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Original Article

Some new operations on fuzzy soft sets and their applications in decision-making

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Abstract

This paper aims to introduce various types of useful operations on fuzzy soft sets (FSSs), such as Einstein sum, Einstein product, algebraic sum, algebraic product, bounded product, bounded sum, and the basic properties of these new operations have been investigated. In addition, on the basis of our newly defined operations, we implement a new machine learning algorithm to solve FSS based real-life decision-making problems (DMPs), and using one real-life example we have demonstrated the viability of our proposed approach in real-world applications.

Keywords: decision-making, soft set, fuzzy set, fuzzy soft set

1. Introduction

Soft set theory (SST) was first proposed by Molodtsov (1999) as a fundamental and useful mathematical method for dealing with complexity, unclear definitions, and unknown objects (elements). Since there are no limitations to the description of elements in SST, researchers may choose the type of parameters that they need, simplified DMPs are easier to make decisions in absence of partial knowledge, and significantly it becomes more effective. While several mathematical tools for modeling uncertainties are available, such as operations analysis, probability theory, game theory, fuzzy set, rough set, and interval-valued fuzzy set, intuitionistic fuzzy set, each of these theories has inherent difficulties. Due to lack of parameterization, these tools are not suitable for solving the problems, especially in the economic, environmental, and social realms. To remove such restrictions, Molodtsov (1999) introduced the popular concept of SST.

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The SST is extremely useful in a variety of situations. Molodtsov (1999) developed the basic results of SST and successfully applied it to various fields, including the smoothness of functions, operations analysis, game theory, Riemann integration, probability, and so on. Later, Maji, Biswas, and Roy (2003) presented several new SST concepts, such as subset, complements, union, and intersection, as well as their implementations in DMPs. Ali, Feng, Liu, Min, and Shabir (2009) identified some more operations on SST and demonstrated that De Morgan's laws are also applicable to these new operations in SST. Thereafter, several researchers doing their innovative research work in this theory and applied in various field. To solve the DMPs, Maji, Roy, and Biswas (2002) used SST for the first time. Very recently, several authors implemented the properties and applications of SST in broader perspectives. Recently, Zhan and Alcantud (2019) introduced the concept of soft rough covering and demonstrated how it can be used in DMPs. Rajput, Thakur, and Dubey (2020) defined soft almost ββ-continuity in soft topological spaces. Dalkılıç (2021) introduced a novel approach to SST-based DM under uncertainty.

Since Zadeh (1965) introduced the idea of fuzzy sets, several new approaches and theories for dealing with imprecision and ambiguity have been proposed. Deng and Jiang (2019) presented the D number theory based game-theoretic framework in adversarial DM under a fuzzy environment. Mandal and Ranadive (2019) introduced the concept of multi-granulation fuzzy probabilistic rough sets and their corresponding three-way decisions over two universes. Thereafter, Wan Mohd, Abdullah, Yusoff, Taib, and Merigo (2019) presented an integrated multi criteria decision making (MCDM) model based on the Pythagorean fuzzy sets for a green supplier development program. Li, Liu, Dai, Chen, and Fujita (2020) described the measures of uncertainty based on the Gaussian kernel for a fully fuzzy information system.

Maji, Biswas, and Roy (2001) described FSSs by combining soft sets and fuzzy sets, which have a lot of potential for solving DMPs. The applications of FSS theory have been gradually concentrated by using these concepts. Kong, Gao, and Wang (2009) proposed an FSS theoretic approach to DMPs and Feng, Jun, Liu, and Li (2010) introduced an adjustable method to solve FSS-based DMPs. Thereafter, Zhu and Zhan (2015) described and presented the t-norm operations on fuzzy parameterized FSSs, as well as their applications in DM. Vimala, Reeta, and Ilamathi (2018) studied the fuzzy soft cardinality in lattice ordered fuzzy soft group and shown its application in DMPs. Peng and Li (2019) proposed an algorithm for hesitant fuzzy soft DM based on revised aggregation operators. Shakila and Selvi (2019) had written a note on fuzzy soft paraopen sets and maps in fuzzy soft topological space (FSTS) and Nihal (2020) studied the concept of pasting lemma on an FSTS with mixed structure. Lathamaheswari, Nagarajan, and Kavikumar (2020) introduced the concept of triangular interval type-2 FSS and also, shown its applications. Petchimuthu, Garg, Kamaci, and Atagün (2020) defined the mean operators and generalized products of fuzzy soft matrices and discussed their applications in MCGDM. Smarandache, Parimala, and Karthika (2020) reviewed the concepts FSTS, intuitionistic FSTS, and neutrosophic soft topological spaces. Paik and Mondal, (2020) introduced a distance-similarity method to solve fuzzy sets and FSSs based DMPs. Paik and Mondal (2021) had shown the representation and application of FSSs in a type-2 environment. Močkoř and Hurtik (2021) used the concept FSSs in image processing applications. Gao and Wu (2021) defined filter and its applications in FSTSs. Dalkılıç and Demirtaş (2021) introduced the idea of bipolar fuzzy soft D-metric spaces. Dalkiliç (2021) defined topology on virtual fuzzy parametrized-FSSs. Bhardwaj and Sharma (2021) described an advanced uncertainty measure using FSSs and shown its application in DMPs. Khan, Mahmood, and Hassan (2019) introduced the idea of multi Q-single valued neutrosophic soft expert set and its application in DMPs. Riaz and Hashmi (2019) defined linear diophantine fuzzy sets as well as applications towards MCDM problems. Lee (2000) developed bipolar valued fuzzy sets and defined their operations. Hussain, Ali, and Mahmood (2020) presented the concepts of Pythagorean fuzzy soft rough sets and their applications in decision-making. Mahmood (2020) proposed a novel approach towards bipolar soft sets and presented their applications in DMPs. Hussain, Ali, Mahmood, and Munir (2020) defined q-Rung orthopair fuzzy soft average aggregation operators and shown their applications in MCDM. Siddique, Mahmood, and Jan (2020) presented double framed soft rings. Riaz and Tehrim (2020) studied some concepts of bipolar fuzzy soft topology with decision-making and Riaz, Davvaz, Fakhar, and Firdous (2020) introduced the idea of hesitant fuzzy soft topology and its applications to multi-attribute group DMPs. Farid & Riaz (2021) introduced some generalized q-Rung orthopair fuzzy Einstein interactive geometric aggregation operators with improved operational laws and Iampan, Garcia, Riaz, Farid, and Chinram (2021) developed linear diophantine fuzzy Einstein aggregation operators for MCDM problems.

In this paper, we have introduced various types of useful operations on FSSs, such as Einstein sum, Einstein product, algebraic sum, algebraic product, bounded product, bounded sum, and the basic properties of these new operations have been investigated. In addition, on the basis of our newly defined operations, we implement a new machine learning algorithm to solve FSS based real-life DMPs, and using one real-life example we have demonstrated the viability of our proposed approach in real-world applications. These new operations are very helpful to solve real-world DMPs, deciding what kind of strategy to use based on the circumstances so that we can make a well decision and the benefits of these operations are not the same. By applying these operations, we can be connected them to numerous fields that contain questionable ties. The following is the structure of this paper: In the preliminary section, various definitions related to this concept are discussed. In section 3, we define various types of FSS operations such as Einstein product, Einstein sum, algebraic sum, algebraic product, bounded product, and bounded sum, and their fundamental properties are to be studied. Finally, we implement a new approach to FSS-based DM in an unpredictable situation using our newly defined operations in section 4.

2. Preliminaries

Let U stand for a nonempty universe, E for a nonempty set of parameters, P(U) for the power set of U and let $A \subseteq E$.

2.1 Definition (**Zadeh, 1965**) A fuzzy set Y on U is a set with the form $Y = \{(u, \mu_Y(u)) : u \in U\}$, where the function $\mu_Y : U \to [0, 1]$ is called the membership function and $\mu_Y(u)$ represents each element's degree of membership. If $\mu_Y(u) = 1, \ \forall u \in U$, then X becomes a regular (crisp) set. The class of all fuzzy sets on U is denoted by FS (U).

2.2 Definition (**Zadeh, 1965**) Let $\Gamma, \Psi \in FS(U)$. Then the union of Γ and Ψ denoted by $\Gamma \bigcup \Psi$, is a fuzzy set defined by

$$\Gamma \bigcup \Psi = \left\{\left(x, \max\left(\mu_\Gamma(x), \mu_\Psi(x)\right)\right) : x \in U\right\}.$$

2.3 Definition (**Zadeh, 1965**) Let $\Gamma, \Psi \in FS(U)$. Then the intersection of Γ and Ψ, denoted by $\Gamma \bigcap \Psi$, is a fuzzy set defined by

$$\Gamma \bigcap \Psi = \left\{\left(x, \min\left(\mu_\Gamma(x), \mu_\Psi(x)\right)\right) : x \in U\right\}$$

2.4 Definition (**Zadeh, 1965**) Let $\Gamma, \Psi \in FS(U)$. Then complement of Γ, denoted by Γ^c, is a fuzzy set defined by

$$\Gamma^c = \left\{\left(x,\ 1\text{-}\mu_\Gamma(x)\right): x \in U\right\}$$

2.5 Definition (**Zadeh, 1965**) Let $\Gamma, \Psi \in FS(U)$. Then Γ is said to be a fuzzy subset of Ψ, denoted by $\Gamma \subseteq \Psi$ if

$$\mu_\Gamma(x) \le \mu_\Psi(x),\ \forall x \in U.$$

2.6. **Definition** (**Molodtsov, 1999**) Soft set on U refers to a couple (*F, A*), where *F: A*→ *P*(*U*) is a function.

2.7. **Definition** (**Maji, Biswas, & Roy, 2001**) An FSS over U is a pair (F, A), where F is a function given by F*:* A→FS(U). Simply put, the set of all FSSs on U as $FSS(U)$.

2.8. **Definition** (**Maji, Biswas, & Roy, 2001**) The union of two FSSs (F, A) and (G, B) over U is an FSS (H, C), where $C = A \cup B$ and $\forall e \in C,\ x \in U$

$$\mu_{H(e)}(x) = \begin{cases} \mu_{F(e)}(x), & \text{if } e \in A\text{-}B \\ \mu_{G(e)}(x), & \text{if } e \in B\text{-}A \\ \max\left(\mu_{F(e)}(x), \mu_{G(e)}(x)\right), & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \bigcup (G, B) = (H, C)$.

2.9 Definition (**Maji, Biswas, & Roy, 2001**) The intersection of two FSSs (F, A) and (G, B) over U is an FSS (H, C), where $C = A \cup B$ and $\forall e \in C,\ x \in U$

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We write $(F, A) \bigcap (G, B) = (H, C)$.

3. Some New Operations on FSSs

In this present section, we define various types of FSS operations such as Einstein product, Einstein sum, algebraic sum, algebraic product, bounded product, and bounded sum, and their fundamental properties are to be studied.

3.1 Definition Let $(\psi, A),\ (\varphi, B) \in FSS(U)$. Then the Einstein product of (ψ, A) and (φ, B), denoted by $(\psi, A) o (\varphi, B)$ is an FSS $(\sigma, A \times B)$, where $\forall (\alpha, \beta) \in A \times B, \forall u \in U$,

$$\mu_{\sigma(\alpha,\beta)}(u) = \frac{\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}{2 - [\mu_{\psi(\alpha)}(u) + \mu_{\varphi(\beta)}(u) - \mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)]}.$$

3.2 Definition Let $(\psi, A),\ (\varphi, B) \in FSS(U)$. Then the Einstein sum of (ψ, A) and (φ, B), denoted by $(\psi, A) o (\varphi, B)$ is an FSS $(\sigma, A \times B)$, where $\forall (\alpha, \beta) \in A \times B$,

$$\mu_{\sigma(\alpha,\beta)}(u) = \frac{\mu_{\psi(\alpha)}(u) + \mu_{\varphi(\beta)}(u)}{1 + \mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)},\ \forall u \in U.$$

3.3 Proposition Let $(\psi, A),\ (\varphi, B) \in FSS(U)$. Then

$[i]\ (\psi, A) o (\varphi, B) = (\varphi, B) o (\psi, A);$

a.

$[ii]\ (\psi, A) \square (\varphi, B) = (\varphi, B) \square (\psi, A).$

3.4. **Proposition** Let $(\psi, A),\ (\varphi, B), (\sigma, C) \in FSS(U)$. Then b.

$[i]\ (\psi,A)\square\big((\varphi,B)\square(\sigma,C)\big)=\big((\psi,A)\square(\varphi,B)\big)\square(\sigma,C);$

$[ii]\ (\psi,A)o\big((\varphi,B)o(\sigma,C)\big)=\big((\psi,A)o(\varphi,B)\big)o(\sigma,C).$

Proof. [i] Let us consider three FSSs $(\psi,A),\ (\varphi,B),(\sigma,C)\in FSS(U)$ and let $(\varphi,B)\square(\sigma,C)=(\Omega,B\times C),$ where $\forall(\beta,\gamma)\in B\times C,$ the membership value of u in $(\Omega,B\times C)$ is $\forall u\in U$.

$$\mu_{\Omega(\beta,\gamma)}(u)=\frac{\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)}{1+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)},\ \text{[using definition 3.2]}$$

Then $(\psi,A)\square\big((\varphi,B)\square(\sigma,C)\big)=(\psi,A)\square(\Omega,B\times C).$ Let $(\psi,A)\square(\Omega,B\times C)=(\Psi,A\times B\times C),$ where $\forall(\alpha,\beta,\gamma)\in A\times B\times C$ $\forall u\in U$

$$\mu_{\Psi(\alpha,\beta,\gamma)}(u)=\frac{\mu_{\psi(\alpha)}(u)+\mu_{\Omega(\beta,\gamma)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\Omega(\beta,\gamma)}(u)}$$

$$=\frac{\mu_{\psi(\alpha)}(u)+\left(\dfrac{\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)}{1+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}\right)}{1+\mu_{\psi(\alpha)}(u).\left(\dfrac{\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)}{1+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}\right)},\quad\left[\text{since } \mu_{\Omega(\beta,\gamma)}(u)=\frac{\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)}{1+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)},\right]$$

$$=\frac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)+\mu_{\psi(\alpha)}(u).\mu_{\sigma(\gamma)}(u)+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}\quad(1)$$

Also, let $(\psi,A)\square(\varphi,B)=(\Gamma,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$

$$\mu_{\Gamma(\alpha,\beta)}(u)=\frac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)},\ \text{[using definition 3.2]}.$$

Therefore, $\big((\psi,A)\square(\varphi,B)\big)\square(\sigma,C)=(\Gamma,A\times B)\square(\sigma,C).$

Let $(\Gamma,A\times B)\square(\sigma,C)=(\Upsilon,A\times B\times C),$ where $\forall(\alpha,\beta,\gamma)\in A\times B\times C,\quad\forall u\in U$

$$\mu_{\Upsilon(\alpha,\beta,\gamma)}(u)=\frac{\mu_{\Gamma(\alpha,\beta)}(u)+\mu_{\sigma(\gamma)}(u)}{1+\mu_{\Gamma(\alpha,\beta)}(u).\mu_{\sigma(\gamma)}(u)}$$

$$=\frac{\left(\dfrac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}\right)+\mu_{\sigma(\gamma)}(u)}{1+\left(\dfrac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}\right).\mu_{\sigma(\gamma)}(u)},\quad\left[\text{since } \mu_{\Gamma(\alpha,\beta)}(u)=\frac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}\right]$$

$$=\frac{\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}{1+\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)+\mu_{\psi(\alpha)}(u).\mu_{\sigma(\gamma)}(u)+\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)}$$

$$=\mu_{\Psi(\alpha,\beta,\gamma)}(u),\ \text{[using (1)]}$$

Thus $(\psi,A)\square\big((\varphi,B)\square(\sigma,C)\big)=\big((\psi,A)\square(\varphi,B)\big)\square(\sigma,C).$

The proof of [ii] is similar to that of [i].

3.5 Proposition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then

$[i]\ \big((\psi,A)o(\varphi,B)\big)^c=(\psi,A)^c\square(\varphi,B)^c;$

$[ii]\ \big((\psi,A)\square(\varphi,B)\big)^c=(\psi,A)^c o(\varphi,B)^c.$

Proof. Let us consider two FSSs $(\psi,A),\ (\varphi,B)\in FSS(U)$ and let $(\psi,A)o(\varphi,B)=(\Gamma,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,$

$$\mu_{\Gamma(\alpha,\beta)}(u)=\frac{\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}{2-[\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)]},\forall u\in U \qquad (2)$$

Then, $\left((\psi,A)o(\varphi,B)\right)^c=(\Gamma,A\times B)^c=(\Gamma^c,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$

$$\mu_{\Gamma^c(\alpha,\beta)}(u)=1-\mu_{\Gamma(\alpha,\beta)}(u)=1-\frac{\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)}{2-[\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)]},\quad [\text{using (2)}]$$

$$=\frac{2-\mu_{\psi(\alpha)}(u)-\mu_{\varphi(\beta)}(u)}{2-[\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)]} \qquad (3)$$

Also, $(\psi,A)^c=(\psi^c,A),(\varphi,B)^c=(\varphi^c,B)$ and let $(\psi,A)^c\square(\varphi,B)^c=(\Omega,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$

$$\mu_{\Omega(\alpha,\beta)}(u)=\frac{1-\mu_{\psi(\alpha)}(u)+1-\mu_{\varphi(\beta)}(u)}{1+(1-\mu_{\psi(\alpha)}(u)).(1-\mu_{\varphi(\beta)}(u))},\quad [\text{using definition 3.2}].$$

$$=\frac{2-\mu_{\psi(\alpha)}(u)-\mu_{\varphi(\beta)}(u)}{2-[\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)]}$$

$$=\mu_{\Gamma^c(\alpha,\beta)}(u),\ [\text{using (3)}]$$

Thus $\left((\psi,A)o(\varphi,B)\right)^c=(\psi,A)^c\square(\varphi,B)^c.$

The proof of [ii] is similar to that of [i].

3.6 Definition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then the algebraic product of (ψ,A) and $(\varphi,B),$ denoted by $(\psi,A)@^{\times}(\varphi,B)$ is an FSS $(\sigma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,\ \mu_{\sigma(\alpha,\beta)}(u)=\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u),\ \forall u\in U.$

3.7 Definition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then the algebraic sum of (ψ,A) and $(\varphi,B),$ denoted by $(\psi,A)@^{+}(\varphi,B)$ is an FSS $(\sigma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,$

$$\mu_{\sigma(\alpha,\beta)}(u)=\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u),\forall u\in U.$$

3.8. Proposition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then

a. $[i]\ (\psi,A)@^{\times}(\varphi,B)=(\varphi,B)@^{\times}(\psi,A);$

$[ii]\ (\psi,A)@^{+}(\varphi,B)=(\varphi,B)@^{+}(\psi,A).$

3.9. Proposition Let $(\psi,A),\ (\varphi,B),(\sigma,C)\in FSS(U)$. Then

b. $[i]\ (\psi,A)@^{\times}\left((\varphi,B)@^{\times}(\sigma,C)\right)=\left((\psi,A)@^{\times}(\varphi,B)\right)@^{\times}(\sigma,C);$

$[ii]\ (\psi,A)@^{+}\left((\varphi,B)@^{+}(\sigma,C)\right)=\left((\psi,A)@^{+}(\varphi,B)\right)@^{+}(\sigma,C).$

Proof. [i] Let us consider three FSSs $(\psi,A),\ (\varphi,B),(\sigma,C)\in FSS(U)$ and let $(\varphi,B)@^{\times}(\sigma,C)=(\Omega,B\times C),$ where $\forall(\beta,\gamma)\in B\times C,$ $\forall u\in U$

$$\mu_{\Omega(\beta,\gamma)}(u)=\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u),\quad [\text{using the Definition 3.6}] \qquad (4)$$

Then $(\psi,A)@^{\times}\left((\varphi,B)@^{\times}(\sigma,C)\right)=(\psi,A)@^{\times}(\Omega,B\times C).$

Let $(\psi,A)@^{\times}(\Omega,B\times C)=(\Psi,A\times B\times C),$ where $\forall(\alpha,\beta,\gamma)\in A\times B\times C$

$$\mu_{\Psi(\alpha,\beta,\gamma)}(u)=\mu_{\psi(\alpha)}(u).\mu_{\Omega(\beta,\gamma)}(u),\quad [\text{From the Definition 3.6}]$$

$$=\mu_{\psi(\alpha)}(u).\left(\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)\right),\ [\text{using (4)}]$$

$$=\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u),\forall u\in U. \qquad (5)$$

Also, let $(\psi,A)@^{\times}(\varphi,B)=(\Gamma,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$

$\mu_{\Gamma(\alpha,\beta)}(u)=\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u),$ [From the Definition 3.6] (6)

Therefore, $\big((\psi,A)@^{\times}(\varphi,B)\big)@^{\times}(\sigma,C)=(\Gamma,A\times B)@^{\times}(\sigma,C).$

Let $(\Gamma,A\times B)@^{\times}(\sigma,C)=(\Upsilon,A\times B\times C),$ where $\forall(\alpha,\beta,\gamma)\in A\times B\times C$

$$\mu_{\Upsilon(\alpha,\beta,\gamma)}(u)=\mu_{\Gamma(\alpha,\beta)}(u).\mu_{\sigma(\gamma)}(u),\quad \text{[From the Definition 3.6]}$$
$$=\big(\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)\big).\mu_{\sigma(\gamma)}(u)\ ,\ \text{[using (6)]}$$
$$=\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u).\mu_{\sigma(\gamma)}(u)$$
$$=\mu_{\Psi(\alpha,\beta,\gamma)}(u),\forall u\in U.\ \text{[From (5)]}$$

Thus $(\psi,A)@^{\times}\big((\varphi,B)@^{\times}(\sigma,C)\big)=\big((\psi,A)@^{\times}(\varphi,B)\big)@^{\times}(\sigma,C).$

The proof of [ii] is similar to that of [i].

3.10. Proposition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then

$[i]$ $\big((\psi,A)@^{\times}(\varphi,B)\big)^{c}=(\psi,A)^{c}@^{+}(\varphi,B)^{c};$

$[ii]$ $\big((\psi,A)@^{+}(\varphi,B)\big)^{c}=(\psi,A)^{c}@^{\times}(\varphi,B)^{c}.$

Proof. Let $(\psi,A)@^{\times}(\varphi,B)=(\Gamma,A\times B),$ where $\forall(\alpha,\beta)\in A\times B,$ $\mu_{\Gamma(\alpha,\beta)}(u)=\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u),\forall u\in U.$

Then, $\big((\psi,A)@^{\times}(\varphi,B)\big)^{c}=(\Gamma,A\times B)^{c}=(\Gamma^{c},A\times B),$ where $\forall(\alpha,\beta)\in A\times B$

$$\mu_{\Gamma^{c}(\alpha,\beta)}(u)=1-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u),\forall u\in U.$$

Also, $(\psi,A)^{c}=(\psi^{c},A),\ (\varphi,B)^{c}=(\varphi^{c},B)$ and let $(\psi,A)^{c}@^{+}(\varphi,B)^{c}=(\Omega,A\times B),$ where $\forall(\alpha,\beta)\in A\times B$

$$\mu_{\Omega(\alpha,\beta)}(u)=(1-\mu_{\psi(\alpha)}(u))+(1-\mu_{\varphi(\beta)}(u))-(1-\mu_{\psi(\alpha)}(u)).(1-\mu_{\varphi(\beta)}(u))$$

$$=1-\mu_{\psi(\alpha)}(u).\mu_{\varphi(\beta)}(u)=\mu_{\Gamma^{c}(\alpha,\beta)}(u),\forall u\in U.$$

Thus $\big((\psi,A)@^{\times}(\varphi,B)\big)^{c}=(\psi,A)^{c}@^{+}(\varphi,B)^{c}.$

The proof of [ii] is similar to that of [i].

3.11 Definition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then the bounded product of (ψ,A) and $(\varphi,B),$ denoted by $(\psi,A)\otimes(\varphi,B)$ is an FSS $(\sigma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,$

$$\mu_{\sigma(\alpha,\beta)}(u)=\max\left\{0,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-1\right\},\ \ \forall u\in U.$$

3.12 Definition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then the bounded sum of (ψ,A) and $(\varphi,B),$ denoted by $(\psi,A)\oplus(\varphi,B)$ is an FSS $(\sigma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,$ $\mu_{\sigma(\alpha,\beta)}(u)=\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)\right\},\forall u\in U.$

3.13 Proposition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then

a. $[i]$ $(\psi,A)\oplus(\varphi,B)=(\varphi,B)\oplus(\psi,A);$

$[ii]$ $(\psi,A)\otimes(\varphi,B)=(\varphi,B)\otimes(\psi,A).$

3.14. Proposition Let $(\psi,A),\ (\varphi,B),(\sigma,C)\in FSS(U)$. Then

b. $[i]$ $(\psi,A)\oplus\big((\varphi,B)\oplus(\sigma,C)\big)=\big((\psi,A)\oplus(\varphi,B)\big)\oplus(\sigma,C);$

$[ii]$ $(\psi,A)\otimes\big((\varphi,B)\otimes(\sigma,C)\big)=\big((\psi,A)\otimes(\varphi,B)\big)\otimes(\sigma,C).$

Proof. [i] Let $(\psi,B)\oplus(\sigma,C)=(\Omega,B\times C),$ where $\forall(\beta,\gamma)\in B\times C,$

$$\mu_{\Omega(\beta,\gamma)}(u)=\min\left\{1,\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)\right\},\forall u\in U.$$

Then $(\psi,A)\oplus((\varphi,B)\oplus(\sigma,C))=(\psi,A)\oplus(\Omega,B\times C)$. Let $(\psi,A)\oplus(\Omega,B\times C)=(\Psi,A\times B\times C)$, where $\forall(\alpha,\beta,\gamma)\in A\times B\times C$,

$$\mu_{\Psi(\alpha,\beta,\gamma)}(u)=\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\Omega(\beta,\gamma)}(u)\right\}=\min\left\{1,\mu_{\psi(\alpha)}(u)+\min\left\{1,\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)\right\}\right\}$$
$$=\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)\right\},\forall u\in U.$$

Also, let $(\psi,A)\oplus(\varphi,B)=(\Gamma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B$

$$\mu_{\Gamma(\alpha,\beta)}(u)=\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)\right\},\forall u\in U.$$

Therefore, $((\psi,A)\oplus(\varphi,B))\oplus(\sigma,C)=(\Gamma,A\times B)\oplus(\sigma,C)$.
Let $(\Gamma,A\times B)\oplus(\sigma,C)=(\Upsilon,A\times B\times C)$, where $\forall(\alpha,\beta,\gamma)\in A\times B\times C$

$$\mu_{\Upsilon(\alpha,\beta,\gamma)}(u)=\min\left\{1,\mu_{\Gamma(\alpha,\beta)}(u)+\mu_{\sigma(\gamma)}(u)\right\}$$
$$=\min\left\{1,\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)\right\}+\mu_{\sigma(\gamma)}(u)\right\}$$
$$=\min\left\{1,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)+\mu_{\sigma(\gamma)}(u)\right\}=\mu_{\Psi(\alpha,\beta,\gamma)}(u),\forall u\in U.$$

Thus $(\psi,A)\oplus((\varphi,B)\oplus(\sigma,C))=((\psi,A)\oplus(\varphi,B))\oplus(\sigma,C)$.
The proof of [ii] is similar to that of [i].

3.15 Proposition Let $(\psi,A),\ (\varphi,B)\in FSS(U)$. Then

$[i]$ $((\psi,A)\otimes(\varphi,B))^c=(\psi,A)^c\oplus(\varphi,B)^c$;

$[ii]$ $((\psi,A)\oplus(\varphi,B))^c=(\psi,A)^c\otimes(\varphi,B)^c$.

Proof. let $(\psi,A)\otimes(\varphi,B)=(\Gamma,A\times B)$, where $\forall(\alpha,\beta)\in A\times B$

$$\mu_{\Gamma(\alpha,\beta)}(u)=\max\left\{0,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-1\right\},\ \forall u\in U.$$

Then, $((\psi,A)\otimes(\varphi,B))^c=(\Gamma,A\times B)^c=(\Gamma^c,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$

$$\mu_{\Gamma^c(\alpha,\beta)}(u)=1-\max\left\{0,\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-1\right\}=\min\left\{1-0,1-(\mu_{\psi(\alpha)}(u)+\mu_{\varphi(\beta)}(u)-1)\right\}$$
$$=\min\left\{1,2-\mu_{\psi(\alpha)}(u)-\mu_{\varphi(\beta)}(u)\right\}.$$

Also, $(\psi,A)^c=(\psi^c,A),\ (\varphi,B)^c=(\varphi^c,B)$ and let $(\psi,A)^c\oplus(\varphi,B)^c=(\Omega,A\times B)$, where $\forall(\alpha,\beta)\in A\times B,\ \forall u\in U$,

$$\mu_{\Omega(\alpha,\beta)}(u)=\min\left\{1,1-\mu_{\psi(\alpha)}(u)+1-\mu_{\varphi(\beta)}(u)\right\}=\min\left\{1,2-\mu_{\psi(\alpha)}(u)-\mu_{\varphi(\beta)}(u)\right\}=\mu_{\Gamma^c(\alpha,\beta)}(u).$$

Thus $((\psi,A)\otimes(\varphi,B))^c=(\psi,A)^c\oplus(\varphi,B)^c$.
The proof of [ii] is similar to that of [i].

4. Application in DM

Now, we define a soft fuzzification operator and construct an updated DM system based on FSS.

4.1 Definition Let $(\psi,A)\in FSS(U)$ and $r\in[0,1]$. Then a soft fuzzification operator S_r for r, denoted by $S_r(\psi,A)$ and defined as $S_r(\psi,A)=\left\{(u,\mu_{S_r(\psi,A)}(u)):u\in(\psi,A)_r\right\}$,

where $\mu_{S_r(\psi,A)}(u)=\dfrac{1}{|A|}\sum_{e\in A}\mu_{\psi(e)}(u),\ (\psi,A)_r=\left\{u\in U:\mu_{\psi(e)}(u)\geq r,\forall e\in A\right\}$.

The various steps of our machine learning algorithm are described below:

Algorithm 1.

Step 1. Input the FSSs $(\psi,A),\ (\varphi,B)\in FSS(U)$.

Step 2. Input the parameter set $C\subseteq A\times B$ that the observer observed.

Step 3. Compute the resultant FSS (σ,C) from (ψ,A) and (φ,B), (Using the Einstein operations or max-product or min-product or algebraic-operations or bounded-operations).

Step 4. Input a fixed $r \in [0,1]$.

Step 5. Compute $S_r(\sigma, C)$

Step 6. If the value of $\mu_{S_r(\sigma,C)}(o)$ is maximized, then the decision is to choose o from U.

Step 7. If u has several values, the investor can choose from any of them.

Remark 1. In the 7th step of algorithm1, one can return to the 2nd and 3rd steps and change the operation or value of $r \in [0,1]$ that was previously used to adjust the best optimal decision, particularly when there are so many "ideal alternatives" to choose from.

Example 1. Suppose $U_1 = \{h_1, h_2, h_3, h_4\}$ is the collection of houses and $E = \{a_1$=modern, a_2=wooden, a_3=beautiful, a_4=expensive, a_5=cheap, a_6=majestic$\}$; be a collection of parameters. Let $A, B \subseteq E$, such that $A = \{a_1, a_2, a_3, a_4\}, B = \{a_1, a_2, a_5, a_6\}$. Assume Mr. A and Miss. B has a financial plan in place to buy a home. If (ψ, A) and (φ, B) have to consider their own FSSs, we choose a house based on a collection of member criteria using the following DM method:

Step 1. Let us consider, Mr. A and Mrs. B each create their own FSSs (ψ, A) and (φ, B) as shown in Tables 1 and 2 respectively.

Step 2. Let us consider the parameter set

$$C = \{(a_1,a_1),(a_1,a_2),(a_2,a_5),(a_2,a_6),(a_3,a_5),(a_3,a_6),(a_4,a_1),(a_4,a_2)\}$$

Step 3. The resultant FSS (σ, C) using the algebraic product of (ψ, A) and (φ, B) as shown in Table 3

Step 4. Let $r = 0.2$

Step 5. Then we find $S_r(\sigma, C)$ as in Table 4

Step 6. Table 4, shows that house h_3 has the greatest membership value $\mu_{S_r(\sigma,C)}(h_3)$ =0.553; hence house h_3 is the best suit for the requirements.

Table 1. FSS (ψ, A)

U	a_1	a_2	a_3	a_4
h_1	0.5	0.8	0.7	0.6
h_2	0.4	0.7	0.8	0.9
h_3	0.8	0.3	1	0.5
h_4	0.7	0.8	0	0.7

Table 2. FSS (φ, B)

U	a_1	a_2	a_3	a_4
h_1	0.8	0.7	0.7	0.3
h_2	0.6	0.8	0.6	0.4
h_3	0.8	1	0.9	0.7
h_4	0.7	0	0.5	0.7

Table 3. FSS (σ, C)

U	(a_1,a_1)	(a_1,a_2)	(a_2,a_5)	(a_2,a_6)	(a_3,a_5)	(a_3,a_6)	(a_4,a_1)	(a_4,a_2)
h_1	0.4	0.35	0.56	0.24	0.49	0.21	0.48	0.42
h_2	0.24	0.32	0.42	0.28	0.48	0.32	0.54	0.72
h_3	0.64	0.8	0.27	0.21	0.9	0.7	0.4	0.5
h_4	0.49	0	0.40	0.56	0	0	0.49	0

Table 4. $S_r(\sigma, C)$, $r = 0.2$

U	(a_1,a_1)	(a_1,a_2)	(a_2,a_5)	(a_2,a_6)	(a_3,a_5)	(a_3,a_6)	(a_4,a_1)	(a_4,a_2)	$\mu_{S_{0.2}(\sigma,C)}(h_i)$
h_1	0.4	0.35	0.56	0.24	0.49	0.21	0.48	0.42	0.394
h_2	0.24	0.32	0.42	0.28	0.48	0.32	0.54	0.72	0.415
h_3	0.64	0.8	0.27	0.21	0.9	0.7	0.4	0.5	0.553
h_4	0.49	0	0.40	0.56	0	0	0.49	0	0

5. Conclusions

We have described different types of operations on FSSs in this paper, and the basic properties of these operations have been studied. These new operations are very helpful to solve real-life DMPs, deciding what kind of strategy to use based on the circumstances so that we can make a well decision and the benefits of these operations are not the same. By applying these operations, we can be connected them to numerous fields that contain questionable ties. In this research work, we also implement a novel approach to FSSs-based DM using our newly defined operations for solving DM in an unpredictable condition. To demonstrate the viability of our approach in real-world applications, we use an example. There are twofold advantages of this method. First, it's clear that this algorithm needs fewer computations, and that by using Algorithm 1, we'll have a smaller number of items to choose from, making it easier to make a decision. Secondly, the best decision depends upon the operations of FSSs and the values of r, Algorithm 1 can be seen as a flexible approach to FSS-based DM. Algorithm 1 is more suitable for many real-world applications because of this adjustable feature. We can see that it can be related to a variety of fields that have dubious relations by means of types of operations. The approach should be expanded in future to address relevant issues such as computer science, software engineering, current life condition, and so on.

Abbreviations

DM	Decision making
DMP	Decision making problem
FS	Fuzzy set
FSS	Fuzzy soft set
FSTS	Fuzzy soft topological space
MCDM	Multi criteria decision making
MCGDM making	Multi criteria group decision
SST	Soft set theory

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