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Education sector assessment using linguistic quadripartitioned single-valued neutrosophic soft sets

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Abstract

Recently uncertainty analysis and its characterization have become more complex due to dark data sets. It has become more crucial in the education sector where a large number of dark datasets are generated, related to measuring student performance. This becomes more complex while dealing with linguistic information and its significance in student performance measurement. One reason for this is that these types of data are based on human quantum Turiyam consciousness and sometimes in an unconscious way. To deal with these types of dark data generated in unconscious way, this paper tries to introduce the linguistic Quadripartitioned single-valued neutrosophic set (LQSVNS). It defines each object of the universe by four independent linguistic variables known as truth, contradiction, unknown, and false. Some operations and properties based on LQSVNNs are extensively studied in this paper using a combination of soft set (SS) known as the linguistic Quadripartitioned single-valued neutrosophic soft set (LQSVNSS). Moreover, a distance similarity measure based model under the LQSVNSSs is investigated with an illustrative example of the education sector.

Keywords: big data, four valued data, linguistic quadripartitioned single-valued neutrosophic set, quadripartitioned single-valued neutrosophic set, turiyam set

1. Introduction

In the last decade, many studies have paid attention to computing with words, and its vagueness. To deal with this problem the algebra of Fuzzy Sets (FS) (Zadeh, 1965) was introduced with the aid of the truth-grade function (μ) getting membership values in $[0,1]$. Atanassov and Stoeva (1986) initiated an intuitionistic fuzzy set (IFS) to represent the acceptance and the rejection part via dependent true and false membership values, μ and ν respectively, satisfying the inequality $0 \leq \mu + \nu \leq 1$. A problem arises with IFS in the case $\mu + \nu > 1$, or when these are independent of each other. To resolve this issue, Pythagorean fuzzy sets (PFSs) (Yager, 2013), and q-rung orthopair fuzzy sets (q-ROFSs) Yager (2016) are introduced. In a PFS and a q-ROFS the true (μ) and false (ν) membership grades of a particular element are restricted with the conditions $0 \leq \mu^2 + \nu^2 \leq 1$ and $0 \leq \mu^q + \nu^q \leq 1$ respectively in various applications (Ejegwa, 2020; Garg, 2016; Thao, 2020; Xiao & Ding, 2019).

A problem now arises when the hesitant part of IFS becomes independent of true and false membership values. To deal with this type of indeterminacy Single-valued Neutrosophic set (SVNS) was introduced by Smarandache (2005), Wang, Smarandache, Zhang and Sunderraman (2010). It is applied in various fields (Broumi *et al*. 2023; Caballero & Broumi 2023; Sivasankar & Broumi 2023). An alternative way is to make a decision through the application of NST. One approach is to use neutrosophic rough sets (Broumi *et al*., 2014; Salama & Broumi, 2014) and another is to use aggregation operators. To extend neutrosophic set to MCDA problems, Bao and Yang (2017) proposed a model integrating single valued neutrosophic refined sets with rough sets while Bo *et al*. (2018) utilized multi-granulation neutrosophic rough sets. Moreover, many mixed models with NST have been proposed, such as n-

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dimensional single valued neutrosophic refined rough sets (Yang *et al*., 2017; Zhao & Zhang, 2018a) and hesitant neutrosophic rough sets (Zhao & Zhang, 2018b, 2020). In addition, there is a combined ITARA with TOPSIS-AL approach based on neutrosophic sets for risk assessment of university sustainability（Lin & Lo, 2023). A problem arises while dealing with contradictory, unknown, or ambiguous data, or on computing its complement as discussed by Belnap (Belnap Jr, 1977). It can be useful to encounter contradictory facts, human super consciousness, and solve various machine intelligence problems based on human cognitive intelligence. Due to this, the SVNS is extended as Quadripartitioned single-valued neutrosophic set (QSVNS) (Chatterjee *et al*. 2016 a, b). It can represent a dependent data set and its membership via n-valued refined neutrosophic set (Smarandache, 2013), which has motivated several authors to study QSVNS (Mohan & Krishnaswamy, 2020a; Mohanasundari & Mohana, 2020; Sinha & Majumdar, 2020; Roy, Lee, Pal, & Samanta, 2020) and its hybrid model (Kamaci, 2021; Mohan & Krishnaswamy, 2020b; Sinha, Majumdar & Broumi, 2022). It becomes more useful when soft set (SS) (Molodtsov, 1999) theory is connected with this logic. The reason is that a soft set offers a more general framework to present uncertainty without any restriction. The hybridization with soft set is utilized in many areas, such as in decision-making problems (Maji, Roy, & Biswas, 2002; Kong, Wang & Wu, 2011) and in medical diagnosis (Muthukumar & Krishnan, 2016; Xiao, 2018). Moreover, a combination of SS and QSVNS provides a new theoretical concept known as quadripartitioned single-valued neutrosophic soft set (QSVNSS) that can be viewed as a special type of quadripartitioned neutrosophic soft set (QNSS) (Radha, Mary, & Smarandache, 2021). This set is utilized successfully in case of uncertainty measurement in interval-valued possibility QSVNSS (Chatterjee *et al*. 2016b), and topological space based on QNSS (Kumar & Mary, 2021). This current paper focused on Quadripartitioned single-valued neutrosophic soft set (LQSVNSS) for dealing with qualitative data.

However, dealing with qualitative data requires human consciousness for the precise representation of contradictory events. Sometimes such data can be represented via linguistic variables in the case of known objects or dependent variables. Consider as an example small, very small, almost small, not small, quite small, not very small, etc. The linguistic setting is a very popular and interesting topic to measure the uncertainty that arises due to human thoughts. In the research article (Zadeh, 1975), Zadeh first introduced the concept of linguistic variables in approximate reasoning. According to him, a linguistic variable is characterized by a quintuple$(\Im, \mathbb{T}(\Im), \mathcal{U}, \mathcal{G}, \mathscr{M})$, where $\Im$ is a variable, $\mathbb{T}(\Im)$ is the term set of $\Im$, $\mathcal{U}$ is a set of the universe, $\mathcal{G}$ is a rule that generates $\mathbb{T}(\Im)$, and $\mathscr{M}$ is a semantic rule associated with a linguistic value. Liu *et al*. (liu & Zhang, 2010) utilized the risk-based linguistic variable in MADM. Herrera *et al*. (Herrera & Herrera-Viedma, 2000) presented a linguistic decision analysis to solve the decision problem. Xu (Xu, 2004) proposed the linguistic aggregate operators for group decision-making. A fuzzy set-based linguistic value is proposed in Bonissone (1980), and Dohnal (1983). Zhang (2014) introduced the linguistic intuitionistic fuzzy set (LIFS) that is characterized by linguistic membership and non-membership degrees. Garg and Kumar (2018) presented some aggregate operators on LIFSs in GDM. He also presented the linguistic Pythagorean fuzzy set (LPFS) (Garg, 2018) to address uncertain linguistic information in a better way. Also, q-rung orthopair fuzzy sets are based on linguistic information studied in Akram, Naz, Edalatpanah, and Mehreen (2021), Lin, Li, and Chen (2020), Liu and Liu (2019), Wang, Ju, and Liu (2019). The fusion of neutrosophic set and its hybrids with linguistic set resulted in linguistic neutrosophic sets (LNSs) in MCDM (Li, Zhang & Wang, 2017), interval complex neutrosophic sets under linguistic information (Dat, Thong, Ali, Smarandache, Abdel-Basset, & Long, 2019), multi-objective linguistic neutrosophic matrix games (Bhaumik, Roy, & Weber, 2021). Kamaci (2021) initiated the linguistic single-valued neutrosophic soft set (LSVNS) and proposed a game theory model via the TOPSIS technique.

Some other methods are proposed for dealing with human cognition (Singh, 2021) in case of unknown or undefined impossible objects using human quantum Turiyam cognition (Singh, 2022a). It represents qualitative data based on four dimensions where the fourth dimension is represented by Human Turiyam consciousness. It is independent of true, false, uncertain, or contradictory membership values of the given event (Singh, 2022b). It is based on time-based measurement, human superconsciousness, or quantum cognition distinct from any of the available sets, as discussed in Singh (2023c). It represents the error measured in dark datasets due to human consciousness rather than unconsciousness value of Quadripartitioned set. It can be represented via the Turiyam matrix (Ani, Mashadi, & Gemawati, 2023) for precise analysis of knowledge-processing tasks based on the Turiyam relation (Ganati, Srinivasa Rao Repalle, & Ashebo, 2023) and its graph (Ganati, Srinivasa, Ashebo, & Amini, 2023). It gives a way to represent the dataset based on human quantum cognition (Singh, 2023a, b, c) for dealing the self-driving cars (Said *et al*., 2022), robotics (Silva, 2022) or mathematical exploration in school teaching (Singh 2023b). A problem arises when datasets and their uncertainty are represented without human cognition beyond the three polar spaces (Naeem & Divvaz, 2023; Naeem, Riaz & Karaaslan, 2021; Naeem, Riaz & Afzal, 2019; Naeem & Said, 2023; Singh, 2023c; Siraj, Siraj, Fatima, Afzal, Naeem & Karaaslan, 2022). This requires a new set theory for dealing qualitative datasets containing static uncertainty, rather than involvement of human consciousness (Singh 2023b). To achieve this goal, the current paper focuses on dealing with uncertainty in known qualitative data and its refinement using linguistic single-valued neutrosophic soft set (LSVNS) rather than human consciousness.

Table 1 gives an overview and distinctions among each of the available approaches.

Other parts of the paper are organized as follows: Section 2 provides brief background on the mathematical concepts. Section 3 provides the basis of weighted aggregation for the proposed method and its illustration. In Section 4, LQSVNSSs and their properties are defined and studied. Application of the present study is briefly discussed in Section 5. Section 6 contains the conclusion followed by acknowledgment and references.

2. Basic Mathematical Concepts

In this section, some basic concepts are reviewed to ease the discussion in the subsequent sections.

Table 1. The distinctions among fuzzy, intuitionistic, neutrosophic, turiyam and quadripartitioned representations

	Fuzzy set	Intuitionistic fuzzy set	Neutrosophic set	Turiyam set	Quadripartitioned
Data	Uncertain	Vague	Indeterminacy	Unknown or impossible	Uncertain and indeterminant
Membership-values	Single-values for true (*t*) and false (*f*)	Dependent values for true (*t*) and false (*f*)	Independent values for true (*t*), false (*f*) and indeterminacy (*i*)	Independent and dependent values for true (*t*), false (*f*), uncertain (*i*) and liberation (*l*)	Independent and dependent values for true (*t*), false (*f*), uncertain(*i*), incompleteness (*l*)
Range	[0, 1]	Dependent: [0, 1]	Independent case: $^{-}[-3, 3]^{+}$ or Dependent case: $^{-}[0, 1]^{+}$	Independent case: $^{-}[-4, 4]^{+}$ or Dependent case: $^{-}[0, 1]^{+}$	Independent case: $^{-}[-4, 4]^{+}$ or Dependent case: $^{-}[0, 1]^{+}$
Undefined objects	No	No	No	Yes	No
Dynamic attribute	No	No	No	Yes	No
Consciousness utilized	No	No	No	Yes	No
Expert to expert	Same representation	Same representation	Same representation	Varies from expert to expert based on their consciousness	Same representation
Application	Natural language processing	Pattern recognition	Image processing, feedback	Robotics, self- driving cars, cognitive science	Qualitative data measurement

2.1 Linguistic term set (LTS)

Definition 2.1 (Zadeh, 1975) Let $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots\ldots, \mathfrak{s}_{\wp}\}$ be a finite linguistic term set with cardinality $\wp + 1$ where $\wp$ is a positive integer. Also, we consider $\mathfrak{s}_{\mathfrak{m}}$ as a possible value of a linguistic variable. Then the LTS must satisfy the following properties:

(i) $\mathfrak{s}_{\mathfrak{m}} \geq \mathfrak{s}_{\mathfrak{n}\, if\ \mathfrak{m} \geq \mathfrak{n}\ and}\ \mathfrak{s}_{\mathfrak{m}} \leq \mathfrak{s}_{\mathfrak{n}\ if\ \mathfrak{m} \leq \mathfrak{n}}$ (Order relation)

(ii) $neg(\mathfrak{s}_{\mathfrak{m}}) = \mathfrak{s}_{\mathfrak{n}}$ where $\mathfrak{m} + \mathfrak{n} = \wp$ (negation)

This concept is based on discrete LTS. To extend this concept to continuous LTS, see the next definition.

Definition 2.2 (Xu, 2004) Let $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots\ldots, \mathfrak{s}_{\wp}\}$ be a finite linguistic term set with cardinality $\wp + 1$ where $\wp$ is a positive integer. Then, $\mathfrak{S}_{[0,\wp]} = \{\mathfrak{s}_{\mathfrak{m}} : \mathfrak{s}_0 \leq \mathfrak{s}_{\mathfrak{m}} \leq \mathfrak{s}_{\wp,\, \mathfrak{m} \in [0,\wp]}\}$ is said to be a continuous LTS for $\mathfrak{S}$.

2.2 Single-valued neutrosophic set (SVNS)

Definition 2.3 (Smarandache, 2005) A single-valued neutrosophic set (SVNS) $\mathscr{H}$ over $\mathscr{L}$ is defined as $\mathscr{H} = \{(\ell_i, < \mu_{\mathscr{H}(\ell_i)}, \vartheta_{\mathscr{H}(\ell_i)}, \nu_{\mathscr{H}(\ell_i)} >) : \ell_i \in \mathscr{L}\}$ where $\mu_{\mathscr{H}(\ell_i)}, \vartheta_{\mathscr{H}(\ell_i)}, \nu_{\mathscr{H}(\ell_i)} \in [0,1]$ represent the degrees of truth, indeterminacy, and falsity memberships respectively with $0 \leq \mu_{\mathscr{H}(\ell_i)} + \vartheta_{\mathscr{H}(\ell_i)} + \nu_{\mathscr{H}(\ell_i)} \leq 3$.

2.3 Linguistic single-valued neutrosophic set (LSVNS)

Definition 2.4 (Li, Zhang, & Wang, 2017) Let $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots\ldots, \mathfrak{s}_{\wp}\}$ be a finite linguistic term set with cardinality $\wp + 1$ where $\wp$ is a positive integer and $\mathfrak{S}_{[0,\wp]} = \{\mathfrak{s}_{\mathfrak{m}} : \mathfrak{s}_0 \leq \mathfrak{s}_{\mathfrak{m}} \leq \mathfrak{s}_{\wp,\, \mathfrak{m} \in [0,\wp]}\}$. Then a LSVNS $\mathscr{g}$ in $\mathscr{L}$ is defined as $\mathscr{g} = \{(\ell_i, < \mathscr{g}_{\mathfrak{s}_\mu}(\ell_i), \mathscr{g}_{\mathfrak{s}_\vartheta}(\ell_i), \mathscr{g}_{\mathfrak{s}_\nu}(\ell_i) >) : \ell_i \in \mathcal{L}\}$ where $\mathscr{g}_{\mathfrak{s}_\mu}(\ell_i), \mathscr{g}_{\mathfrak{s}_\vartheta}(\ell_i), \mathscr{g}_{\mathfrak{s}_\nu}(\ell_i) \in \mathfrak{S}_{[0,\wp]}$ denote the linguistic truth, indeterminacy, and falsity degrees of $\ell_i \in \mathscr{L}$ respectively such that $0 \leq \mu, \vartheta, \nu \leq \wp$ and $0 \leq \mu + \vartheta + \nu \leq 3\wp$.

2.4 Quadripartitioned single-valued neutrosophic set (QSVNSS)

Definition 2.5 (Chatterjee *et al.* 2016 a,b) A QSVNS $\mathcal{Q}$ in $\mathscr{L}$ is defined as $\mathcal{Q} = \{(\ell_i, < \mu_{\mathcal{Q}(\ell_i)}, \xi_{\mathcal{Q}(\ell_i)}, \zeta_{\mathscr{H}(\ell_i)}, \nu_{\mathcal{Q}(\ell_i)} >) : \ell_i \in \mathscr{L}\}$ where $\mu_{\mathcal{Q}(\ell_i)}, \xi_{\mathcal{Q}(\ell_i)}, \zeta_{\mathscr{H}(\ell_i)}, \nu_{\mathcal{Q}(\ell_i)} \in [0,1]$ denote the truth, contradiction, unknown, and falsity membership grades respectively with $0 \leq \mu_{\mathcal{Q}(\ell_i)} + \xi_{\mathcal{Q}(\ell_i)} + \zeta_{\mathscr{H}(\ell_i)} + \nu_{\mathcal{Q}(\ell_i)} \leq 4$. This is distinct from Turiyam where each parameter is independent and the last dimension is based on human Turiyam consciousness. It means Turiyam contains two truth values rather than one in QSVNSS. Also, the Liberal values in Turiyam are independent from true false or uncertainty whereas in Quadripartitioned Single-Valued Neutrosophic this does not hold. In this case, this set theory is helpful in some cases where human quantum cognition is not required. We require membership value in a partitioned way for dealing with the contradictions among opinions.

2.5 Quadripartitioned single-valued neutrosophic soft set (QSVNSS)

Definition 2.6 (Radha, Mary, & Smarandache, 2021) Let $\mathscr{L}$ be an initial universe and Ɛ be a set of parameters where $Å(\neq \varphi) \subseteq$ Ɛ. Let $\wp(\mathscr{L})$ signify the collection of all Quadripartitioned single-valued neutrosophic sets of $\mathscr{L}$. Then the pair $(\mathscr{F}, Å)$ is considered a QSVNSS over $\mathscr{L}$ where $\mathscr{F}: Å \rightarrow \wp(\mathscr{L})$.

3. Linguistic Quadripartitioned Single-Valued Neutrosophic Set (LQSVNS)

In this section dealing with linguistic variables which are qualitative is discussed via LQSVNS. Also, different aggregate operators and distance measures associated with LQSVNS are introduced in this section for dealing with the linguistic static uncertainty that arises unconsciously.

Definition 3.1 Let $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots, \mathfrak{s}_{\wp}\}$ be a continuous finite linguistic term set with cardinality $\wp + 1$ where $\wp$ is a positive integer and $\mathfrak{S}_{[0,\wp]} = \{\mathfrak{s}_\mathfrak{m} : \mathfrak{s}_0 \leq \mathfrak{s}_\mathfrak{m} \leq \mathfrak{s}_\wp, \mathfrak{m} \in [0,\wp]\}$. Then a LQSVNS ð in $\mathscr{L}$ is defined as
$ð = \{(\ell_i, < ð_{\mathfrak{s}_\mu}(\ell_i), ð_{\mathfrak{s}_\xi}(\ell_i), ð_{\mathfrak{s}_\zeta}(\ell_i), ð_{\mathfrak{s}_\nu}(\ell_i) >): \ell_i \in \mathscr{L}\}$ where $ð_{\mathfrak{s}_\mu}(\ell_i), ð_{\mathfrak{s}_\xi}(\ell_i), ð_{\mathfrak{s}_\zeta}(\ell_i), ð_{\mathfrak{s}_\nu}(\ell_i) \in \mathfrak{S}_{[0,\wp]}$ denote the linguistic truth, contradiction, uncertain, and falsity degrees of $\ell_i \in \mathscr{L}$ respectively such that $0 \leq \mu, \xi, \zeta, \nu \leq \wp$ and $0 \leq \mu + \xi + \zeta + \nu \leq 4\wp$.
In short, $ð =< ð_{\mathfrak{s}_\mu}, ð_{\mathfrak{s}_\xi}, ð_{\mathfrak{s}_\zeta}, ð_{\mathfrak{s}_\nu} >$ denotes the linguistic Quadripartitioned single-valued neutrosophic number (LQSVNN). Moreover, the set of all LQSVNNs on $\mathfrak{S}$ is denoted by $\aleph_{[0,\wp]} = \{< ð_{\mathfrak{s}_\mu}, ð_{\mathfrak{s}_\xi}, ð_{\mathfrak{s}_\zeta}, ð_{\mathfrak{s}_\nu} >: ð_{\mathfrak{s}_\mu}, ð_{\mathfrak{s}_\xi}, ð_{\mathfrak{s}_\zeta}, ð_{\mathfrak{s}_\nu} \in \mathfrak{S}_{[0,\wp]}\}$.

Example 3.2 Let $\mathscr{L} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ be a set of alternatives and $\mathfrak{S} =$
$\{\mathfrak{s}_{0=very\ sensitive(VS)}, \mathfrak{s}_{1=quite\ sensitive(QS)}, \mathfrak{s}_{2=high\ sensitive(HS)}, \mathfrak{s}_{3=very\ high\ sensitive(VHS)}, \mathfrak{s}_{4=medium(M)}, \mathfrak{s}_{5=low(L)}\}$ be a LTS.
If we consider the following
$ð = \{(\ell_1, < \mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_0, \mathfrak{s}_1 >), (\ell_2, < \mathfrak{s}_0, \mathfrak{s}_0, \mathfrak{s}_4, \mathfrak{s}_1 >), (\ell_3, < \mathfrak{s}_1, \mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_3 >), (\ell_4, < \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_3, \mathfrak{s}_4 >), (\ell_5, < \mathfrak{s}_5, \mathfrak{s}_0, \mathfrak{s}_3, \mathfrak{s}_3 >)\}$
Then ð denotes the LQSVNS over $\mathscr{L}$.

Definition 3.3 Let $ð =< ð_{\mathfrak{s}_\mu}, ð_{\mathfrak{s}_\xi}, ð_{\mathfrak{s}_\zeta}, ð_{\mathfrak{s}_\nu} > \in \aleph_{[0,\wp]}$ be a LQSVNN. Then the score function associated to ð is defined as

$$\mho(ð) = \frac{ð_{\mathfrak{s}_\mu} + ð_{\mathfrak{s}_\xi} - ð_{\mathfrak{s}_\zeta} - ð_{\mathfrak{s}_\nu}}{2} \text{ Or } \frac{\mu+\xi-\zeta-\nu}{2} \in [-\wp, \wp] \qquad (1)$$

Definition 3.4 Let $ð =< ð_{\mathfrak{s}_\mu}, ð_{\mathfrak{s}_\xi}, ð_{\mathfrak{s}_\zeta}, ð_{\mathfrak{s}_\nu} > \in \aleph_{[0,\wp]}$ be a LQSVNN. Then the accuracy function associated to ð is defined as

$$\mathfrak{H}(ð) = \frac{ð_{\mathfrak{s}_\mu} + ð_{\mathfrak{s}_\xi} + ð_{\mathfrak{s}_\zeta} + ð_{\mathfrak{s}_\nu}}{4} \text{ Or } \frac{\mu+\xi+\zeta+\nu}{4} \in [0, \wp] \qquad (2)$$

Definition 3.5 Let $ð_1 = < ð_{\mathfrak{s}_{\mu_1}}, ð_{\mathfrak{s}_{\xi_1}}, ð_{\mathfrak{s}_{\zeta_1}}, ð_{\mathfrak{s}_{\nu_1}} >$ and $ð_2 = < ð_{\mathfrak{s}_{\mu_2}}, ð_{\mathfrak{s}_{\xi_2}}, ð_{\mathfrak{s}_{\zeta_2}}, ð_{\mathfrak{s}_{\nu_2}} >$ be two LQSVNNs. Based on the score and accuracy function (Definition 3.3 & 3.4), to compare these two LQSVNNs, we define the following:

$a)$ $if\ \mho(ð_1) > \mho(ð_2)\ then\ ð_1 \succ ð_2$ i.e. $ð_1$ is more preferable than $ð_2$

$b)$ $if\ \mho(ð_1) = \mho(ð_2)\ then$

$$i)\ for\ \mathfrak{H}(ð_1) = \mathfrak{H}(ð_2),\ ð_1 = ð_2$$
$$ii)\ for\ \mathfrak{H}(ð_1) > H(ð_2),\ ð_1 \succ ð_2$$

Example 3.6 Let $\mathfrak{S} = \{\mathfrak{s}_\mathfrak{m} : \mathfrak{s}_0 \leq \mathfrak{s}_\mathfrak{m} \leq \mathfrak{s}_7, \mathfrak{m} \in [0,7]\}$ be a LTS. Also, we consider $ð_1 = < \mathfrak{s}_1, \mathfrak{s}_3, \mathfrak{s}_4, \mathfrak{s}_5 >$, $ð_2 = < \mathfrak{s}_0, \mathfrak{s}_4, \mathfrak{s}_2, \mathfrak{s}_3 >$, $ð_3 = < \mathfrak{s}_3, \mathfrak{s}_5, \mathfrak{s}_4, \mathfrak{s}_2 >$, and $ð_4 = < \mathfrak{s}_7, \mathfrak{s}_6, \mathfrak{s}_2, \mathfrak{s}_3 >$ be LQSVNNs derived from $\mathfrak{S}$. By using equation (1), we obtain the following
$\mho(ð_1) = \frac{1+3-4-5}{2}$=-2.5, $\mho(ð_2) = \frac{0+4-2-3}{2}$=-0.5, $\mho(ð_3) = \frac{3+5-4-2}{2}$=1, $\mho(ð_4) = \frac{7+6-2-3}{2}$=4
Thus, we rank the numbers as $ð_4 \succ ð_3 \succ ð_2 \succ ð_1$

3.1 Operational laws on LQSVNNs

Definition 3.7 Let $ð_1 = < ð_{\mathfrak{s}_{\mu_1}}, ð_{\mathfrak{s}_{\xi_1}}, ð_{\mathfrak{s}_{\zeta_1}}, ð_{\mathfrak{s}_{\nu_1}} >$ and $ð_2 = < ð_{\mathfrak{s}_{\mu_2}}, ð_{\mathfrak{s}_{\xi_2}}, ð_{\mathfrak{s}_{\zeta_2}}, ð_{\mathfrak{s}_{\nu_2}} >$ be two LQSVNNs. Then we have the following laws:

a) $ð_1 \Cup ð_2 = < \max(ð_{\mathfrak{s}_{\mu_1}}, ð_{\mathfrak{s}_{\mu_2}}), \max(ð_{\mathfrak{s}_{\xi_1}}, ð_{\mathfrak{s}_{\xi_2}}), \min(ð_{\mathfrak{s}_{\zeta_1}}, ð_{\mathfrak{s}_{\zeta_2}}), \min(ð_{\mathfrak{s}_{\nu_1}}, ð_{\mathfrak{s}_{\nu_2}}) >$
$=< ð_{max(\mathfrak{s}_{\mu_1}, \mathfrak{s}_{\mu_2})}, ð_{max(\mathfrak{s}_{\xi_1}, \mathfrak{s}_{\xi_2})}, ð_{min(\mathfrak{s}_{\zeta_1}, \mathfrak{s}_{\zeta_2})}, ð_{min(\mathfrak{s}_{\nu_1}, \mathfrak{s}_{\nu_2})} >$

b) $ð_1 \Cap ð_2 = < \min(ð_{\mathfrak{s}_{\mu_1}}, ð_{\mathfrak{s}_{\mu_2}}), \min(ð_{\mathfrak{s}_{\xi_1}}, ð_{\mathfrak{s}_{\xi_2}}), \max(ð_{\mathfrak{s}_{\zeta_1}}, ð_{\mathfrak{s}_{\zeta_2}}), \max(ð_{\mathfrak{s}_{\nu_1}}, ð_{\mathfrak{s}_{\nu_2}}) >$
$=< ð_{min(\mathfrak{s}_{\mu_1}, \mathfrak{s}_{\mu_2})}, ð_{min(\mathfrak{s}_{\xi_1}, \mathfrak{s}_{\xi_2})}, ð_{max(\mathfrak{s}_{\zeta_1}, \mathfrak{s}_{\zeta_2})}, ð_{max(\mathfrak{s}_{\nu_1}, \mathfrak{s}_{\nu_2})} >$

$c)\ \eth_1 \geqslant \eth_2 \Rightarrow \eth_{\mathfrak{s}_{\mu'_1}} \geq \eth_{\mathfrak{s}_{\mu'_2}}, \eth_{\mathfrak{s}_{\xi'_1}} \geq \eth_{\mathfrak{s}_{\xi'_2}}, \eth_{\mathfrak{s}_{\zeta'_1}} \leq \eth_{\mathfrak{s}_{\zeta'_2}}, \eth_{\mathfrak{s}_{\nu'_1}} \leq \eth_{\mathfrak{s}_{\nu'_2}}$

$d)\ \eth_1 = \eth_2 \Rightarrow \eth_{\mathfrak{s}_{\mu'_1}} = \eth_{\mathfrak{s}_{\mu'_2}}, \eth_{\mathfrak{s}_{\xi'_1}} = \eth_{\mathfrak{s}_{\xi'_2}}, \eth_{\mathfrak{s}_{\zeta'_1}} = \eth_{\mathfrak{s}_{\zeta'_2}}, \eth_{\mathfrak{s}_{\nu'_1}} = \eth_{\mathfrak{s}_{\nu'_2}}$

$e)\ \eth_1^{\,c} = < \eth_{\mathfrak{s}_{\nu'_1}}, \eth_{\mathfrak{s}_{\zeta'_1}}, \eth_{\mathfrak{s}_{\xi'_1}}, \eth_{\mathfrak{s}_{\mu'_1}} >$ where $\eth_1^{\,c}$ indicates the complement of $\eth_1$.

Theorem 3.8 If $\eth_1 = < \eth_{\mathfrak{s}_{\mu'_1}}, \eth_{\mathfrak{s}_{\xi'_1}}, \eth_{\mathfrak{s}_{\zeta'_1}}, \eth_{\mathfrak{s}_{\nu'_1}} >$ and $\eth_2 = < \eth_{\mathfrak{s}_{\mu'_2}}, \eth_{\mathfrak{s}_{\xi'_2}}, \eth_{\mathfrak{s}_{\zeta'_2}}, \eth_{\mathfrak{s}_{\nu'_2}} >$ be in $\aleph_{[0,\wp]}$ then

1. $(\eth_1 \Cup \eth_2)^c = \eth_1^{\,c} \Cap \eth_2^{\,c}$
2. $(\eth_1 \Cap \eth_2)^c = \eth_1^{\,c} \Cup \eth_2^{\,c}$

Proof: Considering the Definition 3.7 (a) and (b), we can easily obtain the results.

Definition 3.9 Let $\eth = < \eth_{\mathfrak{s}_{\mu'}}, \eth_{\mathfrak{s}_{\xi'}}, \eth_{\mathfrak{s}_{\zeta'}}, \eth_{\mathfrak{s}_{\nu'}} >$, $\eth_1 = < \eth_{\mathfrak{s}_{\mu'_1}}, \eth_{\mathfrak{s}_{\xi'_1}}, \eth_{\mathfrak{s}_{\zeta'_1}}, \eth_{\mathfrak{s}_{\nu'_1}} >$ and $\eth_2 = < \eth_{\mathfrak{s}_{\mu'_2}}, \eth_{\mathfrak{s}_{\xi'_2}}, \eth_{\mathfrak{s}_{\zeta'_2}}, \eth_{\mathfrak{s}_{\nu'_2}} >$ be three LQSVNNs in $\aleph_{[0,\wp]}$, and $\varrho > 0$. Then the following operational laws are defined in the following:

$a)\ \eth_1 \hat{\oplus} \eth_2 = < \eth_{\mathfrak{s}_{\mu'_1} + \mathfrak{s}_{\mu'_2} - \frac{\mathfrak{s}_{\mu'_1}\mathfrak{s}_{\mu'_2}}{\wp}}, \eth_{\mathfrak{s}_{\xi'_1} + \mathfrak{s}_{\xi'_2} - \frac{\mathfrak{s}_{\xi'_1}\mathfrak{s}_{\xi'_2}}{\wp}}, \eth_{\frac{\mathfrak{s}_{\zeta'_1}\mathfrak{s}_{\zeta'_2}}{\wp}}, \eth_{\frac{\mathfrak{s}_{\nu'_1}\mathfrak{s}_{\nu'_2}}{\wp}} >$

$b)\ \eth_1 \hat{\otimes} \eth_2 = < \eth_{\frac{\mathfrak{s}_{\mu'_1}\mathfrak{s}_{\mu'_2}}{\wp}}, \eth_{\frac{\mathfrak{s}_{\xi'_1}\mathfrak{s}_{\xi'_2}}{\wp}}, \eth_{\mathfrak{s}_{\zeta'_1} + \mathfrak{s}_{\zeta'_2} - \frac{\mathfrak{s}_{\zeta'_1}\mathfrak{s}_{\zeta'_2}}{\wp}}, \eth_{\mathfrak{s}_{\nu'_1} + \mathfrak{s}_{\nu'_2} - \frac{\mathfrak{s}_{\nu'_1}\mathfrak{s}_{\nu'_2}}{\wp}} >$

$c)\ \varrho\eth = < \eth_{\wp - \wp\left(1 - \frac{\mathfrak{s}_{\mu'}}{\wp}\right)^{\varrho}}, \eth_{\wp - \wp\left(1 - \frac{\mathfrak{s}_{\xi'}}{\wp}\right)^{\varrho}}, \eth_{\wp\left(\frac{\mathfrak{s}_{\zeta'}}{\wp}\right)^{\varrho}}, \eth_{\wp\left(\frac{\mathfrak{s}_{\nu'}}{\wp}\right)^{\varrho}} >$

$d)\ \eth^{\varrho} = < \eth_{\wp\left(\frac{\mathfrak{s}_{\mu'}}{\wp}\right)^{\varrho}}, \eth_{\wp\left(\frac{\mathfrak{s}_{\xi'}}{\wp}\right)^{\varrho}}, \eth_{\wp - \wp\left(1 - \frac{\mathfrak{s}_{\zeta'}}{\wp}\right)^{\varrho}}, \eth_{\wp - \wp\left(1 - \frac{\mathfrak{s}_{\nu'}}{\wp}\right)^{\varrho}} >$

Example 3.10 Let $\mathfrak{S} = \{\mathfrak{s}_0 = flop(F), \mathfrak{s}_1 = average(A), \mathfrak{s}_2 = below\ average(BA), \mathfrak{s}_3 = above\ average(AA), \mathfrak{s}_4 = hit(H), \mathfrak{s}_5 = semi\ hit(SMH), \mathfrak{s}_6 = super\ hit(SPH), \mathfrak{s}_7 = blockbuster(BBH), \mathfrak{s}_8 = disaster(D)\}$ be a LTS. Let $\eth_1 = < \mathfrak{s}_1, \mathfrak{s}_5, \mathfrak{s}_6, \mathfrak{s}_3 >$ and $\eth_2 = < \mathfrak{s}_3, \mathfrak{s}_0, \mathfrak{s}_2, \mathfrak{s}_5 >$ be two LQSVNNs obtained from $\mathfrak{S}$ and $\varrho = 0.6$. Then by using Definition 3.8, we obtain the following:

1. $\eth_1 \hat{\oplus} \eth_2 = < \eth_{1+3-\frac{1.3}{8}}, \eth_{5+0}, \eth_{\frac{6.2}{8}}, \eth_{\frac{3.5}{8}} > = < \eth_{3.625}, \mathfrak{s}_5, \eth_{1.5}, \eth_{1.87} >$
2. $\eth_1 \hat{\otimes} \eth_2 = < \eth_{\frac{1.3}{8}}, \eth_0, \eth_{6+2-\frac{6.2}{8}}, \eth_{3+5-\frac{3.5}{8}} > = < \eth_{0.375}, \mathfrak{s}_0, \eth_{6.5}, \eth_{6.125} >$
3. $\varrho\eth_1 = < \eth_{8-8\left(1-\frac{1}{8}\right)^{0.6}}, \eth_{8-8\left(1-\frac{5}{8}\right)^{0.6}}, \eth_{8\left(\frac{6}{8}\right)^{0.6}}, \eth_{8\left(\frac{3}{8}\right)^{0.6}} > = < \eth_{0.615}, \eth_{3.55}, \eth_{6.73}, \eth_{4.441} >$
4. $\eth_2^{\,\varrho} = < \eth_{8\left(\frac{3}{8}\right)^{0.6}}, \eth_{8\left(\frac{0}{8}\right)^{0.6}}, \eth_{8-8\left(1-\frac{2}{8}\right)^{0.6}}, \eth_{8-8\left(1-\frac{5}{8}\right)^{0.6}} > = < \eth_{4.441}, \mathfrak{s}_0, \eth_{1.268}, \eth_{3.558} >$

3.2 Weighted aggregation operators of LQSVNNs

Definition 3.11 Let $\eth_{j} = < \eth_{\mathfrak{s}_{\mu'_j}}, \eth_{\mathfrak{s}_{\xi'_j}}, \eth_{\mathfrak{s}_{\zeta'_j}}, \eth_{\mathfrak{s}_{\nu'_j}} > \in \aleph_{[0,\wp]} (j = 1,2,..,k)$ be a class of LQSVNNs, and $\mathfrak{W}_j \in [0,1]$ denote the weight of $\eth_j$ satisfying $\sum_{j=1}^{k} \mathfrak{W}_j = 1$. Then the linguistic Quadripartitioned single-valued weighted average aggregation (LQSVNWAA) operator is defined as

$$LQSVNWAA(\eth_1, \eth_2, \eth_3, \dots, \eth_k) = \sum_{j=1}^{k} \mathfrak{W}_j\ \eth_j$$

$$= < \eth_{\wp - \wp\prod_{j=1}^{k}\left(1 - \frac{\mathfrak{s}_{\mu'_j}}{\wp}\right)^{\mathfrak{W}_j}}, \eth_{\wp - \wp\prod_{j=1}^{k}\left(1 - \frac{\mathfrak{s}_{\xi'_j}}{\wp}\right)^{\mathfrak{W}_j}}, \eth_{\wp\prod_{j=1}^{k}\left(\frac{\mathfrak{s}_{\zeta'_j}}{\wp}\right)^{\mathfrak{W}_j}}, \eth_{\wp\prod_{j=1}^{k}\left(\frac{\mathfrak{s}_{\nu'_j}}{\wp}\right)^{\mathfrak{W}_j}} > \qquad (3)$$

The LQSVNWAA operator satisfies the following properties:

(i) Idempotency: Let $\eth_j = < \eth_{\mathfrak{s}_{\mu'_j}}, \eth_{\mathfrak{s}_{\xi'_j}}, \eth_{\mathfrak{s}_{\zeta'_j}}, \eth_{\mathfrak{s}_{\nu'_j}} > \in \aleph_{[0,\wp]}$ $(j = 1,2,..,k)$ be a collection of LQSVNNs where $\eth_1 = \eth_2 = \eth_3 = \dots = \eth_k = \eth$(say), then $LQSVNWAA(\eth_1, \eth_2, \eth_3, \dots, \eth_k) = \eth$.

(ii) Monotonicity: Let $\eth_j = < \eth_{\mathfrak{s}_{\mu'_j}}, \eth_{\mathfrak{s}_{\xi'_j}}, \eth_{\mathfrak{s}_{\zeta'_j}}, \eth_{\mathfrak{s}_{\nu'_j}} > \in \aleph_{[0,\wp]}$ $(j = 1,2,..,k)$ be a collection of LQSVNNs. If $\eth_j \leq \eth_j^{\,*}$ $for\ j = 1,2,..,k$, then $LQSVNWAA(\eth_1, \eth_2, \eth_3, \dots, \eth_k) \leq LQSVNWAA(\eth_1^{\,*}, \eth_2^{\,*}, \eth_3^{\,*}, \dots, \eth_k^{\,*})$

(iii) Boundedness: Let $\eth_j = < \eth_{\mathfrak{s}_{\mu'_j}}, \eth_{\mathfrak{s}_{\xi'_j}}, \eth_{\mathfrak{s}_{\zeta'_j}}, \eth_{\mathfrak{s}_{\nu'_j}} > \in \aleph_{[0,\wp]}$ $(j = 1,2,..,k)$ be a collection of LQSVNNs. If there exists $\lambda^- = < min_j(\eth_{\mathfrak{s}_{\mu'_j}}), min_j(\eth_{\mathfrak{s}_{\xi'_j}}), max_j(\eth_{\mathfrak{s}_{\zeta'_j}}), max_j(\eth_{\mathfrak{s}_{\nu'_j}}) >$ and $\lambda^+ = < max_j(\eth_{\mathfrak{s}_{\mu'_j}}), max_j(\eth_{\mathfrak{s}_{\xi'_j}}), min_j(\eth_{\mathfrak{s}_{\zeta'_j}}), min_j(\eth_{\mathfrak{s}_{\nu'_j}}) >$ then $\lambda^- \leq LQSVNWAA(\eth_1, \eth_2, \eth_3, \dots, \eth_k) \leq \lambda^+$.

Definition 3.12 Let $\eth_{j} = < \eth_{\mathfrak{s}_{\mu'_j}}, \eth_{\mathfrak{s}_{\xi'_j}}, \eth_{\mathfrak{s}_{\zeta'_j}}, \eth_{\mathfrak{s}_{\nu'_j}} > \in \aleph_{[0,\wp]} (j = 1,2,..,k)$ be a class of LQSVNNs, and $\mathfrak{W}_j \in [0,1]$ denote the weight of $\eth_j$ satisfying $\sum_{j=1}^{k} \mathfrak{W}_j = 1$. Then the linguistic Quadripartitioned single-valued weighted geometric aggregation (LQSVNWGA) operator is defined as

$$LQSVNWAA(\eth_1,\eth_2,\eth_3,\ldots,\eth_k)=\prod_{\jmath=1}^{k}\eth_{\jmath}^{\mathfrak{W}_{\jmath}}$$

$$=<\eth_{\wp\prod_{\jmath=1}^{k}\left(\frac{s_{\mu'_\jmath}}{\wp}\right)^{\mathfrak{W}_\jmath}},\ \eth_{\wp\prod_{\jmath=1}^{k}\left(\frac{s_{\xi'_\jmath}}{\wp}\right)^{\mathfrak{W}_\jmath}},\ \eth_{\wp-\wp\prod_{\jmath=1}^{k}\left(1-\frac{s_{\zeta'_\jmath}}{\wp}\right)^{\mathfrak{W}_\jmath}},\ \eth_{\wp-\wp\prod_{\jmath=1}^{k}\left(1-\frac{s_{\nu'_\jmath}}{\wp}\right)^{\mathfrak{W}_\jmath}}> \quad (4)$$

Considering the way we have defined the properties of the LQSVNWAA operator, we can similarly define the properties of the LQSVNWAA operator, so it is not repeated here.

3.3 Distance measures between two linguistic quadripartitioned single-valued neutrosophic numbers

Definition 3.13 For any two LQSVNNs $\eth_1= <\eth_{s_{\mu'_1}},\eth_{s_{\xi'_1}},\eth_{s_{\zeta'_1}},\eth_{s_{\nu'_1}}>$ and $\eth_2= <\eth_{s_{\mu'_2}},\eth_{s_{\xi'_2}},\eth_{s_{\zeta'_2}},\eth_{s_{\nu'_2}}>$ the Hamming distance between them is defined as

$$\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=\frac{\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_2}}\right|+\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_2}}\right|+\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}\right|+\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}\right|}{4} \quad (5)$$

Example 3.14 Let $\eth_1=<s_2,s_0,s_3,s_2>$ and $\eth_2=<s_3,s_5,s_2,s_3>$ be two LQSVNNs obtained from $\aleph_{[0,5]}$. Then we obtain their Hamming distance in the following way

$$\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=\frac{|2-3|+|0-5|+|3-2|+|2-3|}{4}=2$$

Proposition 3.15 The Hamming distance between two LQSVNNs $\eth_1$ and $\eth_2\in\aleph_{[0,\wp]}$ is denoted by $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)$ and it satisfies the following properties:

1. $0\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)\le\wp$
2. $\eth_1=\eth_2\Leftrightarrow\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=0$
3. $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=\mathfrak{X}_{\mathfrak{h}}(\eth_2,\eth_1)$
4. $If\ \eth_1\le\eth_2\le\eth_3\ for\ \eth_3\in\aleph_{[0,\wp]}\ then\ \mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_3)$ and $\mathfrak{X}_{\mathfrak{h}}(\eth_2,\eth_3)\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_3)$.

Proof.
1. We have
$0\le\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_2}}\right|\le\wp,\ 0\le\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_2}}\right|\le\wp,\ 0\le\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}\right|\le\wp, and\ 0\le\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}\right|\le\wp$. Thus, $0\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)\le\wp$.
2. If $\eth_1=\eth_2$ then it is obvious that $\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_2}}\right|=\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_2}}\right|=\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}\right|=\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}\right|=0$
which implies $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=0$.
On the other hand $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=0\Rightarrow\eth_{s_{\mu'_1}}=\eth_{s_{\mu'_2}},\ \eth_{s_{\xi'_1}}=\eth_{s_{\xi'_2}},\ \eth_{s_{\zeta'_1}}=\eth_{s_{\zeta'_2}}$, and $\eth_{s_{\nu'_1}}=\eth_{s_{\nu'_2}}$ i.e $\eth_1=\eth_2$.
Thus, $\eth_1=\eth_2\Leftrightarrow\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=0$
3. $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=\frac{\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_2}}\right|+\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_2}}\right|+\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}\right|+\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}\right|}{4}$
$=\frac{\left|\eth_{s_{\mu'_2}}-\eth_{s_{\mu'_1}}\right|+\left|\eth_{s_{\xi'_2}}-\eth_{s_{\xi'_1}}\right|+\left|\eth_{s_{\zeta'_2}}-\eth_{s_{\zeta'_1}}\right|+\left|\eth_{s_{\nu'_2}}-\eth_{s_{\nu'_1}}\right|}{4}=\mathfrak{X}_{\mathfrak{h}}(\eth_2,\eth_1)$
4. Let $\eth_1= <\eth_{s_{\mu'_1}},\eth_{s_{\xi'_1}},\eth_{s_{\zeta'_1}},\eth_{s_{\nu'_1}}>$, $\eth_2= <\eth_{s_{\mu'_2}},\eth_{s_{\xi'_2}},\eth_{s_{\zeta'_2}},\eth_{s_{\nu'_2}}>$, and $\eth_3= <\eth_{s_{\mu'_3}},\eth_{s_{\xi'_3}},\eth_{s_{\zeta'_3}},\eth_{s_{\nu'_3}}>$
It is given that $\eth_1\le\eth_2\le\eth_3$ which implies $\eth_{s_{\mu'_1}}\le\eth_{s_{\mu'_2}}\le\eth_{s_{\mu'_3}},\ \eth_{s_{\xi'_1}}\le\eth_{s_{\xi'_2}}\le\eth_{s_{\xi'_3}},\ \eth_{s_{\zeta'_1}}\ge\eth_{s_{\zeta'_2}}\ge\eth_{s_{\zeta'_3}}$, and $\eth_{s_{\nu'_1}}\ge\eth_{s_{\nu'_2}}\ge\eth_{s_{\nu'_3}}$.
Now,
$\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_3)-\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)=\frac{\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_3}}\right|+\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_3}}\right|+\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_3}}\right|+\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_3}}\right|}{4}-\frac{\left|\eth_{s_{\mu'_1}}-\eth_{s_{\mu'_2}}\right|+\left|\eth_{s_{\xi'_1}}-\eth_{s_{\xi'_2}}\right|+\left|\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}\right|+\left|\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}\right|}{4}$
$=\frac{\eth_{s_{\mu'_3}}-\eth_{s_{\mu'_1}}+\eth_{s_{\xi'_3}}-\eth_{s_{\xi'_1}}+\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_3}}+\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_3}}}{4}-\frac{\eth_{s_{\mu'_2}}-\eth_{s_{\mu'_1}}+\eth_{s_{\xi'_2}}-\eth_{s_{\xi'_1}}+\eth_{s_{\zeta'_1}}-\eth_{s_{\zeta'_2}}+\eth_{s_{\nu'_1}}-\eth_{s_{\nu'_2}}}{4}$
$=\frac{\eth_{s_{\mu'_3}}-\eth_{s_{\mu'_2}}+\eth_{s_{\xi'_3}}-\eth_{s_{\xi'_2}}+\eth_{s_{\zeta'_2}}-\eth_{s_{\zeta'_3}}+\eth_{s_{\nu'_2}}-\eth_{s_{\nu'_3}}}{4}\ge0$
Therefore, $\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_2)\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_3)$.
Similarly, we can prove that $\mathfrak{X}_{\mathfrak{h}}(\eth_2,\eth_3)\le\mathfrak{X}_{\mathfrak{h}}(\eth_1,\eth_3)$.

4. Linguistic Quadripartitioned Single-Valued Neutrosophic Soft Sets (LQSVNSSs) and their Properties

In this part, we first give the notion of LQSVNSSs and then study their various properties and operations on them.

Definition 4.1 Let Á be a set of alternatives, Ç be a set of criteria, and $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots, \mathfrak{s}_{\wp}\}$ be a continuous finite linguistic term set with cardinality $\wp + 1$. Then, a LQSVNSS defined on Á under Ç be denoted and defined as $\Upsilon_{Ç} = \left\{\left(\varepsilon_j, \pi_{Ç(\varepsilon_j)}\right) : \varepsilon_{j \in Ç}, \pi_{Ç(\varepsilon_j)} \in \Lambda^{Á}_{[0,\wp]}\right\}$ where $\pi_{Ç}: Ç \to \Lambda^{Á}_{[0,\wp]}$. Furthermore, we represent it in the following manner:

$$\Upsilon_{Ç} = \left\{\left(\varepsilon_j, \left\{\left(a_i, < \mathfrak{s}_{\mu_{ij}}, \mathfrak{s}_{\xi_{ij}}, \mathfrak{s}_{\zeta_{ij}}, \mathfrak{s}_{\nu_{ij}} >\right)\right\}\right) : \varepsilon_{j \in Ç}, a_{i \in Á}, < \mathfrak{s}_{\mu_{ij}}, \mathfrak{s}_{\xi_{ij}}, \mathfrak{s}_{\zeta_{ij}}, \mathfrak{s}_{\nu_{ij}} > \in \Lambda_{[0,\wp]}\right\}$$

For simplicity, the set of all LQSVNSSs on Á under the criteria set Ç and $\mathfrak{S} = \{\mathfrak{s}_0, \mathfrak{s}_1, \mathfrak{s}_2, \ldots, \mathfrak{s}_{\wp}\}$ is denoted by $\Lambda^{Á/Ç}_{[0,\wp]}$.

Example 4.2 Let us assume that $P = \{p_1, p_2, p_3, p_4, p_5\}$ is a set of people whose sputum is collected by a primary health center for diagnostic purposes whenever the given set of persons have the symptoms represented by the set $S = \{s_1 = cough, s_2 = chest\ pain, s_3 = fever, s_4 = breathing\ difficulty\}$.
The linguistic term set is $L = \{l_0 = frequently, l_1 = always, l_2 = never, l_3 = severe, l_4 = more\ than\ one\ week, l_5 = last\ for\ 6\ days\}$. The qualitative measurement of expert is represented by the following LQSVNSS:
$\Upsilon = \{(s_1, \{(p_1, < l_1, l_2, l_3, l_1 >), (p_2, < l_0, l_4, l_3, l_5 >), (p_3, < l_1, l_1, l_3, l_4 >), (p_4, < l_1, l_0, l_2, l_1 >), (p_5, < l_5, l_5, l_2, l_3 >)\}), (s_2, \{(p_1, < l_1, l_0, l_4, l_2 >), (p_2, < l_2, l_2, l_3, l_4 >), (p_3, < l_3, l_0, l_2, l_4 >), (p_4, < l_2, l_2, l_2, l_3 >), (p_5, < l_5, l_2, l_2, l_1 >)\}), (s_3, \{(p_1, < l_1, l_0, l_2, l_2 >), (p_2, < l_4, l_2, l_3, l_5 >), (p_3, < l_1, l_2, l_3, l_3 >), (p_4, < l_0, l_0, l_0, l_5 >), (p_5, < l_4, l_4, l_2, l_1 >)\}), (s_4, \{(p_1, < l_5, l_4, l_3, l_1 >), (p_2, < l_2, l_2, l_4, l_0 >), (p_3, < l_1, l_2, l_0, l_3 >), (p_4, < l_3, l_3, l_4, l_5 >), (p_5, < l_4, l_4, l_3, l_5 >)\})\}$
For a better understanding the above LQSVNS set can be represented in the matrix form given in Table 2.

Table 2. Tabular representation of an LQSVNSS

	s_1	s_2	s_3	s_4
p_1	$< l_1, l_2, l_3, l_1 >$	$< l_1, l_0, l_4, l_2 >$	$< l_1, l_0, l_2, l_2 >$	$< l_5, l_4, l_3, l_1 >$
p_2	$< l_0, l_4, l_3, l_5 >$	$< l_2, l_2, l_3, l_4 >$	$< l_4, l_2, l_3, l_5 >$	$< l_2, l_2, l_4, l_0 >$
p_3	$< l_1, l_1, l_3, l_4 >$	$< l_3, l_0, l_2, l_4 >$	$< l_1, l_2, l_3, l_3 >$	$< l_1, l_2, l_0, l_3 >$
p_4	$< l_1, l_0, l_2, l_1 >$	$< l_2, l_2, l_2, l_3 >$	$< l_0, l_0, l_0, l_5 >$	$< l_3, l_3, l_4, l_5 >$
p_5	$< l_5, l_5, l_2, l_3 >$	$< l_5, l_2, l_2, l_1 >$	$< l_4, l_4, l_2, l_1 >$	$< l_4, l_4, l_3, l_5 >$

Definition 4.3 Let $\Upsilon^1, \Upsilon^2 \in \Lambda^{Á/Ç}_{[0,\wp]}$, then Υ^1 is an LQSVNS subset of Υ^2 if $\Lambda^1_{ij} \le \Lambda^2_{ij}$ for all $i \in \{1,2,\ldots,p\}$ and $j \in \{1,2,\ldots,q\}$ and this is denoted by $\Upsilon^1 \tilde{\subseteq} \Upsilon^2$.
Note: For all $i \in \{1,2,\ldots,p\}$ and $j \in \{1,2,\ldots,q\}$ if $\Lambda^1_{ij} = \Lambda^2_{ij}$ then equality holds.

Proposition 4.4 Let $\Upsilon^1, \Upsilon^2, \Upsilon^3 \in \Lambda^{Á/Ç}_{[0,\wp]}$, then we have the following:
(i) $\Upsilon^1 \tilde{\subseteq} \Upsilon^2$ and $\Upsilon^2 \tilde{\subseteq} \Upsilon^1 \Leftrightarrow \Upsilon^1 = \Upsilon^2$
(ii) $\Upsilon^1 \tilde{\subseteq} \Upsilon^2$ and $\Upsilon^2 \tilde{\subseteq} \Upsilon^3 \Rightarrow \Upsilon^1 \tilde{\subseteq} \Upsilon^3$
Proof. The proofs are straightforward.

Definition 4.5 Let $\Upsilon_{Ç} = \left\{\left(\varepsilon_j, \pi_{Ç(\varepsilon_j)}\right) : \varepsilon_{j \in Ç}, \pi_{Ç(\varepsilon_j)} \in \Lambda^{Á}_{[0,\wp]}\right\}$ be a LQSVNSS. Then its complement is denoted and defined by $\widetilde{\Upsilon_{Ç}} = \{\left(\varepsilon_j, \widetilde{\pi_{Ç(\varepsilon_j)}}\right) : \varepsilon_{j \in Ç}, \widetilde{\pi_{Ç(\varepsilon_j)}} \in \Lambda^{Á}_{[0,\wp]}\} = \{\left(\varepsilon_j, \{(a_i, \widetilde{\pi_{ij}})\}\right) : \varepsilon_{j \in Ç}, \widetilde{\pi_{ij}} \in \Lambda_{[0,\wp]}\}$.

Definition 4.6 Let $\Upsilon_{Ç}{}^1$ and $\Upsilon_{Ç}{}^2$ be two LQSVNSSs. Then their intersection is denoted and defined by

$$\Upsilon_{Ç}{}^1 \tilde{\cap} \Upsilon_{Ç}{}^2 = \{\left(\varepsilon_j, \pi^{\cap}_{Ç}(\varepsilon_j)\right) : \varepsilon_{j \in Ç}, \pi^{\cap}_{Ç}(\varepsilon_j) \in \Lambda^{Á}_{[0,\wp]}\}$$
$$= \{\left(\varepsilon_j, \{(a_i, \pi^{\cap}_{ij})\}\right) : \varepsilon_{j \in Ç}, \pi^{\cap}_{ij} \in \Lambda_{[0,\wp]}\}$$

where $\pi^{\cap}_{ij} = \Upsilon_{ij}{}^1 \cap \Upsilon_{ij}{}^2$ for i=1,2,…,n and j=1,2,….,m.

Definition 4.7 Let $\Upsilon_{Ç}{}^1$ and $\Upsilon_{Ç}{}^2$ be two LQSVNSSs. Then their union is denoted and defined by

$$\Upsilon_{Ç}{}^1 \tilde{\cup} \Upsilon_{Ç}{}^2 = \{\left(\varepsilon_j, \pi^{\cup}_{Ç}(\varepsilon_j)\right) : \varepsilon_{j \in Ç}, \pi^{\cup}_{Ç}(\varepsilon_j) \in \Lambda^{Á}_{[0,\wp]}\}$$
$$= \{\left(\varepsilon_j, \{(a_i, \pi^{\cup}_{ij})\}\right) : \varepsilon_{j \in Ç}, \pi^{\cup}_{ij} \in \Lambda_{[0,\wp]}\}$$

where $\pi^{\cup}_{ij} = \Upsilon_{ij}{}^1 \cup \Upsilon_{ij}{}^2$ for i=1,2,…,n and j=1,2,….,m.

5. Applications

In this section an example of Linguistic Quadripartitioned Single-Valued Neutrosophic Soft Set for various applications is briefly discussed. It is well known that this set is independent of human quantum Turiyam cognition (Singh 2023c). It represents Linguistics and its indeterminacy in four ways as static uncertainty rather than dynamic or involvement of Human consciousness. Let us consider a problem related to India where some of the students demand teaching in the local language. To assess this issue feedback data is collected. Some of the students agreed to teaching in the local language and $\mu_{\mathcal{Q}(\ell_i),} \in [0,1]$ denotes the truth of this. Some of the students from nearby or other states contradict the statement above, because as the local language of my nearby state that language should be considered $\xi_{\mathcal{Q}(\ell_i),} \in [0,1]$. Some of the students from outside India $\zeta_{\mathscr{H}(\ell_i)} \in [0,1]$ who came for a degree wanted to take a degree in any language or were unaware about an issue, and they can be represented via an uncertain degree. The last cluster is educated or intellectual students who disagree since teaching locally is harmful to the career: we should go for a global language. These can be represented as falsity membership grades. These types of datasets can be analyzed using the Linguistic Quadripartitioned Single-Valued Neutrosophic Soft Set. Table 3 presents x representing the number of students (x_n) and y representing the number of subjects (y_m) and the Quadripartitioned Single-Valued Neutrosophic $(< l_{1,} l_2, l_3, l_1 >)$relations show their feedback for teaching in a local language, its contradiction, or uncertainty or rejection. Everyone independently gave their feedback. It will help to analyze the influence of local language in the university, diversity in the university, weak or strong student percentages in the university as well as acceptance of other languages. For more details see Table 3.

We can also compute the Hamming distance to analyze the similarity among feedback of two departments and their students. In this way, the proposed method helps deal with these types of datasets where contradiction exists between two or more opinions beyond the true or false, or uncertainty. However, to explore the unknown or impossible data on teaching in a multilingual system requires human quantum Turiyam cognition, as discussed by Singh (2023c). The authors will try to focus on exploring this area with an illustrative example.

6. Conclusions

This paper focused on dealing with the uncertainty in qualitative data which are known. To deal with uncertainty in these types of data a Linguistic Quadripartitioned Single-Valued Neutrosophic Soft Sets (LQSVNSSs) and its weighted aggregation operator were introduced. The paper introduced some of the basic concepts and examples for dealing with education sector data. Shortly, the authors will focus on introducing other metrics for dealing with these types of data using four-valued logic.

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Table 3. Tabular representation of an LQSVNSS for teaching in Local Language

	y_1	y_2	y_3	…	y_4
X_1	$< l_{1,} l_2, l_3, l_1 >$	$< l_{1,} l_0, l_4, l_2 >$	$< l_{1,} l_0, l_2, l_2 >$	…	$< l_{5,} l_4, l_3, l_1 >$
X_2	$< l_{0,} l_4, l_3, l_5 >$	$< l_{2,} l_2, l_3, l_4 >$	$< l_{4,} l_2, l_3, l_5 >$	…	$< l_{2,} l_2, l_4, l_0 >$
X_3	$< l_{1,} l_1, l_3, l_4 >$	$< l_{3,} l_0, l_2, l_4 >$	$< l_{1,} l_2, l_3, l_3 >$	…	$< l_{1,} l_2, l_0, l_3 >$
X_4	$< l_{1,} l_0, l_2, l_1 >$	$< l_{2,} l_2, l_2, l_3 >$	$< l_{0,} l_0, l_0, l_5 >$	…	$< l_{3,} l_3, l_4, l_5 >$
…					
X_n	$< l_{5,} l_5, l_2, l_3 >$	$< l_{5,} l_2, l_2, l_1 >$	$< l_{4,} l_4, l_2, l_1 >$	…	$< l_{4,} l_4, l_3, l_5 >$

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