tube with the sensing area perpendicular to the penetrate field direction, at the half height of the cylinder, and at distance 6 mm from its inner wall (Probe I) (Arpaia *et al.*, 2014). In Figure 2, the comparison between the penetrate field measured by the Hall probe inside the MgB_2 cylinder (open circle) and the results from our calculations (Equation 3) was

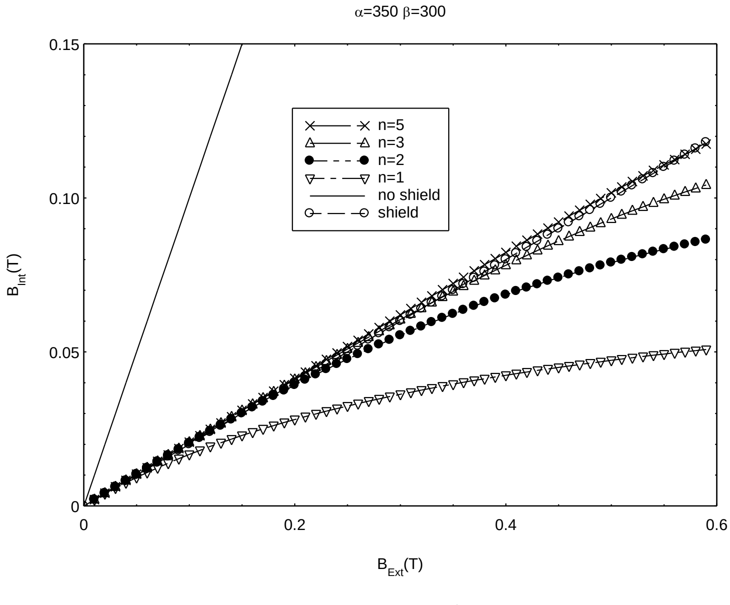

Figure 2. Relationship between internal and external field comparison between results of calculations with experimental data of MgB_2 cylinder (open circle).

illustrated. The solid line represented the field without shielding. The experiment result showed the relationship between internal (penetrate) and external field was linearly increase with increasing the external magnetic field. We could found that our numerical calculation based on n=5 could be fit well with experimental data in the range of 0.1-0.6 T, following parameters were used as $\alpha = 350$ and $\beta = 300$. For ordinary modified Beer-Lambert case (n=1), the result of calculation was diverted from experimental data at all range of external magnetic fields. For n=2 and 3, the result of calculation could be fit well with experimental data when the external field was lower than 0.2 T and 0.4 T respectively. This result was due to the presence of the effect of previous penetrating magnetic field influence with the following field significantly.

Changjan *et al.* (2014) used the modified Beer-Lambert model to describe behavior of magnetic field attenuation by Tl-based superconducting shield. The penetrate field, London penetration depth and the attenuation of applied magnetic field are derived analytically. The experimental data showed nonlinear behavior of the attenuation of applied magnetic fields and our results of calculation could be fit well with experimental data at low field scenery. From our model the relationship between external and internal magnetic field in superconducting shielding was investigated by modified Beer-Lambert model that taking into account the effect of a power-law term of previous penetrate magnetic fields internal materials, the result of calculations showed that the effect of the power law term of internal magnetic field was related to the penetrate fields.

4. Conclusions

The behavior of penetrate magnetic field into cylindrical superconductors was investigated by modified Beer-Lambert model analytically. The effects due to the power law term of previous penetrate magnetic fields internal materials were considered. The penetrate field was calculated and applied to cylindrical MgB_2 superconductors. We found that our calculation results of penetrate field based on n=5, $\alpha = 350$ and $\beta = 300$ could be fit well with experimental data of cylindrical MgB_2 superconductors in the range of 0.1-0.6 T. This result showed that the effect of the power law term of internal magnetic field would be significantly to the penetrate fields especially at low external magnetic field scenery.

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