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Original Article

Two echelon inventory models with the market price, advertisement, and discount sensitive demand in the non-co-operative environment

Shiv Kumar Singh Pundhir[1*], Jitendra Kaushik[2], Anand Kumar Gupta[3], and Sandeep Kumar[4]

[1] *Institute of Business Management, GLA University, Mathura, Uttar Pradesh, 281406 India*

[2] *University School of Business, Chandigarh University, Mohali, Punjab, 140413, India*

[3] *Faculty of Management, United University, Prayagraj, Uttar Pradesh, 211012 India*

[4] *Department of Mathematics, IILM University, Greater Noida, Uttar Pradesh, 201306 India*

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Abstract

In supply chain management manufacturers and retailers are the two most important nodes. Some decision variables are decided by the manufacturer and some are decided by the retailer to maximize their profits. In this work, market demand is considered sensitive to market price, advertising, and discount given by the manufacturer to the retailer. In this work, EOQ and advertising will be decided by the retailer and a discount will be given by the manufacturer to the retailer to motivate the retailer to generate more market demand. Manufacturers' and retailers' profit models are developed in the non-co-operative environment. Models are verified using dummy data, and sensitivity analysis is performed for all the decision variables for both manufacture and retailer Stackelberg models.

Keywords: manufacturer, retailer, discount, advertisement cost, stackelberg models

1. Introduction

In a supply chain model the first link which produces items is known as a manufacturer and other important links are distributors and retailers. In previous works manufacturers and retailers were also represented by sellers and buyers. In this work mathematical models are developed for the two-echelon system. In this work, the models are developed in a non-co-operative environment, which is known as manufacturer Stackelberg and retailer Stackelberg models.

Market demand is sensitive to so many factors like market price, brand value, the discount given on maximum retail prices, etc. In the previous works total annual demand was considered sensitive to market price and marketing expenditure. Prakash (1994), Prakash and Jaggi (2003), considered that as price increases market demand will decrease, and more marketing expenditure increases market demand. A mathematical model considering market demand sensitive to marketing expenditure was also developed by Lee and Daesoo (1993), (1998), Sadjadi, Oroujee, and Aryanezhad (2005). Esmaeili, Bahadur, and Zeephongsekul (2009) developed an inventory model for sellers and buyers in both co-operative and non-co-operative environments considering annual demand sensitive to market price and marketing expenditure. Kunter (2012) studied cost and profit-sharing between seller and buyer. According to him both manufacturer and retailer should share market revenue generated. Leng, and Zhu (2009) suggested that a supply chain model can be improved by the coordination between all the links. A model of annual market demand, sensitive to marketing expenditure was also studied by Yue *et al.*, (2006). As the retailer's profit is also dependent on market demand, they suggested that marketing expenditure also be shared by the retailer. A model for discount policy and price-sensitive demand was developed by Lin and Ho (2011).

*Corresponding author
Email address: shiv.pundhir@gla.ac.in

Models for price-sensitive demand were also studied by Pal, Sana, and Chaudhuri (2015). Papachrstos, and Skouri (2011) also developed a model for the quantity discount. A price discount model was also discussed by Shaikh, Khan, Panda, and Konstantaras (2019). Najafi- Ghobadi, Bagherinejad, and Taleizadeh (2020) studied dynamic pricing and advertising optimization. In their research, they found that the forward-looking behavior of customer decreased the profit of the firm, and planning horizon length had a negative impact on advertising expenditure. Farshbaf-Geranmayeh, Rabbani, and Taleizadeh (2017) studied cooperative advertising between manufacturer and retailer and found that co-operative advertising can increase the profit of manufacturer and retailer. They found that more expenditure by the retailer on advertising leads the customer to early purchase. Taleizadeh, Rabiei, and Noori-Daryan (2018) studied market demand-oriented to quality and price of the product. Taleizadeh, Wee, and Jolai (2013) studied quantity discounts for deteriorating products. They found that Bees Colony Optimization technique and Particle Swarm Optimization methods give better results than other methods for cost and computational time. Taleizadeh, Cheraghi, Cárdenas-Barrón, and Noori-Daryan (2021) studied pricing and marketing decisions for co-operating advertising in a two-echelon supply-chain system in cooperative and Stackelberg environment. Tavakoli and Taleizadeh (2017) developed a mathematical model for EOQ for an advanced payment scheme for the retailer. Taleizadeh, Satarian, and Jamili (2014) developed a model for multi-discount price and order quantity. They found that multiple discounts increase profit in the case of a single discount but too many discounts cannot significantly increase the profit.

In previous work, demand was considered to market price and advertisement cost-sensitive. In some work, the discount was also considered as an important factor for annual market demand. But we cannot neglect the fact that all three parameters (Price, discount given on price, and advertisement cost) are simultaneously important to increase total demand. If the price is high but a discount is given to customers, it will attract a customer to purchase the product. Generally, this discount is given by the manufacturer to the customer at market price. But in the supply chain the manufacturer also gives an attractive commission or discount to retailers. In this work, market demand is considered sensitive to the market price, advertisement cost, and discount given by the manufacturer to the retailer.

2. Materials and Methods

2.1 Assumptions

Following assumptions are considered for models.

1. The planning horizon is infinite.
2. Parameters are deterministic.
3. Total demand is sensitive to the market price, advertisement cost, and discount given by the manufacturer to the retailer.
4. The shortage is not allowed.
5. The production rate is infinite.
6. Retailer determines lot size.
7. Inventory level is finite.

2.2 Notations

All the notations used are defined in this section.

2.2.1 Decision variables

P_{mr}: Price charged by the manufacturer to the retailer.
d: Discount given by the manufacturer to the retailer.
P_{rc}: Price charged by the retailer to the customer.
A_c: Advertisement cost per unit decided by the retailer.
Q: Economic order quantity decided by the retailer

2.2.2 Input variables

z: Inventory level for the manufacturer.
P_{cm}: Production cost (/unit) for manufacturer
S_{cm}: Set up cost for the manufacturer
H_{cm}: Holding cost for the manufacturer
O_{cr}: Ordering cost for the retailer
H_{cr}: Holding cost for the retailer
ψ: Scaling constant for total demand (D_T) function so ψ>0
α: Price sensitivity constant in total demand function.
β: Advertisement cost sensitivity constant in total demand function.
γ: Discount sensitivity constant in total demand function.
r: Per unit time demand rate.

Here researcher considered that total demand D_T is sensitive to market price, advertisement cost, and discount given by the manufacturer to the retailer, so demand function can be defined as:

$$D_T = \psi P_{rc}^{-\alpha} A_c^{\beta} d^{\gamma}$$

2.3 General model formulation

2.3.1 The inventory model for the manufacturer

The manufacturer's profit model will be given by
All costs included for the manufacturer are defined below:

(a) Total Production cost=P_{cm}*D_T

(b) Total Set up cost= $S_{cm} * \frac{D_T}{Q}$

(c) Holding cost= $0.5 * z * \frac{z}{r} * \frac{D_T}{Q} * H_{cm}$

Manufacturer's total profit for time T

Πs = Total sales revenue - Setup cost –Holding cost – Production cost

$$\Pi s = (100 - d)P_{mr}\, D_T - S_{Cm}\frac{D_T}{Q} - 0.5z\frac{z}{r}\frac{D_T}{Q} H_{cm} - P_{Cm}\, D_T$$

By putting the value of function DT=$\psi P_{rc}^{-\alpha} A_c^{\beta} d^{\gamma}$

$$\Pi s = \psi P_{rc}{}^{-\alpha} A_c{}^{\beta} d^{\gamma} [(100-d)P_{mr} - S_{Cm}\frac{1}{Q} - 0.5\frac{z^2}{rQ} H_{cm} - P_{Cm}] \quad (1)$$

For zero profit, Π_S=0 gives

$$P_{mr}(0) = \frac{1}{(100-d)}\left(P_{Cm} + \frac{S_{Cm}}{Q} + 0.5z^2 H_{cm}\frac{1}{r}\frac{1}{Q}\right)$$

Since $P_{mr}(0)$ is a linear function considering K as linear scale constant we can find the optimal value of P_{mr}

$$P_{mr} = KP_{mr}(0)$$

$$P_{mr} = \frac{K}{(100-d)}\left(P_{Cm} + \frac{S_{Cm}}{Q} + 0.5z^2 H_{cm}\frac{1}{r}\frac{1}{Q}\right) \quad (2)$$

Using the first-order partial differential equation for decision variable d, $\frac{\partial \Pi s}{\partial d} = 0$

$$d = \frac{\gamma}{(\gamma+1)}\left(100P_{mr} - P_{Cm} - \frac{S_{Cm}}{Q} - 0.5z^2 H_{cm}\frac{1}{r}\frac{1}{Q}\right) \quad (3)$$

2.3.2 Retailer's model formulation

In this section a model for the retailer's profit will be developed.
considering

Ordering cost= $O_{cr} * \frac{D_T}{Q}$

Holding cost= $0.5 * Q * \frac{Q}{r} * \frac{D_T}{Q} * H_{cr}$

Total Advertisement cost= $A_c * D_T$

So the profit for the retailer will be

Profit=Total Revenue-Purchase cost-Advertisement cost-Ordering cost-Holding cost

$$\Pi_b = P_{rc} D_T - P_{mr} D_T - A_c D_T - O_{Cr}\frac{D_T}{Q} - 0.5Q\frac{Q}{r}\frac{D_T}{Q} H_{cr}$$

By putting the value of

$$D_T = \psi P_{rc}{}^{-\alpha} A_c{}^{\beta} d^{\gamma}$$

$$\Pi_b = \psi P_{rc}{}^{-\alpha} A_c{}^{\beta} d^{\gamma} (P_{rc} - P_{mr} - A_c - O_{Cr}\frac{1}{Q} - 0.5\frac{Q}{r} H_{cr}) \quad (4)$$

To find all decision variables for the retailer

$$\frac{\partial \Pi_b}{\partial P_{rc}} = 0, \frac{\partial \Pi_b}{\partial A_c} = 0, \frac{\partial \Pi_b}{\partial Q} = 0$$

$$P_{rc} = \frac{\alpha}{(\alpha-\beta-1)}(P_{mr} + \frac{O_{Cr}}{Q} + 0.5\frac{Q}{r} H_{cr}) \quad (5)$$

$$A_c = \frac{\beta}{(\alpha-\beta-1)}(P_{mr} + \frac{O_{Cr}}{Q} + 0.5\frac{Q}{r} H_{cr}) \quad (6)$$

$$Q = \sqrt{\frac{2r O_{Cr} D}{H_{cr}}} \quad (7)$$

Theorem 1. 1(a) Πs is a concave function if.

$$A^{\beta}\Psi[P_{mc} - 1] \le 0 \quad (8)$$

1(b) Πb is a concave function if.

$$* \; Q \le \sqrt{\frac{2rQ(A_c + O_{cr} + P_{mr} - P_{rc}) + 2r}{H_{cm}}} \quad (9)$$

The stated above concave condition received Πb with respect to P_{rc}

$$* * Q \le \sqrt{\frac{2rQ(A_c + O_{cr} + P_{mr} - P_{rc}) + 2r}{H_{cm}}} \quad (10)$$

The stated above concave condition received Πb with respect to Q. And

$$* ** A_c^{-1+\beta}\beta \le A^{\beta} \tag{11}$$

The stated above concave condition received Πb with respect to A_c

Proof. 1(a) On partially differentiating Πs with respect to d.

Now on solving $\frac{\partial \Pi s}{\partial d}=0$ for d we get

$$\frac{\partial \Pi s}{\partial d}=\{(A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}\gamma\psi[(-100-d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]$$

$$-P_{mr}(A_c^{\beta} d^{\gamma} P_{rc}^{-\alpha}\psi)'[(100-d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]$$

The RHS of the stated above Expression

$$=\{(A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}\gamma\psi[-(-100+d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]$$

$$-P_{mr}(A_c^{\beta} d^{\gamma} P_{rc}^{-\alpha}\psi)'[-(-100+d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]$$

$$=\{(A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}\gamma\psi[-(-100+d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]$$

$$-A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}P_{mr}\gamma\psi[-(-100+d)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}]\le 0$$

$$=[(d-100)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}][-A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}\gamma\psi+A_c^{\beta} d^{-1+\gamma} P_{rc}^{-\alpha}P_{mr}\gamma\psi]\le 0$$

$$=[(d-100)P_{mr}-P_{cm}-\frac{S_{cm}}{Q}-\frac{H_{cm}Z^2}{2rQ}][\frac{A_c^{\beta}\psi(P_{mr}-1)}{dP_{rc}^{1+\gamma-\alpha}}]\le 0$$

After simplification, we get,

$$A^{\beta}\Psi[P_{mc}-1]\le 0 \quad \Rightarrow P_{mr}\le -1$$

Hence it is a concave function with respect to d.

1(b1) On partially differentiating Πb with respect to P_{rc} ,

Now on solving $\frac{\partial \Pi b}{\partial P_{rc}}=0$ for P_{rc} we get

$$\frac{\partial \Pi b}{\partial P_{rc}}=\{[A_c^{-\beta} d^{\gamma} P_{rc}^{-1-\alpha}\alpha\psi][-\mathrm{A}_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]\}$$

The RHS of the above expression is

$$[A_c^{-\beta} d^{\gamma} P_{rc}^{-1-\alpha}\alpha\psi][-\mathrm{A}_c-O_{cr}-P_{mr}+P_{rc}-\frac{-1}{Q}-\frac{H_{cm}Q}{2r}]$$

$$=[A_c^{-\beta} d^{\gamma} P_{rc}^{-1-\alpha}\alpha\psi][-\mathrm{A}_c-O_{cr}-P_{mr}+P_{rc}+\{\frac{H_{cm}Q^2-2r}{2rQ}\}]\le 0$$

After simplification

$$\Rightarrow Q\le\sqrt{\frac{2rQ(A_c+O_{cr}+P_{mr}-P_{rc})+2r}{H_{cm}}}$$

(b2) On partially differentiating Πb with respect to Q

Now on solving $\frac{\partial \Pi b}{\partial Q}=0$ for Q, we get

$$\frac{\partial \Pi b}{\partial Q}=\{(\frac{1}{Q^2}-\frac{H_{cm}}{2r})(A_c^{\beta} d^{\gamma} P_{rc}^{-\alpha}\psi)'[-\mathrm{A}_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]\}$$

The RHS of the above expression is

$$\{(\frac{1}{Q^2}-\frac{H_{cm}}{2r})(A_c^{\beta}d^{\gamma}P_{rc}^{-\alpha}\psi)^{'}[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]\}\leq 0$$

$$=\frac{H_{cm}Q^2-2r}{2Qr}\leq[A_c+O_{cr}+P_{mr}-P_{rc}]$$

$$\Rightarrow Q\leq\sqrt{\frac{2rQ(A_c+O_{cr}+P_{mr}-P_{rc})+2r}{H_{cm}}}$$

(b3) On partially differentiating Πb with respect to A_c

Now on solving $\frac{\partial \Pi b}{\partial A_c}=0$ for A_c we get

$$\frac{\partial \Pi b}{\partial A_c}=\{(A_c^{-1+\beta}d^{\gamma}P_{rc}^{-\alpha}\beta\psi)[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]$$

$$-(A_c^{\beta}d^{\gamma}P_{rc}\psi)^{'}[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]\}$$

The RHS of the above expression is

$$\{(A_c^{-1+\beta}d^{\gamma}P_{rc}^{-\alpha}\beta\psi)^{'}[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]$$

$$-A_c^{\beta}d^{\gamma}P_{rc}\psi[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}]\}$$

$$=[-A_c-O_{cr}-P_{mr}+P_{rc}-\frac{1}{Q}-\frac{H_{cm}Q}{2r}][A_c^{-1+\beta}d^{\gamma}P_{rc}^{-\alpha}\beta\psi-A_c^{\beta}d^{\gamma}P_{rc}^{-\alpha}\psi]\leq 0$$

$$=d^{\gamma}P_{rc}^{-\alpha}\psi(A_c^{-1+\beta}\beta-A^{\beta})\leq 0$$

After the simplification

$$=A_c^{-1+\beta}\beta\leq A^{\beta}\qquad\Rightarrow\beta\leq 0$$

Since the above condition is met, we can conclude that it is a concave function with respect to A_c

2.4 Non-Cooperative model

2.4.1 Manufacturer Stackelberg model for profit maximization in the non-co-operative environment

The manufacturer's Stackelberg model will be as follows:

Max

$$\Pi s=\psi P_{rc}{}^{-\alpha}A_c{}^{\beta}d^{\gamma}[(100-d)P_{mr}-S_{Cm}\frac{1}{Q}-0.5\frac{z^2}{rQ}H_{cm}-P_{Cm}$$

S.T.

$$P_{rc}=\frac{\alpha}{(\alpha-\beta-1)}(P_{mr}+\frac{O_{Cr}}{Q}+0.5\frac{Q}{r}H_{cr})$$

$$A_c=\frac{\beta}{(\alpha-\beta-1)}(P_{mr}+\frac{O_{Cr}}{Q}+0.5\frac{Q}{r}H_{cr})$$

$$Q=\sqrt{\frac{2r\,O_{Cr}\,D}{H_{cr}}}$$

2.4.2 Retailer's Stackelberg model for the non-cooperative environment

Model for profit maximization for the retailer will be

Max

$$\Pi_b=\psi P_{rc}{}^{-\alpha}A_c{}^{\beta}d^{\gamma}(P_{rc}-P_{mr}-A_c-O_{Cr}\frac{1}{Q}-0.5\frac{Q}{r}H_{cr}$$

S.T.

$$P_{mr}=\frac{K}{(100-d)}\left(P_{Cm}+\frac{S_{Cm}}{Q}+0.5z^2H_{cm}\frac{1}{r}\frac{1}{Q}\right)$$

$$d = \frac{\gamma}{(\gamma+1)}\left(100P_{mr} - P_{Cm} - \frac{S_{Cm}}{Q} - 0.5z^2 H_{cm}\frac{1}{r}\frac{1}{Q}\right)$$

3. Results and Discussion

3.1 Computational results

In this section models are verified using dummy data: α=1.1;β=.04;γ=1.6;O_{cr}=0.8;H_{cr}=1.9;P_{cm}=1.5;S_{cm}=6.5; H_{cm}=1.7;ψ=2000;r=160;K=1.2;z=80 Models are solved with the help of LINGO software, and the following results are obtained.

3.1.1 Seller's model

Seller's model provides the following results: Πs=1933804, d=60.84, P_{mr}= 4.54, A_c=3.12, P_{rc}=85.93, Q=11.6, D_T= 11174

3.1.2 Buyer's model

Buyer's model provides the following results for Π_b=1982052, d=98.32, P_{mr}= 1.62, A_c=1.299, P_{rc}=35.74, Q=52.9, D_T= 60996

3.2 Sensitivity analysis

In this section sensitivity analysis for all the decision variables d, Pmr, Prc, Ac, Q has performed for both manufacturer Stackelberg and retailer Stackelberg models for α, β, γ.

3.2.1 Tabular representation of sensitivity analysis

Table 1. Sensitivity analysis of the manufacturer's and retailer's model when α varies and β=0.06, γ=.08 are constant.

Manufacturer Stackelberg Model					Retailer Stackelberg Model				
α	1.3	1.5	1.7	1.9	α	1.3	1.5	1.7	1.9
P_{mr}	1.02	0.59	0.43	0.34	P_{mr}	2.24	2.24	2.24	2.24
d	42.27	40.68	39.24	37.93	d	98.13	97.9	97.77	97.68
P_{rc}	6.28	2.48	1.5	1.08	P_{rc}	13.02	8.15	6.33	5.39
A_c	0.29	0.1	0.05	0.03	A_c	0.6	0.33	0.22	0.17
Q	11.6	11.6	11.6	11.6	Q	20.48	16.82	15.23	14.33
D_T	3408	8661	15901	25809	D_T	2705	3148	3099	2864
Πm	183869	259173	332058	416188	Π_r	27096.5	17096.5	11549.4	8122.69

Table 2. Sensitivity analysis of the manufacturer's and retailer's model when β varies and α =1.1, γ=1.5 are constant.

Manufacturer Stackelberg Model					Retailer Stackelberg Model				
β	0.01	0.03	0.05	0.07	β	0.01	0.03	0.05	0.07
P_{mr}	3	3.84	5.33	8.84	P_{mr}	1.66	1.66	1.66	1.66
d	59	59.22	59.43	59.66	d	98.1	98.3	98.39	98.52
P_{rc}	38.41	62.49	120.5	329.22	P_{rc}	23.57	30.79	44.2	77.37
A_c	0.34	1.7	5.48	20.95	A_c	0.21	0.84	2	4.92
Q	11.6	11.6	11.6	11.6	Q	41.68	47.22	56.09	73.53
D_T	16210	9800	5128	1940	D_T	59299	44705	31304	18297
Πm	1916439	1485468	1085080	682208	Π_r	1270518	1251231	1258073	1286842

Table 3. Sensitivity analysis of manufacturer's and retailer's model when γ varies and α=1.1, β=0.04 are constant.

Manufacturer Stackelberg Model					Retailer Stackelberg Model				
γ	0.8	1	1.2	1.4	γ	0.8	1	1.2	1.4
P_{mr}	3.87	4.04	4.2	4.38	P_{mr}	2.24	1.99	1.83	1.71
d	43.87	49.38	53.9	57.67	d	98.6	98.5	98.42	98.37
P_{rc}	73.43	76.57	79.69	82.81	P_{rc}	45.55	41.4	38.77	36.99
A_c	2.67	2.78	2.9	3.01	A_c	1.66	1.5	1.4	1.35
Q	11.6	11.6	11.6	11.6	Q	36.72	41.14	45.27	49.18
D_T	380	871	2022	4738	D_T	1204	3333	8940	23509
Π_m	80516.5	173676	382323	854673	Π_r	49852.4	125458	315135	790639

3.2.2 Graphical analysis of decision variables P_{mr}, d, A_c, P_{rc}, Q, and profit to α, β, and γ

3.2.2.1 Effect of parameters α, β, and γ on decision variables for manufacture Stackelberg model

In Figure 1, Figure 2, Figure 3 three decision variables d, P_{rc}, and Q are represented on the primary axis and two decision variables P_{mr} and A_c are represented on the secondary axis.

Figure 1 represents the effect of sensitivity constant α on all the decision variables, while β and γ are constant for the manufacturer Stackelberg model. From Figure 1, we can see that the values of four decision variables d, P_{rc}, P_{mr}, and A_c decrease while EOQ (Q) is constant. It indicates that if the price sensitivity constant of the retailer to customer increases, the price of the retailer to customer, advertising cost, price of the manufacturer to retailer, and discount given by the manufacturer to the retailer will decrease.

Figure 2 represents the effect of sensitivity constant β on all the decision variables, while α and γ are constant for the manufacturer Stackelberg model. From Figure 2, we can see that the values of four decision variables d, P_{rc}, P_{mr}, and A_c increase while EOQ (Q) is constant when the value of advertising sensitivity constant β increases. It indicates that if advertising sensitivity constant β of retailer increases, price of the retailer to customer and advertising cost increases exponentially. From Table 2 we can see that for a small change in β from .01 to .07, the price of the retailer to the customer increases from 38.41 to 389.22 while advertising cost increases from 0.34 to 20.95 units. The price of the manufacturer to retailer also increases while there is not a big impact of β on discount given by the manufacturer to the retailer as d is changing slowly.

Figure 3 represents the effect of sensitivity constant γ on all the decision variables while α and β are constants for the manufacturer Stackelberg model. From Figure 3, we can see that when the value of sensitivity constant γ of discount given by the manufacturer to retailer increases, the values of four decision variables d, P_{rc}, P_{mr}, and A_c increase, while EOQ (Q) is constant. It indicates that if γ of manufacturer's discount increases, price of the retailer to customer, price of the manufacturer to retailer and discount given by the manufacturer to retailer increases significantly, while there is not a big impact of γ on advertising cost and it increases slowly.

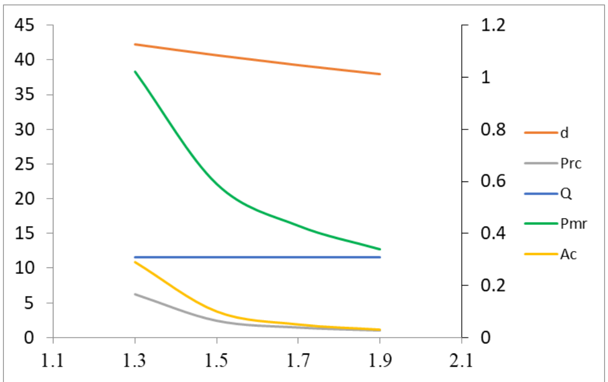

Figure 1. The effect on decision variables when α varies and β=0.06, γ=.08 are constant.

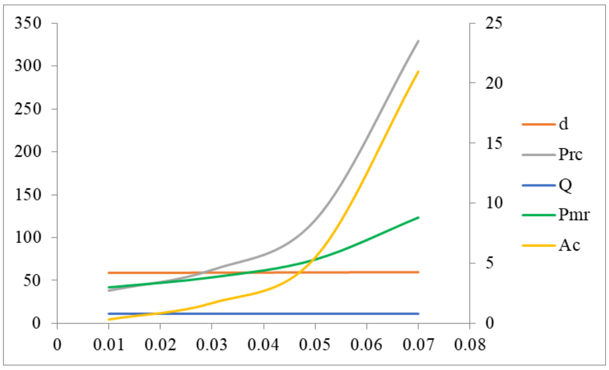

Figure 2. The effect on decision variables when β varies and α=1.1, γ=1.5 are constant.

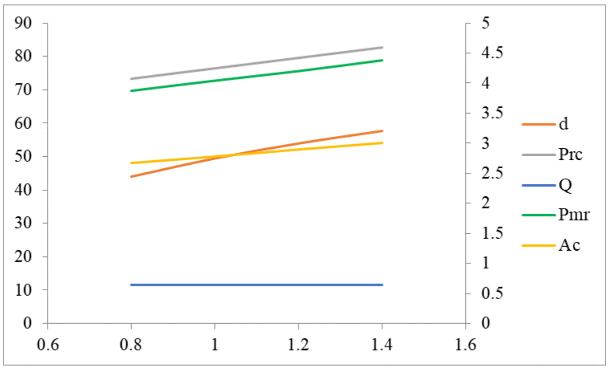

Figure 3. The effect on decision variables when γ varies and α=1.1, β=.04 are constant.

3.2.2.2 Effect of parameters α, β, and γ on decision variables for retailer's Stackelberg model

In Figures 4, Figure5, Figure 6 three decision variables d, P_{rc}, and Q are represented on the primary axis and two decision variables P_{mr} and A_c are represented on the secondary axis.

Figure 4 represents the effect of sensitivity constant α on all the decision variables, while β and γ are constant for the retailer Stackelberg model. From Figure 4, we can see that the values of four decision variables d, P_{rc}, Q, and A_c decrease while P_{mr} is constant. It indicates that if price sensitivity constant α of the retailer to customer increases, price of the retailer to customer, advertising cost of the retailer, discount given by the manufacturer to retailer, and economic order quantity Q decreases. From Table 1 we can see that for the retailer Stackelberg model impact of price sensitivity constant α on decision variables of the manufacturer (d, P_{mr}) is not high as the price of the manufacturer to the retailer is constant and there is a minor change in the value of d. But we can find a significant impact of price sensitivity constant α on economic order quantity Q while EOQ is not affected by price sensitivity constant α for the manufacturer Stackelberg model.

Figure 5 represents the effect of sensitivity constant β on all the decision variables, while α and γ are constant for the retailer Stackelberg model. From Figure 5, we can see that when the value of advertising sensitivity constant β increases the values of decision variables EOQ (Q), P_{rc}, d, and A_c increase while P_{mr}, is constant. It indicates that if advertising sensitivity

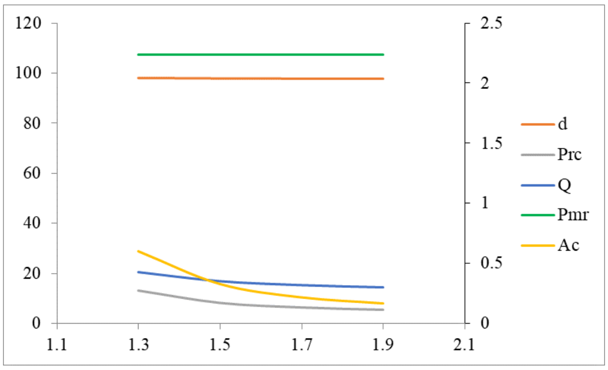

Figure 4. The effect on decision variables when α varies and β=0.06, γ=.08 are constant.

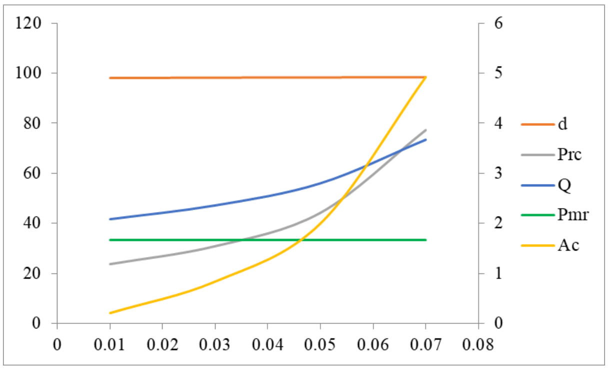

Figure 5. The effect on decision variables when β varies and α=1.1, γ=1.5 are constant.

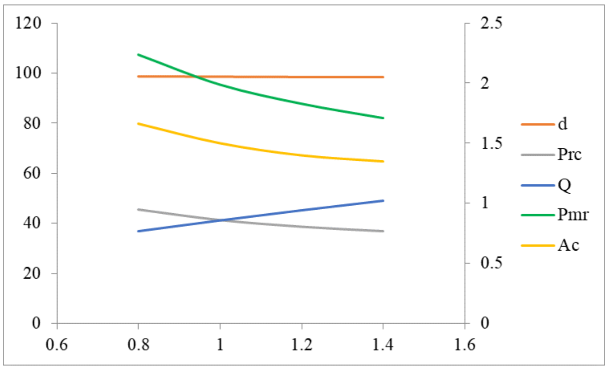

Figure 6. The effect on decision variables when γ varies and α=1.1, β=.04 are constant.

constant β of retailer increases, price of advertising cost increases exponentially. From Table 2 we can see that for a small change in β from .01 to .07, advertising cost increases from 0.21 to 4.92 units. From Table 2 we can see that for the retailer Stackelberg model impact of advertising sensitivity constant β on decision variables of the manufacturer(d, P_{mr}) is not high as the price of the manufacturer to the retailer is constant and there is a minor change in the value of d. But we can find a significant impact of advertising sensitivity constant β on economic order quantity Q while EOQ is not affected by advertising sensitivity constant β for manufacturer Stackelberg model.

Figure 6 represents the effect of sensitivity constant γ on all the decision variables while α and β are constants for the retailer Stackelberg model. From Figure 6, we can see that when the value of sensitivity constant γ of discount given by the manufacturer to retailer increases, the values of four decision variables d, P_{rc}, P_{mr}, and A_c decreases while EOQ (Q) is increasing. It indicates that if γ of manufacturer's discount increases, price of the retailer to customer, price of the manufacturer to retailer and advertising cost increases significantly, while there is not a big impact of γ on discount given by the manufacturer to retailer.

From Table 3 we can find a significant impact of γ on economic order quantity Q while EOQ is not by γ for manufacturer Stackelberg model.

3.2.2.3 Effect of parameter α, β, and γ on profits

Figure 7, Figure 8, Figure 9 represents the impact of sensitivity constants α, β, and γ on manufacturer's and retailer's profits for Stackelberg models.

Figure 7 represents the impact of sensitivity constant α on manufacturer's and retailer's profit while β and γ are constant. From Figure 7 we can see that profit of the manufacturer increases while the profit of the retailer decreases when α increases and β, γ are constant. It indicates that if the sensitivity constant of price charged by the retailer to customer increases, profit of manufacturer increases while profit of retailer will decrease in the non-co-operative environment.

Figure 8 represents the impact of sensitivity constant β on manufacturer's and retailer's profit while α and γ are constant. From Figure 8 we can see that profit of manufacture decreases when β increases and α, γ are constant, while profit of retailer decreases till β is .05 and then start increasing.

Figure 9 represents the impact of sensitivity constant γ on manufacturer's and retailer's profit while α and β are constant. From Figure 9 we can see that profit of manufacture and retailer increases when γ increases and α, β are constant. From Figure 9 and Table 3 we can see that profits of manufacturer and retailer increase rapidly when sensitivity constant γ of discount given by the manufacturer to retailer increases.

4. Conclusions

In this work manufacturer Stackelberg and retailer Stackelberg, models are studied in the non-co-operative environment. Total demand is considered sensitive to price, advertisement cost, and discount given by the manufacturer to the retailer. Here decision variables, the price for the manufacturer to retailer and discount given by the manufacturer to the retailer are decided by the manufacturer, while market price, advertising cost, and economic order quantity are decided by the retailer. In the supply chain, since the retailer is the first face of any product for the consumer, retailers play a crucial role to increase more market demand. So in this work, the third parameter discount given by the manufacturer to the retailer is also considered. Mathematical Stackelberg models are developed for both manufacturer and retailer and solved by the LINGO software. In the next section, models are also verified numerically. Sensitivity analysis is performed for all the three parameters α, β, and γ for all the decision variables. We can see from sensitivity analysis that total demand D_T and profit of both

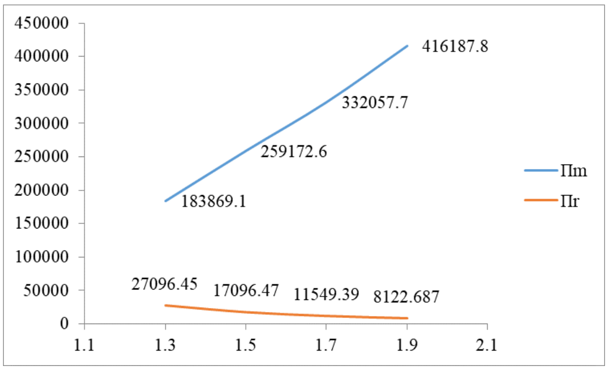

Figure 7. The effect on profits when α varies and β=0.06, γ=.08 are constant.

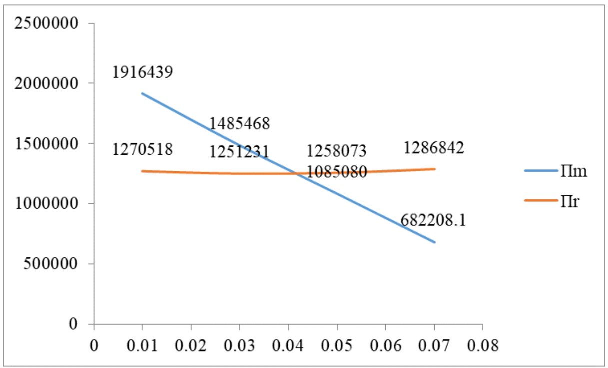

Figure 8. The effect on profits when β varies and α=1.1, γ=1.5 are constant.

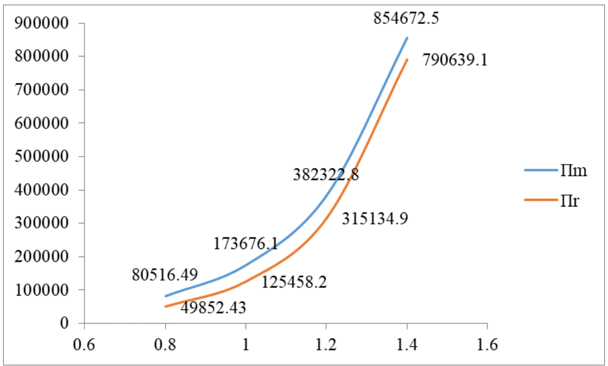

Figure 9. The effect on profits when γ varies and α=1.1, β=.04 are constant.

manufacture and retailer increases when the sensitivity coefficient of discount γ increases.

Future work can also be done in both co-operative and non-cooperative environments. In this work, the shortage was not permitted, while the shortage may also be incorporated from the manufacturer's side. For future work advertisement cost also may be shared by both the manufacturer and the retailer. In this work, scrapped items are not considered while it may be an important factor also. So, the concept of damaged items may also be incorporated. In this work inventory level is also known, while inventory level may also be a decision variable for the manufacturer.

References

Avakoli, S., & Taleizadeh, A. A. (2017). An EOQ model for decaying item with full advanced payment and conditional discount. *Annals of Operations Research*, *259*, 415-436.

Chan, C. K., & Kingsman, B. G. (2007). Coordination in a single-vendor multi-buyer supply chain by synchronizing delivery and production cycles. *Transportation Research Part E*, *43*, 90–111.

Chiang, C. W., Fitzsimmons, J., Huang, Z., & Li, S. X. (1994). A game-theoretic approach to quantity discount problem. *Decision Sciences*, *25*(1), 153–168.

Corbett, C. J., & Groote, D. X. (2000). A supplier's optimal quantity discount policy under asymmetric information. *Management Science*, *46*(3), 444–450.

Daesoo, K., & Lee, W. J. (1998). Optimal joint pricing and lot sizing with constant and variable capacity. *European Journal of Operational Research*, *109*(1), 212–227.

Dai, T., & Qi, X. (2007). An acquisition policy for a multi-supplier system with a finite-time horizon. *Computers and Operations Research*, *34*, 2758–2773.

Datta, T. K., & Paul, K. (2001). An inventory system with a stock-dependent, price-sensitive demand rate. *Production Planning and Control*, *12*, 1, 13–20

Esmaeili, M., Bahadur. M., & Zeephongsekul, P. (2009) A game theory approach in manufacturer- Retailer supply chain. *European Journal of Operational Research*, *125*(2), 442-448.

Farshbaf-Geranmayeha, A., Rabbania, M., & Taleizadeha, A. A. (2019). Cooperative advertising to induce strategic customers for purchase at the full price. *International Transactions in Operational Research*, *26*(6), 1-33.

Freeland, J. R. (1982). Coordination strategies for production and marketing in a functionally decentralized firm. *AIIE Transactions*, *12*, 126–132.

Ghobadi, S. N., Bagherinejad, J., & Taleizadeh, A. A. (2021). A two-generation new product model by considering forward-looking customers: Dynamic pricing and advertising optimization. *Journal of Retailing and Consumer Services*, 63.

He, X., Krishnamoorthy, A., Prasad, A., & Sethi, S. P. (2011). Retail competition and cooperative advertisement. *Operations Research Letters*, *39*, 11-16

He, B., Pan, W., & Yang, Y, (2017). Joint pricing and overbooking policy in a full payment presale mechanism of new products. *International Transactions in Operational Research*, 1-18

Heuvel, W. V. D., Borm, P., & Hamers, H. (2007). Economic lot-size in games. *European Journal of Operational Research*, *176*, 1117–1130.

Huang, Z. L., & Li, S.X. (2001). Coop advertisement models in manufacturer retailer supply chains: A game theory. *European Journal of Operational Research*, *135*, 527–544

Jinfeng, Y., Jill, A., Min-Chiang, W., & Zhimin, H. (2006). Coordination of cooperative advertisement in a two-level supply chain when manufacturer offers a discount. *European Journal of Operational Research*, *168*(1), 65–85.

Jung, H. C., & Klein, M. (2005). Optimal inventory policies for an economic order quantity model with decreasing cost functions. *European Journal of Operational Research*, *165*(1), 108–126.

Jüttner, U., Christopher, M., & Baker, S. (2007). Demand chain management-integrating marketing and supply chain management. *Industrial Marketing Management*, *36*, 377-392

Kunter, M. (2012). Coordination via cost and revenue sharing in manufacturer and retailer channels. *European Journal of Operational Research*, *216*, 477–486

Lee, W. J., & Daesoo, K. (1993). Optimal and heuristic decision strategies for integrated product and marketing planning. *Decision Sciences*, *24*(6), 1203–1213

Lee, W. J., & Daesoo, K. (1998). Optimal coordination strategies for production and marketing decisions. *Operations Research Letters*, *22*, 41–47.

Leng, M. & Zhu, A. (2009). Side payment contracts in two-person non zero supply chain games: Review, discussion, and applications. *European Journal of Operational Research*, *196*, 600-618.

Lin, Y. J. & Ho, C. H. (2011). Integrated inventory model with a quantity discount and price-sensitive demand. *TOP, 19*(1), 177–188.

Pal, S., Mahapatra, G. S., & Samanta, G. P. (2013). An inventory model of price and stock dependent demand rate with deterioration under-inflation and delay in payment. *International Journal of System Assurance Engineering and Management, 5*(4), 591–601.

Papachrstos, S., & Skouri, K. (2011). An inventory model with a deteriorating items quantity discount, pricing, and time-dependent partial backlogging. *International Journal of Production Economics*, *83*(3), 247-256.

Pal, B., Sana, S. S., & Chaudhuri, K. (2015). Two-echelon manufacturer–retailer supply chain strategies with price, quality, and promotional effort-sensitive demand. *International Transactions in Operational Research, 22*, 6, 1071–1095.

Prakash, A. L. (1994). Supplier pricing and lot sizing when demand is price sensitive. *European Journal of Operational Research, 78*, 334–354.

Prakash, A. L., & Jaggi, C. K. (2003). Joint approach for setting unit price and the length of the credit period for a manufacturer when end demand is price sensitive. *International Journal of Production Economics*, *83*, 115–122.

Sadjadi, S., Oroujee, M., & Aryanezhad, M. B. (2005). Optimal production and marketing planning. *Computational Optimization and Applications*, *30*(2), 195-203

Shaikh, A. A., Khan, A. A., Panda G. C. & Konstantaras, I. (2019). Price discount facility in an EOQ model for deteriorating items with stock dependent demand and partial backlogging. *International transactions in Operations Research*, 1-31.

Sarkar, B. (2011). An EOQ model with delay in payments and stock-dependent demand in the presence of imperfect production. *Applied Mathematics and Computation, 218*(17).

Sarmah, S. P., Acharya, D., & Goyal, S. K. (2006). Buyer vendor coordination models in supply chain management. *European Journal of Operational Research*, *175*, 1-15

Soni, H. N., & Shah, N. H. (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. *European Journal of Operational Research*, *184*(1), 91-100

Taleizadeh, A. A., Hazarkhani, B., & Moon, L. (2020). Joint pricing and inventory decisions with carbon emission considerations, partial back ordering, and planned discounts. *Annals of Operations Research*, *290*(1), 95-113,

Taleizadeh, A. A., Rabiei, N., & Noori-Daryan, M. (2019). Coordination of a two-echelon supply chain in presence of market segmentation, credit payment, and quantity discount policies. *International Transactions in Operational Research*, *26*, 1576-1605

Taleizadeh, A. A., Haghighi, F., & Niaki, S. T. A. (2019). Modeling and solving a sustainable closed loop supply chain problem with pricing decisions and discounts on returned products. *Journal of Cleaner Production*, *207*, 163-181.

Taleizadeh, A. A., Satarian, F., & A. Jamili. (2016). optimal multi discount selling prices schedule for deteriorating product. *Scientia Iranica*, *22*(6), 2595-2603.

Taleizadeh, A. A., Stojkovska, I., & Pentico, D. W. (2015). An economic order quantity model with partial backordering and incremental discount. *Computer and Industrial Engineering*, *82*, 21-32.

Taleizadeh, A. A., Stojkovska, I., & Pentico, D. W. (2014). An economic order quantity model with partial backordering and all-units discount. *International Journal of Production Economic*, *155*, 172-184,

Taleizadeh, A. A., Moghadasi, H., Niaki , S. T. A. & Eftekhari, A. (2009). An EOQ-joint replenishment policy to supply expensive imported raw materials with payment in advance, *Journal of Applied Science*, *8*(23), 4263-4273.

Taleizadeh, A. A., Cheraghi, Z., Cárdenas-Barrón, L. E. & Noori-Daryan, M. (2021). Studying the effect of noise on pricing and marketing decisions of new products under co-op advertising strategy in supply chains. *Mathematics*, *9*(11).

Taleizadeh, A. A., Maryam, K. A., Ali , T. S. (2018). A possibilistic closed-loop supply chain: pricing, advertising, and remanufacturing optimization. *Neural Computing and Applications*, *32*, 1195-1215.

Yue, J., Austin, J., Wang, M. C., Huang, Z. (2006). Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount. *European Journal of Operational Research*, *168*(1), 65–85.