

Original Article

Some novel correlation coefficients of spherical fuzzy sets with their application in pattern recognition

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Abstract

A spherical fuzzy set is a generalization of picture fuzzy sets, intuitionistic fuzzy sets, and fuzzy sets in which the square sum of the membership, non-membership, and neutrality values is at most one. The correlation coefficient is a crucial tool in fuzzy/non-standard fuzzy theory and has been applied in various fields such as clustering, pattern recognition, medical diagnosis, decision-making, etc. The existing correlation coefficients for spherical fuzzy sets give only the correlation degree and do not express the nature or direction of correlation between the spherical fuzzy sets. So, in this study, we propose two correlation coefficients for spherical fuzzy sets, which not only give the strength of correlation between two spherical fuzzy sets but also tell us whether the two spherical fuzzy sets are positively correlated or negatively correlated. We also discuss several properties of these correlation coefficients. We apply these correlation coefficients to solve a pattern recognition problem in the spherical fuzzy environment and compare the results with some existing measures.

Keywords: correlation coefficient, picture fuzzy set, spherical fuzzy set, linguistic hedge, pattern recognition

1. Introduction

The concept of the fuzzy set (FS) theory was put forward by Zadeh (1965) for handling imprecise and vague information. In an FS, each element is assigned a membership value lying between 0 and 1, indicating its degree of belongingness to the set. FSs have been applied in many fields such as pattern recognition, medical diagnosis, clustering, etc. Since in an FS, the non-membership value of an element cannot be chosen independently, so Atanassov (1986) introduced the concept of intuitionistic fuzzy sets (IFSs). In an intuitionistic fuzzy set (IFS), each element has a membership value and a non-membership value lying in the interval $[0, 1]$ with their sum less or equal to one. This restriction on the sum of membership values limits the scope of IFSs and so Yager (2013) proposed the concept of Pythagorean fuzzy sets (PFSs). In a PFS, each element has a membership value and a non-membership value lying in the interval $[0, 1]$ with their square sum less or equal to one. Though PFSs are more robust

than FSs and IFSs they cannot handle the situations in which the square sum of membership grades exceeds one. So Yager (2017) introduced generalized orthopair fuzzy sets and termed them as q -rung orthopair fuzzy sets (q -ROPFSs). In a q -rung orthopair fuzzy set (q -ROPFS) each element has a membership value and a non-membership value lying in the interval $[0, 1]$ with their q^{th} power sum less or equal to one. But all of these extensions of FSs lack an important concept i.e., degree of neutrality which plays an important role in many decision-making problems such as medical diagnosis, personnel selection, human voting, etc. So, realizing this a new generalization of FSs known as picture fuzzy sets (PIFSs) were suggested by Cuong and Kreinovich (2013). In a picture fuzzy set (PIFS) each element has a membership value, a non-membership value, and a neutrality value lying in the interval $[0, 1]$ with their sum less or equal to one.

Like IFSs, the scope of PIFSs is also limited due to restriction on the sum of membership values. Realizing this, Mahmood, Ullah, Khan, and Jan (2019) proposed the concept of spherical fuzzy sets (SFSs). In a spherical fuzzy set (SFS), each element is characterized by a membership, non-membership, and neutrality degree with the square sum of the membership, non-membership, and neutrality less or equal to

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one. This means that the space of SFSs is relatively broader than the space of FSs, IFs, PFSs, q -ROPFSs, and PIFSs. The spherical fuzzy (SF) TOPSIS along with its utility in Hospital location selection was developed by Kahraman, Kutlu, Cevik, and Oztaysi (2019). Liu, Zhu, and Wang (2019) proposed a new multi-attribute decision-making (MADM) method in SF environment for a research and development project problem. Kutlu and Kahraman (2020a) introduced the Quality Function Development (QFD) in SF setting. The classical Analytical Hierarchical Process (AHP) was extended to SF environment by Gundogdu and Kahraman (2020b). A SF entropy measure and its application in MADM was proposed by Aydogdu and Gul (2020). By combining the AHP and TOPSIS, Mathew, Chakraborty, and Ryan (2020) introduced a novel method in SF environment for an advanced manufacturing system selection problem. The classical VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method was extended to SF environment by Kutlu and Kahraman (2019). Kutlu (2020) extended the traditional Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) to SF settings.

Application of some SF distance and similarity measures are available in the literature (Khan, Kumam, Deebani, Kumam, & Shah, 2020; Shishavan, Kutlu, Farrokhizadeh, Donyatalab, & Kahraman, 2020; Wei, Wang, Lu, Wu, & Wei, 2019). Ashraf, Abdullah, and Mahmood (2020) introduced some Dombi aggregation operators for SFSs and applied them in group decision-making. Some entropy-based Logarithmic aggregation operators in SF theory were proposed by Jin, Wu, Sun, Zeng, Luo, and Peng (2019). Some SF t -norms and t -conorms were introduced by Ashraf, Abdullah, Aslam, Qiyas, and Kutbi (2019). Some similarity measures and information measures for SFSs along with their applications were introduced by Mahmood, Ilyas, Ali, and Gumaei (2021). The application of some SF similarity measures in decision-making was shown by Rafiq, Ashraf, Abdullah, Mahmood, and Muhammad (2019). Some SF aggregation operators with their applicability in decision-making were suggested by Khan, Mahmood, and Ullah (2021). Mahmood *et al.*, (2019) also proposed the concept of T-spherical fuzzy sets (T-SFSs) in which the sum of t^{th} power of membership, non-membership, and neutrality values is at most one. Some recent studies related to T-SFSs and their diverse applications are available in the literature (Mahmood, & Jan, 2018; Ullah, Hassan, Mahmood, Jan, & Hassan, 2019; Ullah, Ullah, Mahmood, Jan, & Ahmad, 2020; Zedam, Jan, Rak, Mahmood, & Ullah, 2020). The present study is related to the correlation coefficient for SFSs. The main contributions of this study are:

1. We propose two correlation coefficients for SFSs that receive their values in $[-1, 1]$ and therefore give both the strength of correlation as well as the nature of the correlation between the SFSs.

2. We discuss their various properties.

3. We compare the suggested correlation coefficients with the existing correlation coefficients in the SF environment through the linguistic hedge aspect.

4. We demonstrate the application of the suggested correlation coefficients in pattern recognition and contrast the performance with the existing SF correlation coefficients.

2. Correlation Coefficients and Their Applications

In this section, we study the importance of correlation coefficients and their applications.

The correlation coefficient is used to describe the relationship between two objects. In fuzzy theory, the correlation coefficient is vital due to its application in MADM, pattern recognition, clustering, etc. Chiang and Lin (1999) introduced a correlation coefficient for FSs based on a statistical viewpoint. This correlation coefficient provides both strength as well as the nature of the correlation between two FSs. Chaudhuri and Bhattacharya (2001) introduced Spearman's rank-type correlation coefficient for FSs. By using Pearson's correlation coefficient, Wu and Hung (2016) introduced a method for calculating the correlation of interval fuzzy data. A generalized fuzzy correlation coefficient with its application in the ranking of Primary Health Centers was given by Sharma and Singh (2019).

The informational energy and correlation coefficient for IFs was introduced by Gerstenkorn and Manko (1991). But this correlation coefficient for IFs gives only the degree of correlation and not the nature of correlation. A correlation coefficient for IFs indicating both nature and degree was proposed by Hung (2001). Some other studies concerning the IF correlation coefficients with their applicability in medical diagnosis, pattern recognition, clustering, etc. are available in the literature (Ejegwa & Onyeke, 2020; Hong & Hwang, 1995; Huang & Guo, 2019; Hung & Wu, 2002; Kumar, 2019; Liu, Shen, Mu, Chen, & Chen, 2016; Mitchell, 2004; Thao, Ali, & Smarandache, 2019).

Du (2019) proposed q -ROPFSs correlation coefficient on both bounded and unbounded continuous universes. Recently, Singh and Ganie (2020) introduced some robust correlation coefficients in the Pythagorean fuzzy environment and demonstrated their application in MADM, medical diagnosis, clustering, and pattern recognition. Based on a statistical viewpoint, Singh and Ganie (2021a) introduced some correlation coefficients for q -ROPFSs.

In the picture fuzzy (PIF) theory, Singh (2015) introduced two correlation coefficients with their applications in bidirectional approximate reasoning and clustering analysis. However, the PIF correlation measures due to Singh (2015) indicate only the strength of correlation between two PIFSs and do not tell us whether the two PIFSs are positively or negatively correlated. So, to find the nature of the correlation between two PIFSs, Ganie, Singh, and Bhatia (2020) proposed two correlation coefficients that express both nature and degree of correlation between two PIFSs. They also utilized these correlation coefficients for solving some problems related to pattern recognition, clustering, and medical diagnosis. A novel PIF correlation with its application in classification and decision-making was proposed by Singh and Ganie (2021b). For SFSs, Mahmood *et al.* (2021) developed correlation coefficients and applied them in pattern recognition and medical diagnosis.

Some novel methods of constructing the correlation coefficients from similarity and dissimilarity functions were suggested by Batyrshin (2019). The definitions of association measures that generalize Pearson's correlation coefficient as well as the general methods for constructing such measures

were proposed by Batyrshin (2015b). Some similarity-based correlation coefficients for binary data were introduced by Batyrshin, Ramirez, Batyrshin, and Solovyev (2020). Batyrshin (2015a, 2019) studied the similarity measures and construction of association measures on $[0, 1]$.

Most of the existing correlation coefficients in fuzzy/non-standard fuzzy theory receive their value in $[0, 1]$ and thus indicate only the degree of correlation and remain silent about the nature of correlation. From an application point of view, a correlation coefficient indicating both degree and nature of correlation is more suitable than the correlation coefficient indicating only the degree of correlation. Also, the correlation coefficients for SFSs given by Mahmood *et al.*

(2021) do not inform us about the nature of the correlation between two SFSs. So, keeping these points in mind, we, in this study, propose two novel correlation coefficients for SFSs. These correlation coefficients indicate both nature as well as the strength of correlation between two SFSs.

The content of the paper is structured as follows. Section 3 covers the preliminary concepts. Section 4 introduces two novel correlation coefficients in SF settings. A comparative analysis based on linguistic hedges is shown in Section 5. The application and advantage of the proposed correlation coefficients in pattern recognition are shown in Section 6. Finally, Section 7 concludes the paper.

3. Preliminaries

In this section, we introduce the basic definitions of fuzzy and non-standard FSs.

Definition 1. (Zadeh, 1965) A fuzzy set G in a universe of discourse $T = \{t_1, t_2, \dots, t_l\}$ is defined as

$$G = \{(t_k, m_G(t_k)), t_k \in T, k = 1, 2, \dots, l.\}$$

where $m_G : T \rightarrow [0, 1]$ is a membership function, and $m_G(t_k)$ is the membership degree of $t_k \in T$ in the set G .

Definition 2. (Atanassov, 1986) An intuitionistic fuzzy set G in a universe of discourse $T = \{t_1, t_2, \dots, t_l\}$ is given as

$$G = \{(t_k, m_G(t_k), n_G(t_k)), t_k \in T, k = 1, 2, \dots, l.\}$$

where $m_G : T \rightarrow [0, 1]$ and $n_G : T \rightarrow [0, 1]$ are the membership and non-membership functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree of the element $t_k \in T$ in the set G , and $0 \leq m_G(t_k) + n_G(t_k) \leq 1$.

Definition 3. (Cuong & Kreinovich, 2013) A picture fuzzy set G in a universe of discourse $T = \{t_1, t_2, \dots, t_l\}$ is given as

$$G = \{(t_k, m_G(t_k), n_G(t_k), h_G(t_k)), t_k \in T, k = 1, 2, \dots, l.\}$$

where $m_G : T \rightarrow [0, 1]$, $n_G : T \rightarrow [0, 1]$ and $h_G : T \rightarrow [0, 1]$ are the membership, non-membership, and neutrality functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree, $h_G(t_k)$ represents the neutrality degree of the element $t_k \in T$ in the set G , and $0 \leq m_G(t_k) + n_G(t_k) + h_G(t_k) \leq 1$.

Definition 4. (Kutlu Gündoğdu & Kahraman, 2020b) A spherical fuzzy set G in a universe of discourse $T = \{t_1, t_2, \dots, t_l\}$ is given as

$$G = \{(t_k, m_G(t_k), n_G(t_k), h_G(t_k)), t_k \in T, k = 1, 2, \dots, l.\}$$

where $m_G : T \rightarrow [0, 1]$, $n_G : T \rightarrow [0, 1]$ and $h_G : T \rightarrow [0, 1]$ are the membership, non-membership, and neutrality functions respectively. Here, $m_G(t_k)$ represents the membership degree, $n_G(t_k)$ represents the non-membership degree, $h_G(t_k)$ represents the neutrality degree of the element $t_k \in T$ in the set G and $0 \leq (m_G(t_k))^2 + (n_G(t_k))^2 + (h_G(t_k))^2 \leq 1$. Also, the degree of refusal of an element $t_k \in T$ in the spherical fuzzy set G is given by

$$e_G(t_k) = \sqrt{1 - (m_G(t_k))^2 - (n_G(t_k))^2 - (h_G(t_k))^2}.$$

Definition 5. (Mahmood *et al.*, 2021) The two correlation coefficients for spherical fuzzy sets are given as:

$$C_{MIAG1} = \frac{\sum_{k=1}^l \left\{ (m_G(t_k)m_H(t_k))^2 + (n_G(t_k)n_H(t_k))^2 + (h_G(t_k)h_H(t_k))^2 + (e_G(t_k)e_H(t_k))^2 \right\}}{\left(\sum_{k=1}^l \left((m_G(t_k))^4 + (n_G(t_k))^4 + (h_G(t_k))^4 + (e_G(t_k))^4 \right) \right)^{\frac{1}{2}} \times \left(\sum_{k=1}^l \left((m_H(t_k))^4 + (n_H(t_k))^4 + (h_H(t_k))^4 + (e_H(t_k))^4 \right) \right)^{\frac{1}{2}}} \tag{1}$$

$$C_{MIAG2} = \frac{\sum_{k=1}^l \left\{ (m_G(t_k)m_H(t_k))^2 + (n_G(t_k)n_H(t_k))^2 \right.}{\max \left\{ \sum_{k=1}^l \left((m_G(t_k))^4 + (n_G(t_k))^4 + (h_G(t_k))^4 + (e_G(t_k))^4 \right), \right.} \left. \left. \left. \left. + (h_H(t_k)h_H(t_k))^2 + (e_H(t_k)e_H(t_k))^2 \right\} \right\} \right\} \left. \left. \left. \left. + (m_H(t_k))^4 + (n_H(t_k))^4 + (h_H(t_k))^4 + (e_H(t_k))^4 \right) \right\} \right\} \right\} \quad (2)$$

4. New Correlation Coefficients for SFSSs

In this section, we introduce some novel SF correlation coefficients.

Let $V(T)$ denote the set of all SFSSs in the universe of discourse $T = \{t_1, t_2, \dots, t_l\}$ throughout this paper. Here we propose two correlation coefficients for SFSSs, which, together with strength, also express the nature of the correlation between two SFSSs.

Definition 5. For any $G, H \in V(T)$, a correlation coefficient based on membership, non-membership, and neutrality degree is defined as

$$C_{G1}(G, H) = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3), \quad (3)$$

where

$$\theta_1 = \frac{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)((m_H(t_k))^2 - (\overline{m_H})^2)}{\sqrt{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m_H})^2)^2}},$$

$$\theta_2 = \frac{\sum_{k=1}^l ((n_G(t_k))^2 - (\overline{n_G})^2)((n_H(t_k))^2 - (\overline{n_H})^2)}{\sqrt{\sum_{k=1}^l ((n_G(t_k))^2 - (\overline{n_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((n_H(t_k))^2 - (\overline{n_H})^2)^2}},$$

$$\theta_3 = \frac{\sum_{k=1}^l ((h_G(t_k))^2 - (\overline{h_G})^2)((h_H(t_k))^2 - (\overline{h_H})^2)}{\sqrt{\sum_{k=1}^l ((h_G(t_k))^2 - (\overline{h_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((h_H(t_k))^2 - (\overline{h_H})^2)^2}}, \quad \overline{m_G} = \frac{1}{l} \sum_{k=1}^l m_G(t_k), \quad \overline{n_G} = \frac{1}{l} \sum_{k=1}^l n_G(t_k),$$

$$\overline{h_G} = \frac{1}{l} \sum_{k=1}^l h_G(t_k), \quad \overline{m_H} = \frac{1}{l} \sum_{k=1}^l m_H(t_k), \quad \overline{n_H} = \frac{1}{l} \sum_{k=1}^l n_H(t_k), \quad \overline{h_H} = \frac{1}{l} \sum_{k=1}^l h_H(t_k).$$

Definition 6. For any $G, H \in V(T)$, a correlation coefficient based on membership, non-membership, neutrality, and refusal degree is defined as

$$C_{G2}(G, H) = \frac{1}{4}(\theta_1 + \theta_2 + \theta_3 + \theta_4), \quad (4)$$

where $\theta_4 = \frac{\sum_{k=1}^l ((e_G(t_k))^2 - (\overline{e_G})^2)((e_H(t_k))^2 - (\overline{e_H})^2)}{\sqrt{\sum_{k=1}^l ((e_G(t_k))^2 - (\overline{e_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((e_H(t_k))^2 - (\overline{e_H})^2)^2}}, \quad \overline{e_G} = \frac{1}{l} \sum_{k=1}^l e_G(t_k), \quad \overline{e_H} = \frac{1}{l} \sum_{k=1}^l e_H(t_k)$, and the

rest of the terms are the same as in Equation (3).

Next, we discuss the properties of the proposed SF correlation coefficients $C_{G1}(G, H)$, and $C_{G2}(G, H)$ in the following:

Theorem 1. For any $G, H \in V(T)$, we have,

- (1) $C_{G1}(G, H) = C_{G1}(H, G)$
- (2) $-1 \leq C_{G1}(G, H) \leq 1$
- (3) $C_{G1}(G, H) = 1$, if $G = H$.

Proof. (1) It is obvious.

(2) By Cauchy Schwarz inequality, we have

$$\begin{aligned} (\theta_1)^2 &= \left(\frac{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)((m_H(t_k))^2 - (\overline{m_H})^2)}{\sqrt{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m_H})^2)^2}} \right)^2 \\ &= \frac{\left(\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)((m_H(t_k))^2 - (\overline{m_H})^2) \right)^2}{\left(\sqrt{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m_G})^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m_H})^2)^2} \right)^2} \end{aligned}$$

$$\begin{aligned} &\leq \frac{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m}_G)^2)^2 \sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2}{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m}_G)^2)^2 \sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2} = 1. \\ &\Rightarrow |\theta_1| \leq 1. \end{aligned}$$

Similarly, $|\theta_2| \leq 1$ and $|\theta_3| \leq 1$.

$$\text{So, } |C_{G1}(G, H)| = \left| \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) \right| \leq \frac{1}{3}\{|\theta_1| + |\theta_2| + |\theta_3|\} \leq \frac{1}{3}(1 + 1 + 1) = 1,$$

i.e., $-1 \leq C_{SG2}(G, H) \leq 1$.

(3) Let $G = H$, i.e., $m_G(t_k) = m_H(t_k), n_G(t_k) = n_H(t_k)$ and $h_G(t_k) = h_H(t_k)$, then we have

$$\begin{aligned} \theta_1 &= \frac{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m}_G)^2)((m_H(t_k))^2 - (\overline{m}_H)^2)}{\sqrt{\sum_{k=1}^l ((m_G(t_k))^2 - (\overline{m}_G)^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2}} \\ &= \frac{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)((m_H(t_k))^2 - (\overline{m}_H)^2)}{\sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2}} \\ &= \frac{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)((m_H(t_k))^2 - (\overline{m}_H)^2)}{\sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2} \times \sqrt{\sum_{k=1}^l ((m_H(t_k))^2 - (\overline{m}_H)^2)^2}} = 1. \end{aligned}$$

Similarly, $\theta_2 = 1$ and $\theta_3 = 1$. Therefore, $C_{G1}(G, H) = \frac{1}{3}(\theta_1 + \theta_2 + \theta_3) = \frac{1}{3}(1 + 1 + 1) = 1$.

Theorem 2. For any $G, H \in V(T)$, we have,

- (1) $C_{G2}(G, H) = C_{G2}(H, G)$
- (2) $-1 \leq C_{G2}(G, H) \leq 1$
- (3) $C_{G2}(G, H) = 1$, if $G = H$.

Proof. On the same lines as that of Theorem 1.

5. A Comparative Study based on Linguistic Hedges

In this section, we compare the suggested SF correlation coefficients with the existing SF correlation coefficients by using linguistic hedges.

Definition 10. (Mathew *et al.*, 2020) For any $G \in V(T)$, the n^{th} power of G , where n is any positive real number is defined as

$$G^n = \left\{ \left(\begin{array}{l} t_k, (m_G(t_k))^n, \sqrt{1 - (1 - (n_G(t_k))^2)^n}, \\ \sqrt{(1 - (n_G(t_k))^2)^n - (1 - (n_G(t_k))^2 - (h_G(t_k))^2)^n} \end{array} \right) \mid t_k \in T \right\}, n > 0.$$

Example 1. Consider an SFS G in $T = \{t_1, t_2, t_3, t_4, t_5\}$ as

$$G = \left\{ (t_1, 0.3, 0.1, 0.1), (t_2, 0.3, 0.4, 0.1), (t_3, 0.3, 0.2, 0.2), (t_4, 0.1, 0.1, 0), (t_5, 0.4, 0.5, 0.3) \right\}$$

By considering the above definition of the modifier of an SFS, we define the SFSs

LARGE = G ,

Very LARGE = G^2 ,

Very very LARGE = G^4 ,

Not very LARGE = $G^{2'}$ and More or less LARGE = $G^{\frac{1}{2}}$.

We use these SFSs to compare our proposed methods with some existing ones for calculating the correlation coefficients. The comparison results are given in Tables 1-3. The following notations are used in Tables 1-3.

LARGE: L.

Very LARGE: V.L.

Very very LARGE: V.V.L.

More or less LARGE: M.L.L.

Not Very LARGE: N.V.L.

From the characterization of linguistic variables, a correlation coefficient K should satisfy the following requirements.

$$\left. \begin{aligned}
 &K(M.L.L., L.) > K(M.L.L., V.L.) > K(M.L.L., V.V.L.) \\
 &K(V.V.L., V.L.) > K(V.V.L., L.) > K(V.V.L., M.L.L.) \\
 &K(L., M.L.L.) > K(L., V.L.) > K(L., V.V.L.) \\
 &K(V.L., L.) > K(V.L., V.V.L.) > K(V.L., M.L.L.)
 \end{aligned} \right\} \tag{5}$$

From Tables 2-3, we observe that

$$\begin{aligned}
 &C_{MIAGi}(M.L.L., L.) > C_{MIAGi}(M.L.L., V.L.) > C_{MIAGi}(M.L.L., V.V.L.) \\
 &C_{MIAGi}(V.V.L., V.L.) > C_{MIAGi}(V.V.L., L.) > C_{MIAGi}(V.V.L., M.L.L.) \\
 &C_{MIAGi}(L., M.L.L.) < C_{MIAGi}(L., V.L.) > C_{MIAGi}(L., V.V.L.) \\
 &C_{MIAGi}(V.L., L.) < C_{MIAGi}(V.L., V.V.L.) > C_{MIAGi}(V.L., M.L.L.), i = 1, 2. \\
 &C_{Gi}(M.L.L., L.) > C_{Gi}(M.L.L., V.L.) > C_{Gi}(M.L.L., V.V.L.) \\
 &C_{Gi}(V.V.L., V.L.) > C_{Gi}(V.V.L., L.) > C_{Gi}(V.V.L., M.L.L.) \\
 &C_{Gi}(L., M.L.L.) > C_{Gi}(L., V.L.) > C_{Gi}(L., V.V.L.) \\
 &C_{Gi}(V.L., L.) > C_{Gi}(V.L., V.V.L.) > C_{Gi}(V.L., M.L.L.), i = 1, 2.
 \end{aligned}$$

Thus, we observe that only the suggested SF correlation coefficients C_{G1} and C_{G2} satisfy all the conditions given in Equation (5). On the other hand, the existing SF correlation coefficients C_{MIAG1} and C_{MIAG2} fail to satisfy all the requirements given in Equation (5). This shows that from a linguistic hedge aspect, the suggested SF correlation coefficients are more effective than the available SF correlation coefficients.

Table 1. Values of the correlation coefficients C_{MIAG1} and C_{MIAG2} .

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9636	0.8798	0.8020	0.9566
L.	0.9636	1.0000	0.9728	0.9187	0.9596
V.L.	0.8798	0.9728	1.0000	0.9811	0.8976
V.V.L.	0.8020	0.9187	0.9811	1.0000	0.8157
N.V.L.	0.9566	0.9596	0.8976	0.8157	1.0000

Table 2. Values of the correlation coefficient C_{G1}

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9917	0.9448	0.8409	0.8455
L.	0.9917	1.0000	0.9785	0.8972	0.8768
V.L.	0.9448	0.9785	1.0000	0.9641	0.8982
V.V.L.	0.8409	0.8972	0.9641	1.0000	0.8623
N.V.L.	0.8455	0.8768	0.8982	0.8623	1.0000

Table 3. Values of the correlation coefficient C_{G2}

	M.L.L.	L.	V.L.	V.V.L.	N.V.L.
M.L.L.	1.0000	0.9840	0.9375	0.8580	0.8630
L.	0.9840	1.0000	0.9810	0.9162	0.9047
V.L.	0.9375	0.9810	1.0000	0.9709	0.9236
V.V.L.	0.8580	0.9162	0.9709	1.0000	0.8946
N.V.L.	0.8630	0.9047	0.9236	0.8946	1.0000

6. Application in Pattern Recognition

In this section, we discuss the application of our proposed SF correlation coefficients in pattern recognition. Classifying an unknown pattern into some known patterns is referred to as pattern recognition. Here we utilize our proposed SF correlation coefficients for solving some problems of pattern recognition and contrast the results with the existing SF correlation coefficients. First, we formulate a pattern recognition problem in the SF environment.

Problem: Given some known patterns $G_i (i = 1, 2, \dots, m)$ and an unknown pattern H in the form of SFSs as

$$G_i = \{(t_k, m_{G_i}(t_k), n_{G_i}(t_k), h_{G_i}(t_k)) | t_k \in T, k = 1, 2, \dots, l\}, i = 1, 2, \dots, m.$$

$$H = \{(t_k, m_H(t_k), n_H(t_k), h_H(t_k)) | t_k \in T, k = 1, 2, \dots, l\}.$$

Aim: To classify the unknown pattern H into one of the known patterns $G_i (i = 1, 2, \dots, m)$.

Recognition principle: The unknown pattern H can be assigned to the known pattern with which it has maximum correlation. Now, we solve some pattern recognition problems in view of SF information with the help of our proposed SF correlation measures given in Equations (3) and (4). Furthermore, we compare the results with the existing SF correlation measures given in the Equations (1)-(2).

Example 1. (Ganie *et al.*, 2020) Consider three known patterns G_1, G_2, G_3 , and an unknown pattern H in the form of SFSs as

$$G_1 = \{(t_1, 0.4, 0.3, 0.1), (t_2, 0.5, 0.3, 0.2), (t_3, 0.4, 0.3, 0.0), (t_4, 0.7, 0.0, 0.2), (t_5, 0.6, 0.1, 0.1)\},$$

$$G_2 = \{(t_1, 0.7, 0.1, 0.1), (t_2, 0.2, 0.3, 0.4), (t_3, 0.2, 0.1, 0.5), (t_4, 0.1, 0.5, 0.2), (t_5, 0.3, 0.3, 0.3)\},$$

$$G_3 = \{(t_1, 0.1, 0.3, 0.4), (t_2, 0.4, 0.3, 0.1), (t_3, 0.3, 0.4, 0.2), (t_4, 0.2, 0.5, 0.3), (t_5, 0.5, 0.3, 0.1)\} \text{ and}$$

$$H = \{(t_1, 0.6, 0.2, 0.1), (t_2, 0.3, 0.4, 0.2), (t_3, 0.4, 0.3, 0.2), (t_4, 0.7, 0.1, 0.0), (t_5, 0.4, 0.2, 0.2)\}.$$

The calculated values of the SF correlation measures between the known patterns $G_i, i = 1, 2, 3$, and the unknown pattern H are summarized in Table 4 and also shown in Figure 1.

Table 4. Calculated values of various SF correlation measures regarding Example 1.

	(G_1, H)	(G_2, H)	(G_3, H)	Result
C_{MIAG1} (Mahmood <i>et al.</i> , 2021)	0.9815	0.9432	0.9373	G_1
C_{MIAG2} (Mahmood <i>et al.</i> , 2021)	0.9776	0.9410	0.9329	G_1
C_{G1} (Proposed)	0.3281	0.2549	-0.5880	G_1
C_{G2} (Proposed)	0.3634	0.2932	-0.3626	G_1

From Table 4, it is clear that all the SF correlation measures given in the Equations (1)-(4) assign the unknown pattern H to the known pattern G_1 . This shows that our proposed SF correlation measures are consistent with the existing SF correlation measures. We also note that the pattern H has a negative correlation with the pattern G_3 as indicated by the proposed correlation coefficients C_{G1} , and C_{G2} .

Example 2. Let

$$G_1 = \{(t_1, 0.40, 0.67, 0.22), (t_2, 0.41, 0.74, 0.41), (t_3, 0.67, 0.58, 0.44), (t_4, 0.55, 0.56, 0.37)\},$$

$$G_2 = \{(t_1, 0.44, 0.65, 0.50), (t_2, 0.50, 0.59, 0.59), (t_3, 0.59, 0.80, 0.00), (t_4, 0.59, 0.48, 0.39)\},$$

$$G_3 = \{(t_1, 0.67, 0.51, 0.39), (t_2, 0.74, 0.67, 0.00), (t_3, 0.66, 0.67, 0.31), (t_4, 0.16, 0.82, 0.47)\}$$

be some known patterns and

$$H = \{(t_1, 0.38, 0.80, 0.44), (t_2, 0.59, 0.50, 0.38), (t_3, 0.38, 0.59, 0.31), (t_4, 0.38, 0.74, 0.50)\}$$

be an unknown pattern.

The calculated values of the SF correlation measures between the known patterns $G_i, i = 1, 2, 3$, and the unknown pattern H are summarized in Table 5 and also shown in Figure 2.

Table 5. Calculated values of various SF correlation measures regarding Example 2.

	(G_1, H)	(G_2, H)	(G_3, H)	Result
C_{MIAG1} (Mahmood <i>et al.</i> , 2021)	0.7639	0.7639	0.7510	Not Classified
C_{MIAG2} (Mahmood <i>et al.</i> , 2021)	0.7477	0.7289	0.7477	Not Classified
C_{G1} (Proposed)	-0.4240	-0.2230	0.5179	G_3
C_{G2} (Proposed)	-0.5492	-0.3512	0.2793	G_3

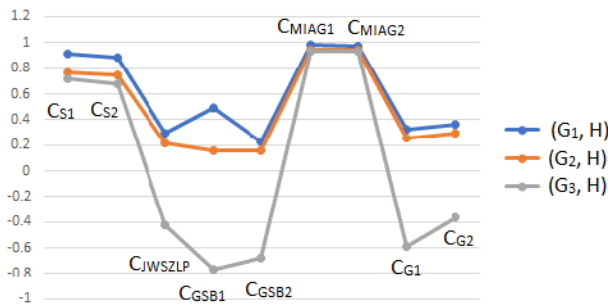


Figure 1. Correlation coefficient between the patterns regarding Example 1

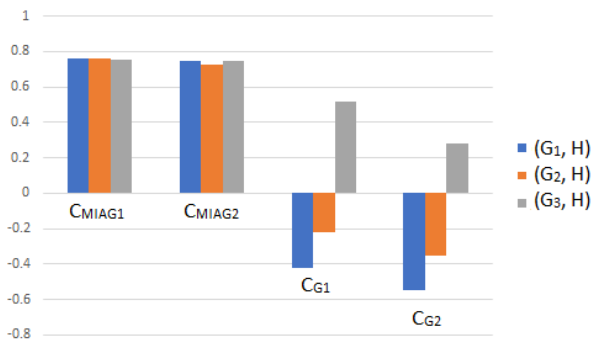


Figure 2. Correlation coefficient between the patterns regarding Example 2

From Table 5, it is clear that the existing SF correlation measures given in Equations (1)-(2) fail to classify the pattern H to one of the patterns G_i , $i = 1, 2, 3$. Our proposed SF correlation coefficients given in Equations (3) and (4) classify the pattern H into the pattern G_3 .

Thus, from Examples 1 and 2, we conclude that the suggested SF correlation coefficients are better than the SF correlation coefficients due to Mahmood *et al.* (2021) in computing the correlation degree and determining the nature of correlation.

7. Conclusions

In this study, we have proposed two correlation coefficients for SFs. It has been observed that the proposed correlation coefficients indicate the degree as well as the nature of correlation (positive or negative correlation) in two SF data sets. Also, we found that from a linguist hedge viewpoint the suggested correlation coefficients are robust than the existing correlation coefficients for SFs. We also applied our proposed SF correlation coefficients for solving some pattern recognition problems. We found that our proposed correlation methods perform better than the existing SF correlation methods.

In the future, we will study the application of the suggested SF correlation coefficients in decision-making problems, clustering analysis, medical diagnosis, etc., and extend them to an interval-valued SF environment.

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