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Original Article

Classes of combined population mean estimators utilizing transformed variables under double sampling with application to air pollution in Chiang Rai, Thailand

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Abstract

The transformation method can be used to increase the efficiency of the variable of interest for estimating population mean. Two classes of combined population mean estimators utilizing transformation on an auxiliary variable and on both an auxiliary variable and a study variable have been proposed under double sampling. The formulas of the biases and mean square errors of the proposed estimators are obtained. Simulation studies and an application to fine particulate matter 2.5 and nitrogen dioxide pollution data in Chiang Rai, Thailand have been investigated to assess performance of the proposed estimators compared to other existing estimators. Under certain conditions, the results indicate that the proposed combined estimators perform much better than other existing estimators.

Keywords: air pollution, bias, mean square error, ratio estimators, transformed variables

1. Introduction

Auxiliary information has been utilized to estimate the possible outcomes of variables of interest in survey research for several decades. For example, we can use the concentration of fine particulate matter 2.5 (PM2.5) to estimate the concentration of nitrogen dioxide (NO_2) in the area of study because there is a correlation between PM2.5 and NO_2 (Wu, Xu, & Wang, 2016). A usual ratio estimator utilizing auxiliary information is very effective when the correlation between the auxiliary and study variables is positive ($\rho > 0$). On the other hand, with a negative relationship between these two variables ($\rho < 0$), a product estimator is successfully used to estimate the population mean and these were proposed by Cochran (1940) and Murthy (1967), respectively. After that, known parameters of the auxiliary variable, for example, correlation coefficient (ρ) or coefficient of variation (C_x), were utilized to improve the precision of estimation of the population mean of the study variable by several researchers (see e.g. Singh and Tailor (2003), Kadilar and Cingi (2004)).

In 1938, a double sampling technique, which is a beneficial method for estimating population parameters when the population parameters of the auxiliary variable are not available, was suggested by Neyman (1938). Let $(X_i, Y_i)(i = 1, 2, 3, . . . , N)$ be the pairs of observations of the auxiliary variable X_i and the study variable Y_i. A random sample of size $\acute{\eta}$ is drawn from a population of size N to observe the auxiliary variable using simple random sampling without replacement (SRSWOR) in the first phase of sampling. Then, a random sample of size $n(n < \acute{\eta})$ is drawn from the sample of size $\acute{\eta}$ to measure both the auxiliary and study variables by SRSWOR in the second phase of sampling. Many researchers proposed estimators for estimating population mean under double sampling. For example, Malik and Tailor (2013) modified Singh and Tailor's (2003) estimator which utilized the correlation coefficient between the auxiliary and study variables. Amin, Shahbaz, and Kadilar (2016) modified the estimators proposed by Kadilar and Cingi (2004) under simple random sampling for use in the double sampling scheme. Other estimators employing auxiliary information under double sampling can be found in Kiregyera (1980), Mohanty (1967), and Singh and Khalid (2015, 2019).

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The transformation technique has been applied to improve the performance of the population mean estimator. Under simple random sampling, Srivenkataramana (1980) suggested the transformation technique to transform an auxiliary variable to increase the efficiency of the population mean estimator, and many other authors have also utilized this technique (see e.g. Bandyopadhyaya (1986), Thongsak and Lawson (2022). Later, Adewara (2006) proposed to increase the performance of the population mean estimator by transforming both auxiliary and study variables (see also Adewara, Singh, and Kumar (2012), Singh, Malik, and Smarandache (2016). Under the double sampling scheme, the transformation of an auxiliary variable was used to obtain a dual to ratio estimator, which was proposed by Kumar and Bahl (2006). Consider the variable transformation in Equation (1),

$$x_i^{*d} = \frac{n'\bar{x}' - nx_i}{n' - n}; i = 1, 2, 3, \ldots, N. \tag{1}$$

Then, the associated sample means of the auxiliary variable and study variable were obtained as in Equation (2) and (3),

$$\bar{x}^{*d} = \frac{n'\bar{x}' - n\bar{x}}{n' - n} = (1+\pi)\bar{x}' - \pi\bar{x}, \tag{2}$$

$$\bar{y}^{*d} = \frac{n'\bar{y}' - n\bar{y}}{n' - n} = (1+\pi)\bar{y}' - \pi\bar{y}, \tag{3}$$

where $\pi = \dfrac{n}{n' - n}$, $\bar{y}'$ and $\bar{y}$ are the sample means of the study variable based on the first phase and second phase samples, respectively, $\bar{x}'$ and $\bar{x}$ are the sample means of the auxiliary variable based on the first phase and second phase samples, respectively.

Later, the transformation technique was also applied to increase the efficiency of the population mean estimator under double sampling in many studies (Choudhury & Singh, 2015; Singh & Choudhury, 2012). Recently, Thongsak and Lawson (2022) proposed four classes of population mean estimators applying the transformation technique on an auxiliary variable, and on both auxiliary and study variables. Thongsak and Lawson showed that the proposed estimators are better than the non-transformed estimators in terms of smaller mean square errors (MSEs). Some researchers also used the combination technique to increase the efficiency of the population mean estimator, and this can be found in Thongsak and Lawson (2022).

The reviewed estimators can be useful in estimating the population mean of the variable of interest when there is a relationship between the variable of interest and the auxiliary variable. Several studies have found that ambient air pollution has been associated with numerous types of cancer (Hasegawa, Tsukahara, and Nomiyama, 2021; Seifi *et al.*, 2019;). Hasegawa *et al.* (2021) found a positive correlation between NO_2 and colorectal cancer, and a correlation between PM2.5 and lung cancer for both sexes. Among males, sulphur dioxide (SO_2) and PM2.5 are associated with liver cancer. In females, there is a significant association between nitric oxide (NO) and lung cancer, and they also found a correlation between NO_2 and breast cancer. Having knowledge of air pollution can be useful to estimating the incidence of cancer and assist in treatment planning.

Following the work of Thongsak and Lawson (in press) and motivated by Thongsak and Lawson (submitted), we proposed two classes of the combined population mean estimators utilizing the transformation of an auxiliary variable and of both auxiliary and study variables under double sampling. The formulas of the biases and mean square errors of the proposed estimators were established. The proposed estimators were compared with the usual ratio estimator utilizing the percentage relative efficiencies (PREs) as a criterion, through simulation studies and in an application to air pollution data in Chiang Rai, Thailand, which is a province with the population having a high risk of cancer (Thongsak *et al.*, 2016).

2. Existing Estimators

In 1938, Neyman (1938) proposed a nowadays usual ratio estimator under double sampling. The estimator is

$$\hat{\bar{Y}}_{\text{Neyman}} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right), \tag{4}$$

The bias and MSE of $\hat{\bar{Y}}_{\text{Neyman}}$ are defined by

$$Bias\left(\hat{\bar{Y}}_{\text{Neyman}}\right) \cong \bar{Y}\left(\gamma - \gamma^*\right)\left(C_x^2 - \rho C_x C_y\right), \tag{5}$$

$$MSE\left(\hat{\bar{Y}}_{\text{Neyman}}\right) \cong \bar{Y}^2\left[\gamma C_y^2 + \left(\gamma - \gamma^*\right)\left(C_x^2 - 2\rho C_x C_y\right)\right], \tag{6}$$

where $\gamma = \dfrac{1}{n} - \dfrac{1}{N}$, $\gamma^* = \dfrac{1}{n'} - \dfrac{1}{N}$, ρ is the correlation coefficient between the auxiliary and study variables, and C_x, C_y are the coefficients of variation of the auxiliary variable and study variable, respectively.

Recently, Thongsak and Lawson (2022) recommended use of the transformation technique to improve the efficiency of the population mean estimator under double sampling. The Thongsak and Lawson (2022) estimators are given by

$$\hat{\bar{Y}}_{\text{R1}} = \bar{y}\left(\frac{A\bar{x}^{*d} + D}{A\bar{x}' + D}\right), \tag{7}$$

$$\hat{\bar{Y}}_{\text{Reg1}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}^{*d} + H}{G\bar{x}' + H}\right), \tag{8}$$

$$\hat{\bar{Y}}_{\text{R2}} = \bar{y}^{*d}\left(\frac{A\bar{x}' + D}{A\bar{x}^{*d} + D}\right), \tag{9}$$

$$\hat{\bar{Y}}_{\text{Reg2}} = \left[\bar{y}^{*d} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}' + H}{G\bar{x}^{*d} + H}\right). \tag{10}$$

The biases and MSEs of $\hat{\hat{\bar{Y}}}_{\mathrm{R1}}$, $\hat{\hat{\bar{Y}}}_{\mathrm{Reg1}}$, $\hat{\hat{\bar{Y}}}_{\mathrm{R2}}$, and $\hat{\hat{\bar{Y}}}_{\mathrm{Reg2}}$ are given by

$$Bias\left(\hat{\hat{\bar{Y}}}_{\mathrm{R1}}\right) \cong -\bar{Y}\left(\gamma-\gamma^{*}\right)\pi\theta_1\rho C_x C_y, \tag{11}$$

$$Bias\left(\hat{\hat{\bar{Y}}}_{\mathrm{Reg1}}\right) \cong -\bar{Y}\left(\gamma-\gamma^{*}\right)\left[\beta K\theta_2\pi^2 C_x^2+\pi\theta_2\rho C_x C_y\right], \tag{12}$$

$$Bias\left(\hat{\hat{\bar{Y}}}_{R2}\right) \cong \pi^2\bar{Y}\left(\gamma-\gamma^{*}\right)\left(\theta_1^2 C_x^2-\theta_1\rho C_x C_y\right), \tag{13}$$

$$Bias\left(\hat{\hat{\bar{Y}}}_{\mathrm{Reg2}}\right) \cong \pi^2\bar{Y}\left(\gamma-\gamma^{*}\right)\left[\left(\theta_2^2+\beta K\theta_2\right)C_x^2-\theta_2\rho C_x C_y\right], \tag{14}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\mathrm{R1}}\right) \cong \bar{Y}^2\left[\gamma C_y^2+\left(\gamma-\gamma^{*}\right)\left(\theta_1^2\pi^2 C_x^2-2\theta_1\pi\rho C_x C_y\right)\right], \tag{15}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\mathrm{Reg1}}\right) \cong \bar{Y}^2\left[\gamma C_y^2+\left(\gamma-\gamma^{*}\right)\left(\left(\theta_2-\beta K\right)^2\pi^2 C_x^2-2\left(\theta_2-\beta K\right)\pi\rho C_x C_y\right)\right], \tag{16}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\mathrm{R2}}\right) \cong \bar{Y}^2\left[\gamma^{*} C_y^2+\pi^2\left(\gamma-\gamma^{*}\right)\left(C_y^2+\theta_1^2 C_x^2-2\theta_1\rho C_x C_y\right)\right], \tag{17}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\mathrm{Reg2}}\right) \cong \bar{Y}^2\left[\gamma^{*} C_y^2+\pi^2\left(\gamma-\gamma^{*}\right)\left(C_y^2+\left(\theta_2+\beta K\right)^2 C_x^2-2\left(\theta_2+\beta K\right)\rho C_x C_y\right)\right], \tag{18}$$

where $\theta_1=\dfrac{A\bar{X}}{A\bar{X}+D}$, $\theta_2=\dfrac{G\bar{X}}{G\bar{X}+H}$, $K=\dfrac{\bar{X}}{\bar{Y}}$, $\beta=\dfrac{\rho S_y}{S_x}$.

Some members of $\hat{\hat{\bar{Y}}}_{\mathrm{R1}}$, $\hat{\hat{\bar{Y}}}_{\mathrm{Reg1}}$, $\hat{\hat{\bar{Y}}}_{\mathrm{R2}}$, and $\hat{\hat{\bar{Y}}}_{\mathrm{Reg2}}$ with some auxiliary parameters applied are shown in Table 1.

Table 1. Some existing estimators

Estimator		A or G	D or H
$\hat{\hat{\bar{Y}}}_{\mathrm{R11}}=\bar{y}\left(\dfrac{\bar{x}^{*d}}{\bar{x}'}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg11}}=\left[\bar{y}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}^{*d}}{\bar{x}'}\right)$	1	0
$\hat{\hat{\bar{Y}}}_{\mathrm{R12}}=\bar{y}\left(\dfrac{\bar{x}^{*d}+C_x}{\bar{x}'+C_x}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg12}}=\left[\bar{y}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}^{*d}+C_x}{\bar{x}'+C_x}\right)$	1	C_x
$\hat{\hat{\bar{Y}}}_{\mathrm{R13}}=\bar{y}\left(\dfrac{\bar{x}^{*d}+\rho}{\bar{x}'+\rho}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg13}}=\left[\bar{y}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}^{*d}+\rho}{\bar{x}'+\rho}\right)$	1	ρ
$\hat{\hat{\bar{Y}}}_{\mathrm{R21}}=\bar{y}^{*d}\left(\dfrac{\bar{x}'}{\bar{x}^{*d}}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg21}}=\left[\bar{y}^{*d}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'}{\bar{x}^{*d}}\right)$	1	0
$\hat{\hat{\bar{Y}}}_{\mathrm{R22}}=\bar{y}^{*d}\left(\dfrac{\bar{x}'+C_x}{\bar{x}^{*d}+C_x}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg22}}=\left[\bar{y}^{*d}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'+C_x}{\bar{x}^{*d}+C_x}\right)$	1	C_x
$\hat{\hat{\bar{Y}}}_{\mathrm{R23}}=\bar{y}^{*d}\left(\dfrac{\bar{x}'+\rho}{\bar{x}^{*d}+\rho}\right)$	$\hat{\hat{\bar{Y}}}_{\mathrm{Reg23}}=\left[\bar{y}^{*d}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'+\rho}{\bar{x}^{*d}+\rho}\right)$	1	ρ

3. The Estimation of Population Mean

3.1 Proposed classes of combined estimators

Motivated by Thongsak and Lawson (2022) using the idea of transformed variables and the work of Thongsak and Lawson (2022) using the idea of the combined technique to increase the efficiency of the population mean estimator, we proposed two classes of combined population mean estimators utilizing transformed variables under double sampling. The proposed estimators are given by

$$\hat{\hat{Y}}_{\text{C1}} = \alpha\hat{\hat{Y}}_{\text{R1}} + (1-\alpha)\hat{\hat{Y}}_{\text{Reg1}}, \tag{19}$$

$$\hat{\hat{Y}}_{\text{C2}} = \alpha\hat{\hat{Y}}_{\text{R2}} + (1-\alpha)\hat{\hat{Y}}_{\text{Reg2}}, \tag{20}$$

Where α is a constant that minimizes MSE.

To obtain the biases and MSEs of the proposed estimators, the following notations are defined: $\varepsilon_0 = (\bar{y}-\bar{Y})/\bar{Y}$ then $\bar{y} = (1+\varepsilon_0)\bar{Y}$, $\varepsilon_1 = (\bar{x}-\bar{X})/\bar{X}$ then $\bar{x} = (1+\varepsilon_1)\bar{X}$, $\varepsilon_2 = (\bar{x}'-\bar{X})/\bar{X}$ then $\bar{x}' = (1+\varepsilon_2)\bar{X}$ and $\bar{x}^{*d} = (1+\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)\bar{X}$, $\varepsilon_3 = (\bar{y}'-\bar{Y})/\bar{Y}$ then $\bar{y}' = (1+\varepsilon_3)\bar{Y}$ and $\bar{y}^{*d} = (1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}$ such that $E(\varepsilon_0)=E(\varepsilon_1)=E(\varepsilon_2)=E(\varepsilon_3)=0$, $E(\varepsilon_0^2)=\gamma C_y^2$, $E(\varepsilon_1^2)=\gamma C_x^2$, $E(\varepsilon_2^2)=E(\varepsilon_1\varepsilon_2)=\gamma^* C_x^2$, $E(\varepsilon_3^2)=E(\varepsilon_0\varepsilon_3)=\gamma^* C_y^2$, $E(\varepsilon_0\varepsilon_1)=\gamma\rho C_x C_y$, $E(\varepsilon_0\varepsilon_2)=E(\varepsilon_1\varepsilon_3)=$ $E(\varepsilon_2\varepsilon_3)=\gamma^*\rho C_x C_y$

Rewriting Equation (19) and Equation (20) in terms of ε_0, $\varepsilon_1, \varepsilon_2$, ε_3 we have:

$$\begin{aligned}\hat{\hat{Y}}_{\text{C1}} &= \alpha(1+\varepsilon_0)\bar{Y}\left(\frac{(A\bar{X}+D)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)A\bar{X}}{(A\bar{X}+D)+\varepsilon_2 A\bar{X}}\right)\\ &\quad+(1-\alpha)\begin{bmatrix}(1+\varepsilon_0)\bar{Y}\\ +b(\varepsilon_1-\varepsilon_2)\pi\bar{X}\end{bmatrix}\left(\frac{(G\bar{X}+H)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)G\bar{X}}{(G\bar{X}+H)+\varepsilon_2 G\bar{X}}\right)\end{aligned} \tag{21}$$

$$\begin{aligned}\hat{\hat{Y}}_{\text{C2}} &= \alpha(1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}\left(\frac{(A\bar{X}+D)+\varepsilon_2 A\bar{X}}{(A\bar{X}+D)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)A\bar{X}}\right)\\ &+(1-\alpha)\begin{bmatrix}(1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}\\ +b(\pi\varepsilon_1-\pi\varepsilon_2)\bar{X}\end{bmatrix}\left(\frac{(G\bar{X}+H)+\varepsilon_2 G\bar{X}}{(G\bar{X}+H)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)G\bar{X}}\right)\end{aligned} \tag{22}$$

The biases and MSEs of $\hat{\hat{Y}}_{\text{C1}}$ and $\hat{\hat{Y}}_{\text{C2}}$ up to the first degree of approximation are as follows

$$Bias\left(\hat{\hat{Y}}_{\text{C1}}\right) \cong \bar{Y}(\gamma-\gamma^*)\left\{(\alpha-1)\theta_2\pi^2\beta K C_x^2 + ((\alpha-1)\theta_2-\alpha\theta_1)\pi\rho C_x C_y\right\}, \tag{23}$$

$$Bias\left(\hat{\hat{Y}}_{\text{C2}}\right) \cong \pi^2\bar{Y}(\gamma-\gamma^*)\left\{\left(\alpha\theta_1^2+(1-\alpha)(\beta K\theta_2+\theta_2^2)\right)C_x^2 - (\alpha\theta_1+(1-\alpha)\theta_2)\rho C_x C_y\right\}, \tag{24}$$

$$MSE\left(\hat{\hat{Y}}_{\text{C1}}\right) \cong \bar{Y}^2\left(\gamma C_y^2 + (\gamma-\gamma^*)\begin{Bmatrix}(\alpha\theta_1+(1-\alpha)(\theta_2-\beta K))^2\pi^2 C_x^2\\ -2\pi(\alpha\theta_1+(1-\alpha)(\theta_2-\beta K))\rho C_x C_y\end{Bmatrix}\right), \tag{25}$$

$$MSE\left(\hat{\hat{Y}}_{\text{C2}}\right) \cong \bar{Y}^2\left(\gamma^* C_y^2 + (\gamma-\gamma^*)\pi^2\begin{Bmatrix}C_y^2+(\alpha\theta_1+(1-\alpha)(\beta K+\theta_2))^2 C_x^2\\ -2(\alpha\theta_1+(1-\alpha)(\beta K+\theta_2))\rho C_x C_y\end{Bmatrix}\right), \tag{26}$$

where the difference $E(b)-\beta$ was omitted (Cochran, 1977).

3.2 Optimum choice of scalar α

In order to minimize MSE of the proposed classes of combined estimators $\hat{\hat{Y}}_{\text{C1}}$ and $\hat{\hat{Y}}_{\text{C2}}$ in Equation (19) and Equation (20), we must find the optimum value of α by taking a partial derivative of the MSE in Equation (25) and Equation (26) with respect to α and equating it to zero.

The optimal α for $\hat{\hat{Y}}_{\text{C1}}$ is

$$\alpha = \frac{\rho C_y - \pi(\theta_2-\beta K)C_x}{\pi(\theta_1-\theta_2+\beta K)C_x} = \alpha_{\text{C1}}^{\text{opt}}, \ (\text{say}). \tag{27}$$

Substituting Equation (27) into Equation (19), the optimum $\hat{\hat{Y}}_{\text{C1}}$ is

$$\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}} = \alpha_{\mathrm{C1}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R1}} + \left(1-\alpha_{\mathrm{C1}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg1}}. \tag{28}$$

Substituting Equation (27) into Equation (25), the optimum MSE of estimator $\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}}$ is

$$MSE_{\min}\left(\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}}\right) \cong \bar{Y}^2\left(\gamma C_y^2 - \left(\gamma-\gamma^*\right)\rho^2 C_y^2\right). \tag{29}$$

Similarly, the optimal α for $\hat{\bar{\bar{Y}}}_{\mathrm{C2}}$ is

$$\alpha = \frac{\rho C_y - \left(\theta_2 + \beta K\right)C_x}{\left(\theta_1 - \theta_2 - \beta K\right)C_x} = \alpha_{\mathrm{C2}}^{\mathrm{opt}},\ \text{(say)}. \tag{30}$$

Substituting Equation (30) into Equation (20), the optimum $\hat{\bar{\bar{Y}}}_{\mathrm{C2}}$ is

$$\hat{\bar{\bar{Y}}}_{\mathrm{C2}}^{\mathrm{opt}} = \alpha_{\mathrm{C2}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R2}} + \left(1-\alpha_{\mathrm{C2}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg2}}. \tag{31}$$

Substituting Equation (30) into Equation (26), the optimum MSE of estimator $\hat{\bar{\bar{Y}}}_{\mathrm{C2}}^{\mathrm{opt}}$ is

$$MSE_{\min}\left(\hat{\bar{\bar{Y}}}_{\mathrm{C2}}^{\mathrm{opt}}\right) \cong \bar{Y}^2 C_y^2\left(\gamma^* + \left(1-\rho^2\right)\left(\gamma-\gamma^*\right)\pi^2\right). \tag{32}$$

3.3 Some members of the proposed estimators

Some members of the proposed classes of combined estimators are shown in Table 2.

Table 2. Some members of the proposed classes of combined estimators

Class I	Class II
$\hat{\bar{\bar{Y}}}_{\mathrm{C11}} = \alpha_{\mathrm{C11}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R11}} + \left(1-\alpha_{\mathrm{C11}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg11}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C21}} = \alpha_{\mathrm{C21}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R21}} + \left(1-\alpha_{\mathrm{C21}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg21}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C12}} = \alpha_{\mathrm{C12}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R11}} + \left(1-\alpha_{\mathrm{C12}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg12}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C22}} = \alpha_{\mathrm{C22}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R21}} + \left(1-\alpha_{\mathrm{C22}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg22}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C13}} = \alpha_{\mathrm{C3}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R11}} + \left(1-\alpha_{\mathrm{C13}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg13}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C23}} = \alpha_{\mathrm{C23}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R21}} + \left(1-\alpha_{\mathrm{C23}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg23}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C14}} = \alpha_{\mathrm{C14}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R12}} + \left(1-\alpha_{\mathrm{C14}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg11}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C24}} = \alpha_{\mathrm{C24}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R22}} + \left(1-\alpha_{\mathrm{C24}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg21}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C15}} = \alpha_{\mathrm{C15}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R12}} + \left(1-\alpha_{\mathrm{C15}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg12}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C25}} = \alpha_{\mathrm{C25}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R22}} + \left(1-\alpha_{\mathrm{C25}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg22}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C16}} = \alpha_{\mathrm{C16}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R12}} + \left(1-\alpha_{\mathrm{C16}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg13}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C26}} = \alpha_{\mathrm{C26}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R22}} + \left(1-\alpha_{\mathrm{C26}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg23}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C17}} = \alpha_{\mathrm{C17}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R13}} + \left(1-\alpha_{\mathrm{C17}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg11}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C27}} = \alpha_{\mathrm{C27}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R23}} + \left(1-\alpha_{\mathrm{C27}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg21}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C18}} = \alpha_{\mathrm{C18}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R13}} + \left(1-\alpha_{\mathrm{C18}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg12}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C28}} = \alpha_{\mathrm{C28}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R23}} + \left(1-\alpha_{\mathrm{C28}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg22}}$
$\hat{\bar{\bar{Y}}}_{\mathrm{C19}} = \alpha_{\mathrm{C19}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R13}} + \left(1-\alpha_{\mathrm{C19}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg13}}$	$\hat{\bar{\bar{Y}}}_{\mathrm{C29}} = \alpha_{\mathrm{C29}}^{\mathrm{opt}}\hat{\bar{\bar{Y}}}_{\mathrm{R23}} + \left(1-\alpha_{\mathrm{C29}}^{\mathrm{opt}}\right)\hat{\bar{\bar{Y}}}_{\mathrm{Reg23}}$

4. Efficiency Comparisons

To compare the efficiency of the proposed estimators with the usual ratio estimator ($\hat{\bar{\bar{Y}}}_{\mathrm{Neyman}}$) and Thongsak and Lawson's (2022) estimators ($\hat{\bar{\bar{Y}}}_{\mathrm{R1}}, \hat{\bar{\bar{Y}}}_{\mathrm{R2}}, \hat{\bar{\bar{Y}}}_{\mathrm{Reg1}}$, and $\hat{\bar{\bar{Y}}}_{\mathrm{Reg2}}$) under the double sampling scheme, the MSEs are used as a criterion.

1) $\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}}$ performs better than $\hat{\bar{\bar{Y}}}_{\mathrm{Neyman}}$ under the certain condition

$$\left(C_x - \rho C_y\right)^2 > 0 \tag{33}$$

We can see from Equation (33) that the inequality is always satisfied. We can conclude that the proposed estimator $\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}}$ is more efficient than $\hat{\bar{\bar{Y}}}_{\mathrm{Neyman}}$.

2) $\hat{\bar{\bar{Y}}}_{\mathrm{C1}}^{\mathrm{opt}}$ performs better than $\hat{\bar{\bar{Y}}}_{\mathrm{R1}}$ and $\hat{\bar{\bar{Y}}}_{\mathrm{Reg1}}$ under the certain condition

$$\left(\omega C_x - \rho C_y\right)^2 > 0 \tag{34}$$

We can see from Equation (34) that the inequality is always satisfied. We can conclude that the proposed estimator $\hat{\hat{Y}}_{C1}^{opt}$ is more efficient than $\hat{\hat{Y}}_{R1}$ and $\hat{\hat{Y}}_{Reg1}$.

3) $\hat{\hat{Y}}_{C2}^{opt}$ performs better than $\hat{\hat{Y}}_{Neyman}$ under the certain condition

$$\pi^2 < \frac{C_y^2 + C_x^2 - 2\rho C_x C_y}{C_y^2\left(1-\rho^2\right)} \tag{35}$$

4) $\hat{\hat{Y}}_{C2}^{opt}$ performs better than $\hat{\hat{Y}}_{R2}$ and $\hat{\hat{Y}}_{Reg2}$ under the certain condition

$$\left(\omega C_x - \rho C_y\right)^2 > 0 \tag{36}$$

We can see from Equation (36) that the inequality is always satisfied. We can conclude that the proposed estimator $\hat{\hat{Y}}_{C2}^{opt}$ is more efficient than $\hat{\hat{Y}}_{R2}$ and $\hat{\hat{Y}}_{Reg2}$.

5. Simulation Studies

To assess the efficiency of the proposed classes of combined estimators compared to the existing estimators, the paired variable (X, Y) from the bivariate normal distribution population was generated with the following parameters: N = 2,000, $\bar{Y} = 50$, $\bar{X} = 40$, C_y = 0.5, C_x = 1.5, ρ = 0.8

In the first phase of sampling, samples of sizes $ή$ = 700, 800, and 900 units were selected from N population units under the SRSWOR scheme, and then samples of sizes n(n=245 and 280 for $ή$ = 700, n = 280 and 320 for $ή$ = 800, and n = 315 and 360 for $ή$ = 900) were selected from the $ή$ units under the SRSWOR scheme and this was repeated 10,000 times using the R program (R Core Team (2021)). The PREs of the proposed and existing estimators with respect to the usual ratio estimator are calculated from

$$PRE\left(\hat{\hat{Y}}, \hat{\hat{Y}}_{Neyman}\right) = \frac{MSE\left(\hat{\hat{Y}}_{Neyman}\right)}{MSE\left(\hat{\hat{Y}}\right)} \times 100, \tag{37}$$

where $MSE\left(\hat{\hat{Y}}_{Neyman}\right) = \frac{1}{10,000}\sum_{i=1}^{10,000}\left(\hat{\hat{Y}}_{Neyman_i} - \bar{Y}\right)^2, MSE\left(\hat{\hat{Y}}\right) = \frac{1}{10,000}\sum_{i=1}^{10,000}\left(\hat{\hat{Y}}_i - \bar{Y}\right)^2.$

The PREs of the proposed and existing estimators are summarized in Table 3.

Table 3. PREs of the proposed estimators and existing estimators

Estimator	PRE					
	n'=700		n'=800		n'=900	
	n=245	n=280	n=280	n=320	n=315	n=360
$\hat{\hat{Y}}_{Neyman}$	100	100	100	100	100	100
$\hat{\hat{Y}}_{R11}$	425	262	429	262	432	266
$\hat{\hat{Y}}_{R12}$	459	285	464	286	467	290
$\hat{\hat{Y}}_{R13}$	442	274	447	274	450	278
$\hat{\hat{Y}}_{Reg11}$	679	465	695	472	707	485
$\hat{\hat{Y}}_{Reg12}$	721	507	739	516	753	531
$\hat{\hat{Y}}_{Reg13}$	701	486	718	495	731	509
$\hat{\hat{Y}}_{R21}$	314	214	311	210	309	210
$\hat{\hat{Y}}_{R22}$	341	233	339	229	336	229
$\hat{\hat{Y}}_{R23}$	328	224	325	220	323	220
$\hat{\hat{Y}}_{Reg21}$	192	129	189	126	187	126

Table 3. Continued.

Estimator	PRE					
	n'=700		n'=800		n'=900	
	n=245	n=280	n=280	n=320	n=315	n=360
$\hat{\hat{Y}}_{\mathrm{Reg22}}$	205	138	203	135	201	135
$\hat{\hat{Y}}_{\mathrm{Reg23}}$	199	134	196	131	194	131
$\hat{\hat{Y}}_{\mathrm{C11}}$	806	694	828	723	859	758
$\hat{\hat{Y}}_{\mathrm{C12}}$	819	713	841	741	870	774
$\hat{\hat{Y}}_{\mathrm{C13}}$	813	704	835	733	865	767
$\hat{\hat{Y}}_{\mathrm{C14}}$	796	679	819	709	851	745
$\hat{\hat{Y}}_{\mathrm{C15}}$	814	704	836	733	866	767
$\hat{\hat{Y}}_{\mathrm{C16}}$	806	693	828	722	860	757
$\hat{\hat{Y}}_{\mathrm{C17}}$	801	688	824	717	856	752
$\hat{\hat{Y}}_{\mathrm{C18}}$	817	709	839	738	869	771
$\hat{\hat{Y}}_{\mathrm{C19}}$	810	699	832	728	863	763
$\hat{\hat{Y}}_{\mathrm{C21}}$	1324	1003	1411	1077	1426	1097
$\hat{\hat{Y}}_{\mathrm{C22}}$	1301	986	1389	1061	1404	1080
$\hat{\hat{Y}}_{\mathrm{C23}}$	1312	995	1400	1069	1415	1089
$\hat{\hat{Y}}_{\mathrm{C24}}$	1337	1013	1424	1086	1439	1108
$\hat{\hat{Y}}_{\mathrm{C25}}$	1327	1006	1415	1080	1429	1100
$\hat{\hat{Y}}_{\mathrm{C26}}$	1332	1009	1419	1083	1434	1104
$\hat{\hat{Y}}_{\mathrm{C27}}$	1333	1011	1421	1084	1435	1105
$\hat{\hat{Y}}_{\mathrm{C28}}$	1319	999	1407	1074	1421	1093
$\hat{\hat{Y}}_{\mathrm{C29}}$	1326	1005	1413	1079	1428	1099

According to Table 3, we found that the proposed classes of combined estimators are more efficient than all existing estimators in terms of a larger PRE for all sample sizes. The PREs of all estimators increased with the sample size. The proposed class of combined estimators utilizing both transformed auxiliary and study variables $\hat{\hat{Y}}_{C2}$ performed much better than the other estimators.

6. Application to Air Pollution in Chiang Rai, Thailand

To demonstrate the performance of the proposed classes of combined estimators we also applied them to air pollution data from Chiang Rai, Thailand. The obtained data are PM2.5 levels (μg/m^3) and NO_2 levels (mg/m^2) recorded once per month (monthly average) from the Copernicus Atmosphere Monitoring Service (CAMS), The European Centre for Medium-Range Weather Forecasts (ECMWF) in 2003-2020 (Inness *et al.*, 2019). The data belong to a population of 216 units. We considered the concentration of NO_2 as the study variable Y and the concentration of PM2.5 is the auxiliary variable X. The population parameters are given as:

$$N = 216,\ \bar{Y} = 2.214,\ \bar{X} = 50.570,\ C_y = 0.410,\ C_x = 1.531,\ \rho = 0.921$$

In the first phase of sampling, a sample of size n'=75 is selected from the population size N = 216 using the SRSWOR scheme. In the second phase of sampling a sample of size n = 20 is selected from n'=75 using the SRSWOR scheme. The PREs of the proposed estimators and existing estimators with respect to the usual ratio estimator are presented in Table 4.

According to Table 4, the proposed estimators have higher PREs than the other existing estimators. It can be deduced that the combined estimators can be used to estimate the population mean better than the existing estimators under the double sampling scheme, especially the class of $\hat{\hat{Y}}_{C2}^{opt}$ which utilized transformation on both study and auxiliary variables performed the best in this situation, giving a substantial improvement by achieving larger PREs than the other estimators.

Table 4. PREs of the proposed estimators and existing estimators when applied to air pollution data

Estimator	PRE	Estimator	PRE	Estimator	PRE
$\hat{\hat{Y}}_{\mathrm{Neyman}}$	100	$\hat{\hat{Y}}_{\mathrm{Reg22}}$	413	$\hat{\hat{Y}}_{\mathrm{C21}}$	2066
$\hat{\hat{Y}}_{\mathrm{R11}}$	494	$\hat{\hat{Y}}_{\mathrm{Reg23}}$	409	$\hat{\hat{Y}}_{\mathrm{C22}}$	2156
$\hat{\hat{Y}}_{\mathrm{R12}}$	511	$\hat{\hat{Y}}_{\mathrm{C11}}$	785	$\hat{\hat{Y}}_{\mathrm{C23}}$	2120
$\hat{\hat{Y}}_{\mathrm{R13}}$	505	$\hat{\hat{Y}}_{\mathrm{C12}}$	780	$\hat{\hat{Y}}_{\mathrm{C24}}$	1987
$\hat{\hat{Y}}_{\mathrm{Reg11}}$	686	$\hat{\hat{Y}}_{\mathrm{C13}}$	782	$\hat{\hat{Y}}_{\mathrm{C25}}$	2049
$\hat{\hat{Y}}_{\mathrm{Reg12}}$	715	$\hat{\hat{Y}}_{\mathrm{C14}}$	787	$\hat{\hat{Y}}_{\mathrm{C26}}$	2025
$\hat{\hat{Y}}_{\mathrm{Reg13}}$	705	$\hat{\hat{Y}}_{\mathrm{C15}}$	781	$\hat{\hat{Y}}_{\mathrm{C27}}$	2012
$\hat{\hat{Y}}_{\mathrm{R21}}$	529	$\hat{\hat{Y}}_{\mathrm{C16}}$	783	$\hat{\hat{Y}}_{\mathrm{C28}}$	2083
$\hat{\hat{Y}}_{\mathrm{R22}}$	546	$\hat{\hat{Y}}_{\mathrm{C17}}$	786	$\hat{\hat{Y}}_{\mathrm{C29}}$	2055
$\hat{\hat{Y}}_{\mathrm{R23}}$	540	$\hat{\hat{Y}}_{\mathrm{C18}}$	781		
$\hat{\hat{Y}}_{\mathrm{Reg21}}$	402	$\hat{\hat{Y}}_{\mathrm{C19}}$	783		

7. Conclusions

We suggested use of the transformation technique on the auxiliary variable and on both auxiliary and study variables to create two new classes of combined population mean estimators under double sampling, to ameliorate the efficiency of the population mean estimator. The biases and MSEs of the proposed estimators were obtained. The results from simulation studies and an application to air pollution data showed that all the proposed estimators had higher PREs than the existing estimators in these specific scenarios. Using the transformation technique especially on both the auxiliary and study variables can lead to bigger percentage relative efficiencies of the population mean estimator. Therefore, this is a powerful method to be used as an alternative way to create a high-performance estimator. The proposed classes of estimators can also be applied using some other known auxiliary parameters to gain a higher efficiency for estimating the population mean of the variable of interest.

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References

Adewara, A. A. (2006). Effect of improving both the auxiliary and variable of interest in ratio and product estimators. *Proceedings of the Pakistan Academy of Sciences*, *43*(4), 275-278.

Adewara, A. A., Singh, R., & Kumar, M. (2012). Efficiency of some modified ratio and product estimators using known value of some population parameters. *International Journal of Applied Science and Technology*, *2*(2), 76-79.

Amin, M. N. U., Shahbaz, M. Q., & Kadilar, C. (2016). Ratio estimators for population mean using robust regression in double sampling. *Gazi University Journal of Science*, *29*(4), 793-798.

Bandyopadhyaya, S. (1980). Improved ratio and product estimators. *Sankhya, Series C*, *42*, 45-49.

Choudhury, S., & Singh, B. K. (2015). Study of dual to ratio-cum-product estimator of finite population mean under double sampling in sample surveys. *Journal of Statistical Theory and Applications*, *14*(2), 214-221.

Cochran, W. G. (1940). *Sampling techniques* (3rd ed.). India: Wiley Eastern Limited.

Cochran, W. G. (1977). *Sampling techniques*. New York, NY: John Wiley and Sons.

Hasegawa, K., Tsukahara, T., & Nomiyama, T. (2021). Association between air pollutants and cancer incidence rates in Japan: An ecological study. *Medical Sciences Forum*, *4*(1), 15.

Inness, A., Ades, M., Agustí-Panareda, A., Barré, J., Benedictow, A., Blechschmidt, A. M., . . . Suttie, M. (2019). CAMS Global Reanalysis (EAC4) monthly averaged fields, Copernicus Atmosphere Monitoring Service, The European Centre for Medium-Range Weather Forecasts, Retrieved from https: //ads.atmosphere.copernicus.eu/cdsapp#!/dataset/cams-global-reanalysis-eac4-monthly?tab=overview(1 March 2022).

Kadilar, C., & Cingi, H. (2004). Ratio estimators in simple random sampling. *Applied Mathematics and Computation*, *151*(3), 893-902.

Kiregyera, B. (1980). A chain ratio-type estimator in finite population double sampling using two auxiliary variables. *Metrika*, *27*, 217-223.

Kumar, M., & Bahl, S. (2006). Class of dual to ratio estimators for double sampling. *Statistical Papers*, *47*, 319-326.

Malik, K. A., & Tailor, R. (2013). Ratio type estimator of population mean in double sampling. *International Journal of Advanced Mathematics and Statistics*, *1*(1), 34-39.

Mohanty, S. (1967). Combination of regression and ratio estimate. *Journal of the Indian Statistical Association*, *5*, 16-19.

Murthy, M. N. (1967). *Sampling theory and methods*. Calcutta, India: Statistical Publishing Society.

Neyman, J. (1938). Contribution to the theory of sampling human populations. *Journal of the American Statistical Association*, *33*, 101-116.

R Core Team (2021). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. Retrieved from https://www.R-project.org/.

Seifi, M., Niazi, S., Johnson, G., Nodehi, V., & Yunesian, M. (2019). Exposure to ambient air pollution and risk of childhood cancers: A population-based study in Tehran, Iran. *Science of the Total Environment*, *646*, 105-110.

Singh, H. P., & Tailor, R. (2003). Use of known correlation co-efficient in estimating the finite population means. *Statistics in Transition*, *6*(4), 555-560.

Singh, B. K., & Choudhury, S. (2012). Dual to product estimator for estimating population mean in double sampling. *International Journal of Statistics and Systems*, *7*, 31-39.

Singh, G. N., & Khalid, M. (2015). Exponential chain dual to ratio and regression type estimators of population mean in two-phase sampling. *Statistica*, *75*(4), 379-389.

Singh, R., Malik, S., & Smarandache, F. (2016). Some ratio and product estimators using known value of population parameters. In S. Malik, N. Kumar, & F. Smarandache (Eds.), *Uses of sampling techniques and inventory control with capacity constraints* (pp. 19-29). Belgium: Pons Editions.

Singh, G. N., & Khalid, M. (2019). Effective estimation strategy of population variance in two-phase successive sampling under random non-response. *Journal of Statistical Theory and Practice*, *13*, 4. Retrieved https://doi.org/10.1007/s42519-018-0010-y

Srivenkataramana, T. (1980). A dual to ratio estimator in sample surveys. *Biometrika*, *67*, 199-204.

Thongsak, N., Chitapanarux, I., Suprasert, P., Prasitwattanaseree, S., Bunyatisai, W., Sripan, P., & Traisathit, P. (2016). Spatial and temporal analyses of cervical cancer patients in upper Northern Thailand. *Asian Pacific Journal of Cancer Prevention*, *17*(11), 5011-5017.

Thongsak, N., & Lawson, N. (2022). A combined family of dual to ratio estimators using a transformed auxiliary variable. *Lobachevskii Journal of Mathematics*, *43*, 2621-2633.

Thongsak, N., & Lawson, N. (in press). Classes of population mean estimators using transformed variables in double sampling. *Gazi University Journal of Science.*

Wu, J., Xu, C., & Wang, Q. (2016). Potential sources and formations of the PM2.5 pollutions in urban Hangzhou. *Atmosphere*, *7*(8), 100.