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Properties of rough fuzzy prime ideals in Γ rings

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Abstract

Rough Set (RS) theory is a useful mathematical strategy to handle uncertainty. In 1982, Pawlak presented the idea, and numerous authors have undertaken in-depth studies on RS in both ordinary cases and fuzzy situations. In terms of both the theoretical investigations and the practical applications, progress in this field of RS theory has yielded favorable outcomes over the last three decades and it is typically considered to be an extension of classical sets. A universe in RS is separated by two subsets known as lower and upper approximations. Upper approximations are nonempty intersections of equivalence classes, whereas lower approximations are subsets of the set. In this study, rough sets are examined when the universe set has a ring structure. The main contribution of this study is to focus on rough fuzzy prime and semi-prime ideals of the gamma ring structure and explain certain respects of its upper and lower approximations. The goal of this research is to investigate some of the characterizations of prime and semi-prime ideals and prove some related results. Moreover, the study discusses rough fuzzy ideals (RFI) in Γ Residue class and gives some findings.

Keywords: Γ rings, fuzzy set, rough set, rough fuzzy prime ideal, rough fuzzy semi prime ideal

1. Introduction

Crisp set is used in classical mathematics to compute the membership of components in a set. It only aids in finding whether a component belongs to the set or not and whether a statement is either true or false. The middle of a complicated problem often arises in real life. To deal with such challenges, several researchers have come up with theories such as fuzzy set theory, RS theory, and soft set theory, applying their tireless efforts. To explain the ambiguity and fuzziness of information, Zadeh (1965) invented the concept of the fuzzy set. In recent years, fuzzy mathematics has become a significant branch in mathematics and fuzzy algebra is one of the most vital concepts in fuzzy mathematics. Naturally, the question arises, what happens if we substitute an algebraic structure in place of the universe set. Numerous applications exist in mathematics that depend on the use of fuzzy sets with algebraic structures. As a result, this concept has been utilized in a variety of fields and researchers are highly motivated to investigate ideas and findings from the field of abstract algebra and apply them to broader fuzzy situations. Several researchers have extensively studied the fuzzy set theoretic approach with algebraic structures (1993). There are several types of algebraic structures including gamma rings. Nobusawa (1964) proposed the notion of a gamma ring, which is more common than other rings, and Barnes (1966) slightly weakened Nobusawa's notion of the gamma rings. After Nobusawa, Luh (1969) investigated the concept of gamma rings.

Linesawat and Lekkoksung (2022) examined a connection between ideals and anti-hybrid ideals. Suebsung, Wattanatripop, and Chinram (2019) studied A-ideals and fuzzy A-ideals of ternary semigroups. Nakkhasen and Pibaljommee (2019) illustrated the m-bi-hyper ideals in semi-hyper rings. Jun *et al*. (1992, 2003) investigated the concept of fuzzy sets in gamma ring theory. Kyuno (1978) explored prime ideals in a gamma ring structure. Homomorphism and endomorphism on T- interval valued fuzzy sub-algebras are discussed by Hemavathi, Muralikrishna, Palanivel, and Chinram (2022). Later, Fuzzy prime ideals of rings were extensively studied by Mukherjee and Sen (1987) and Dutta and Chanda (2007). Yiarayong (2019) investigated the relationship between fuzzy prime and weakly fuzzy quasi-prime ideals in rings. Atanassov introduced the IFS to solve the non-determinacy issue in fuzzy sets. Palaniappan *et al*. (2010, 2011) examined intuitionistic

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fuzzy prime ideals in the gamma ring structure. The intuitionistic prime ideals of BCK-algebras were explored by Abdullah (2014) who also investigated some of their characteristics. Lee (2000) discussed intuitionistic fuzzy points and intuitionistic fuzzy neighborhoods. Pawlak (1982, 1991) proposed RS theory, which offers a novel theoretical framework for resolving complex problems in ambiguous circumstances. Researchers and practitioners in various fields of science and technology have been interested in this theory. The algebraic structure is a key component of RS theory that studies the intelligent systems that contain incomplete and insufficient information as an extension of set theory. This theory has been applied to a wide variety of fields and it is an emerging area of uncertainty mathematics closely relevant to fuzzy set theory. Kazanci and Davvaz (2008) examined the rough prime ideals and the upper and lower approximations of their homomorphism images. The algebraic and topological properties of generalized rough sets have been investigated by Ali, Davvaz, and Shabir (2013).

To extend the concept of a sub-ring in a ring, Davvaz (2004, 2018) introduced the idea of a rough sub-ring concerning the ideal of a ring and also discussed methods and ideas about rough algebraic structures. Bonikowaski (1994) conducted a study on RS algebraic and set-theoretical characteristics. When RS has been combined with other theoretical methods or technologies, it results in a multitude of research with extensions. Fuzzy set theory and RS theory are two popular methodologies for dealing with data ambiguity and imprecision. Even if these ideas differ, they result in highly beneficial ways when combined. Thus, the notion of RFS was developed by combining RS and fuzzy sets. RFS is an approximation of a fuzzy set in a crisp approximation space. A fuzzy rough set is an approximate representation of a crisp set in a fuzzy approximation space. Dubois and Parade (1990) discussed the difference between an RFS and a fuzzy rough set. In gamma ring theory, the present researcher used the idea of rough fuzzy sets.

Several studies have investigated the usefulness of RFS for dealing with uncertainty, particularly vagueness. Subha *et al*. (2019, 2020) studied the rough fuzzy prime ideals in semi-groups. Hussain, Mahmood and Ali (2019a, 2019b) investigated rough fuzzy ideals in semigroups. Durgadevi and Ezhilmaran (2022a, 2022b) examined some of the characteristics of RFI in gamma rings. Malik *et al*. (2019, 2023) explored rough fuzzy set with algebraic structure and also discussed application in rough fuzzy bipolar soft sets. As per the research survey, many academicians have investigated the RFS with several algebraic structures, and no previous study was found on RFPI and RFSPI in gamma rings. As a result, the purpose of this work is to bridge the gap in previous research by establishing the concept of RFPI and RFSPI in the gamma ring structure. This article is organized into the following Sections. The required basic definitions are discussed in Section 2. RFPI are examined in Section 3. In Section 4 RFSPI in gamma rings are defined and related theorems are proved. Section 5, investigates RFI in Γ Residue Class and discusses their respective properties. The conclusion is presented in Section 6.

2. Preliminaries

This section discusses some fundamental concepts such as gamma ring, rough set, and rough fuzzy set.

Definition 2.1: (Barnes, 1966) Let (N, +) and (Γ, +) be additive Abelian groups. If there exists a mapping $N \times \Gamma \times N \to N$ [the image of (p, α, q) is denoted by $p\alpha q$ for $p, q, r \in N$ and $\alpha \in \Gamma$] satisfying the following identities:
(1) $p\alpha q \in N$,
(2) $(p + q)\alpha r = p\alpha r + q\alpha r$, $p(\alpha + \beta)q = p\alpha q + p\beta q, p\alpha(q + r) = p\alpha q + p\alpha r$,
(3) $(p\alpha q)\beta r = p(\alpha q\beta)r = p\alpha(q\beta r)$
for all $p, q, r \in N$ and $\alpha, \beta \in \Gamma$, then N is called a Γ Ring.
If these axioms are strengthened to
(1') $p\alpha q \in N, \alpha p\beta \in \Gamma$,
(2')$(p + q)\alpha r = p\alpha q + q\alpha r, p(\alpha + \beta)q = p\alpha q + p\beta q, p\alpha(q + r) = p\alpha q + p\alpha r$,
(3')$(p\alpha q)\beta r = p(\alpha q\beta)r = p\alpha(q\beta r)$,
(4') $p\alpha q = 0$ for all $p, q \in N$ implies $\alpha = 0$,
We then have a Γ Ring in the sense of Nobusawa.
Note that, it follows from (1) - (3) that $0\alpha q = p0q = p\alpha 0 = 0$ for all $p, q \in N$ and $\alpha \in \Gamma$.

Definition 2.2: (Pawlak, 1991) Suppose the knowledge base K = (U, R). For each subset $P \subseteq U$, and an equivalence relation $R \in IND(K)$ we associate the two subsets $\underline{apr}(P) = \cup\{Y \in U/R \mid Y \subseteq P\}$ and $\overline{apr}(P) = \cup\{Y \in U/R \mid Y \cap P \neq \phi\}$ that are apr-lower and apr-upper approximations of P respectively.

Definition 2.3: (Dubois & Parade, 1990) Let $X \subseteq U$ be a set, R be an equivalence relation on U and P be a fuzzy subset in U. The upper and lower approximation of a fuzzy subsets P by R are the fuzzy set U/R with membership function defined by
$\mu_{\overline{apr}(P)}(X_i) = sup\{\mu_P(x)|\omega(X_i) = [x]_R\}$ and
$\mu_{\underline{apr}(P)}(X_i) = inf\{\mu_P(x)|\omega(X_i) = [x]_R\}$,
where $\mu_{\overline{apr}(P)}(X_i)$ (resp. $\mu_{\underline{apr}(P)}(X_i)$) is the degree of membership of X_i in $\overline{apr}(P)(resp. \underline{apr}(P)). \overline{apr}(P)$, $\underline{apr}(P)$) is called an RFS.

3. Rough Fuzzy Prime Ideals in Γ Rings

In this section, we investigate a few theorems about rough fuzzy prime ideals in gamma rings and discuss some related results.

Definition 3.1: (Pushpanathan & Devarasan, 2022) An upper (resp. lower) RFS $\lambda = < \overline{apr}_\lambda, \underline{apr}_\lambda >$ in N is said to be a RFLI (resp. RFRI) of a Γ Ring N if
(i) $\overline{apr}_\lambda(p-q) \geq \{\overline{apr}_\lambda(p) \wedge \overline{apr}_\lambda(q)\}$, $\overline{apr}_\lambda(p\gamma q) \geq \overline{apr}_\lambda(q)[resp.\ \overline{apr}_\lambda(p\gamma q) \geq \overline{apr}_\lambda(p)]$;
(ii) $\underline{apr}_\lambda(p-q) \leq \{\underline{apr}_\lambda(p) \vee \underline{apr}_\lambda(q)\}$, $\underline{apr}_\lambda(p\gamma q) \leq \underline{apr}_\lambda(q)[resp.\ \underline{apr}_\lambda(p\gamma q) \leq \underline{apr}_\lambda(p)]$.
for all $p, q \in N$ and $\gamma \in \Gamma$.

Example 3.2: (Pushpanathan & Devarasan, 2022)
Let N = {a, b, c, d} and $\alpha = \{e, f, g, h\}$ be two sets and define the operations - and α as follows

$$\overline{apr}_\lambda(p) = \begin{cases} 0.5 & \text{if } p = a, e \\ 0.6 & \text{if } p = b, f \\ 0.6 & \text{if } p = c, d, g, h \end{cases}, \quad \underline{apr}_\lambda(p) = \begin{cases} 0.5 & if\ p = a, e \\ 0.4 & \text{if } p = b, f \\ 0.3 & \text{if } p = c, d, g, h \end{cases}$$

-	a	b	c	d
a	a	b	c	d
b	b	b	d	c
c	c	d	d	c
d	d	c	c	c

α	e	f	g	h
e	e	f	g	h
f	f	f	h	g
g	g	h	h	g
h	h	g	g	g

Figure 1. Example of rough fuzzy ideals in γ rings

By applying the upper and lower approximation values in the RFI condition. By routine calculation, clearly N is a RFI.

Definition 3.3: (Barnes, 1966) An ideal λ of the Γ Ring N is said to be prime if for any ideals P and Q of N, $P\Gamma Q \subseteq \lambda$ implies $P \subseteq \lambda$ or $Q \subseteq \lambda$.

Theorem 3.4: (Mukherjee & Sen, 1987) Let λ be an ideal of a Γ Ring N. Then the following conditions are equivalent
(i) λ is a prime ideal of N
(ii) For all $p, q \in N$, $p\Gamma N \Gamma q \subseteq \lambda$ implies $p \in \lambda$ or $q \in \lambda$.

Definition 3.5: (Mukherjee & Sen, 1987) A fuzzy ideal μ of a Γ Ring N is said to be prime
if (i) μ is a non-constant function
(ii) For any two ideals ϕ, ψ in N, $\phi\Gamma\psi \subseteq \mu$ implies, $\phi \subseteq \mu$ or $\psi \subseteq \mu$.

Definition 3.6: Let λ be a RFI of N. Then λ is said to be prime if λ is not a constant mapping and for any RFI P, Q of a Γ Ring N, $P\Gamma Q \subseteq \lambda$ implies $P \subseteq \lambda$ or $Q \subseteq \lambda$.

Definition 3.7: (Kumar, 1993) A fuzzy subset μ of a Γ Ring N is called fuzzy point if $\mu(p) \in [0,1]$ for some $p \in N$ and $\mu(q)=0$ for all $q \in N \backslash \{x\}$. If $\mu(p) = \beta$, then the fuzzy point μ is denoted by p_β.

Definition 3.8: (**Lee & Lee, 2000**). Let $(x, y) \in \mathscr{J} * \mathscr{J}$ with $x + y \leq 1$. Then a RFP $p_{(x,y)}$ of N is the RFS in N defined as follows for each $q \in N$.

$$p_{(x,y)}(q) = \begin{cases} (x, y) & q = p \\ (0,1) & q \neq p \end{cases}$$

In this case, p is called the support of $p_{(x,y)}$ and x, y is the value and non value of $p_{(x,y)}$ respectively. An RFP $p_{(x,y)}$ is said to belong to an RFS A= $< \overline{apr}_A, \underline{apr}_A >$ in N, denoted by $p_{(x,y)} \in A$ if $x \leq \overline{apr}_A(p)$ and $\underline{apr}_A(p) \leq y$.

Result 3.9: Let A and B be two rough fuzzy subsets in N. Then $A \subset B$ iff for each $p_{(x,y)} \in RFP(N)$, $p_{(x,y)} \in A$ implies $p_{(x,y)} \in B$.

Result 3.10: Let $p_{(x,y)}, q_{(t,s)}$ be rough fuzzy point in N. Then $p_{(x,y)}\Gamma q_{(t,s)} = (p\Gamma q)_{(x \wedge t,\ y \vee s)}$

Definition 3.11: (Mukherjee & Sen, 1987) A Γ Ring N is called commutative if $p\gamma q = q\gamma p$ for all $p, q \in N$ and $\gamma \in \Gamma$.

Theorem 3.12: Let N be a commutative Γ Ring and λ be a RFI of M. Then the below conditions are identical
(i) $p_{(x,y)}\Gamma\, q_{(t,s)} \subseteq \lambda \Rightarrow p_{(x,y)} \subseteq \lambda$ or $q_{(t,s)} \subseteq \lambda$, where $p_{(x,y)}, q_{(t,s)}$ are two RFP of N.
(ii) λ is a RFPI of N.

Proof: (i) implies (ii).
Assume $p_{(x,y)}\Gamma q_{(t,s)} \subseteq \lambda \Rightarrow p_{(x,y)} \subseteq \lambda$ (or) $q_{(t,s)} \subseteq \lambda$. To prove λ is a RFPI of N. Let ϕ and ψ be RFI of N such that $\phi\Gamma\psi \subseteq \lambda$. Suppose $\phi \nsubseteq \lambda$. Then there exists $p \in N$ such that $\overline{apr}_{\phi}(p) > \overline{apr}_{\lambda}(p)$ and $\underline{apr}_{\phi}(p) < \underline{apr}_{\lambda}(p)$. Let $\overline{apr}_{\phi}(p) = a$ and $\underline{apr}_{\phi}(p) = b$ and $q \in N$ such that $\overline{apr}_{\psi}(q) = c$ and $\underline{apr}_{\psi}(q) = d$. If $r = p\gamma q$ for some $\gamma \in \Gamma$, then $p_{(x,y)}\Gamma q_{(t,s)}(r) = (a \wedge c, b \vee d)$. Hence $\overline{apr}_{\lambda}(r) = \overline{apr}_{\lambda}(p\gamma q) = \overline{apr}_{\phi\Gamma\psi}(p\gamma q) \geq \{\overline{apr}_{\phi}(p) \wedge \overline{apr}_{\psi}(q)\} = \{a \wedge c\} = \overline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(p\gamma q)$. Similarly, to find the lower approximation we finally obtain $\underline{apr}_{\lambda}(r) = \{b \vee d\} = \underline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(p\gamma q)$. If $\overline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(r) = 0$ and $\underline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(r) = 1$ then $\overline{apr}_{\lambda}(r) \geq \overline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(r)$ and $\underline{apr}_{\lambda}(r) \leq \underline{apr}_{p_{(x,y)}\Gamma q_{(t,s)}}(r)$. Hence $p_{(x,y)}\Gamma\, q_{(t,s)}(r) \subseteq \lambda$ By (i) either $p_{(x,y)} \subseteq \lambda$ or $q_{(t,s)} \subseteq \lambda$. That is, either $a \leq \overline{apr}_{\lambda}(p)$, $b \geq \underline{apr}_{\lambda}(p)$ (or) $c \leq \overline{apr}_{\lambda}(q)$ and $d \geq \underline{apr}_{\lambda}(q)$. Since $a \nleq \overline{apr}_{\lambda}(p)$, $b \ngeq \underline{apr}_{\lambda}(p)$, $\overline{apr}_{\psi}(q) = c \leq \overline{apr}_{\lambda}(q)$, $\underline{apr}_{\psi}(q) = d \geq \underline{apr}_{\lambda}(q)$. So $\psi \subseteq \lambda$. Thus λ is a RFPI of N.
(ii) implies (i). Assume that λ is a RFPI of a Γ Ring N. To prove $p\Gamma q \subseteq \lambda \Rightarrow p_{(x,y)} \subseteq \lambda$ (or) $q_{(t,s)} \subseteq \lambda$. Let $p_{(x,y)}$ and $q_{(t,s)}$ be two RFP of N such that $p_{(x,y)}\Gamma q_{(t,s)} \subseteq \lambda$. Proof of ((i) ⇒(ii)) $(p_{(x,y)}\Gamma q_{(t,s)})(p\gamma q) = (x \wedge t,\ y \vee s)$, where $x \wedge t \leq \overline{apr}_{\lambda}(p\gamma q)$ and $y \vee s \geq \overline{apr}_{\lambda}(p\gamma q)$ for all $\gamma \in \Gamma$. Let the RFS ϕ and ψ be defined by

$$\phi(r) = \begin{cases} (x,y) & r \in \langle p\rangle, \\ (0,1) & \text{otherwise}, \end{cases} \quad , \quad \psi(r) = \begin{cases} (t,s) & r \in \langle q\rangle, \\ (0,1) & \text{otherwise}, \end{cases}$$

Clearly ϕ and ψ are RFI of N. Now, $(\phi\Gamma\psi)(Z) = \{\vee_{r=u\gamma v}\,[\phi(u) \wedge \psi(v)], \wedge_{r=u\gamma v}[\phi(u) \vee$ $\vee\, \psi(v)]\} = \{x \wedge t, y \vee s\}$ where $u \in \langle p\rangle$ and $v \in \langle q\rangle$. Hence $\overline{apr}_{\phi\Gamma\psi}(r) = x \wedge t \leq \overline{apr}_{A}(u\gamma v)$ and $\underline{apr}_{\phi\Gamma\psi}(r) = y \vee s \geq \underline{apr}_{\lambda}(u\gamma v)$. Proof of ((i) ⇒ (ii)), when $r = u\gamma v$ where $u \in \langle p\rangle$, $v \in \langle q\rangle$. Otherwise $(\phi\Gamma\psi)(r) = (0,1)$. That is $\phi\Gamma\psi \subseteq \lambda$. As λ is a Prime, $\phi \subseteq \lambda$ or $\psi \subseteq \lambda$. Then $p_{(x,y)} \subseteq \phi \subseteq \lambda$ (or) $q_{(t,s)} \subseteq \psi \subseteq \lambda$. Thus $p_{(x,y)}\Gamma q_{(t,s)} \subseteq \lambda$ implies that either $p_{(x,y)} \subseteq \lambda$ or $q_{(t,s)} \subseteq \lambda$.

Theorem 3.13: Let $\mathscr{J}$ be an ideal of a Γ Ring N, $\alpha, \beta \in [0,1]$ and λ be an rough fuzzy subset of N defined by

$$\overline{apr}_{\lambda}(p) = \begin{cases} 1 & p \in \mathscr{J}, \\ \alpha & p \notin \mathscr{J}, \end{cases} \quad , \underline{apr}_{\lambda}(p) = \begin{cases} 0 & p \in \mathscr{J}, \\ \beta & p \notin \mathscr{J}, \end{cases}$$

Then λ is a RFPI of N iff $\mathscr{J}$ is a prime ideal of N.

Proof: Let $\mathscr{J}$ be an ideal of a Γ Ring N. Assume that $\mathscr{J}$ is a prime ideal of N. To prove λ is a RFPI of N let $\mathscr{J}$ be an prime ideal. Obviously λ is non constant. If $\{\overline{apr}_{\lambda}(a) \wedge \overline{apr}_{\lambda}(b)\} = \alpha$ and $\{\underline{apr}_{\lambda}(a) \vee \underline{apr}_{\lambda}(b)\} = \beta$ then $\overline{apr}_{\lambda}(a-b) \geq \{\overline{apr}_{\lambda}(a) \wedge \overline{apr}_{\lambda}(b)\}$, and $\underline{apr}_{\lambda}(a-b) \leq \{\underline{apr}_{\lambda}(a) \vee \underline{apr}_{\lambda}(b)\}$. If $\{\overline{apr}_{\lambda}(a) \wedge \overline{apr}_{\lambda}(b)\} = 1$ and $\{\underline{apr}_{\lambda}(a) \vee \underline{apr}_{\lambda}(b)\} = 0$, then $\overline{apr}_{\lambda}(a) = \overline{apr}_{\lambda}(b) = 1$ and $\underline{apr}_{\lambda}(a) = \underline{apr}_{\lambda}(b) = 0$. So $a, b \in \mathscr{J}$ which implies $a - b \in \mathscr{J}$. Therefore $\overline{apr}_{\lambda}(a-b) = 1$ and $\underline{apr}_{\lambda}(a-b) = 0$. Hence for all $p,q \in N$, $\overline{apr}_{\lambda}(p-q) \geq \{\overline{apr}_{\lambda}(p) \wedge \overline{apr}(q)\}$ and $\underline{apr}_{\lambda}(p-q) \leq \{\underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q)\}$. Similarly $\overline{apr}_{\lambda}(p\gamma q) \geq \overline{apr}_{\lambda}(p)$[resp. $\overline{apr}_{\lambda}(p\gamma q) \geq \overline{apr}_{\lambda}(q)$]and $\underline{apr}_{\lambda}(p\gamma q) \leq \underline{apr}_{\lambda}(p)$[resp $\underline{apr}_{\lambda}(p\gamma q) \leq \underline{apr}_{\lambda}(q)$]. Thus λ is a RFI of N. Let the RFI ϕ, ψ of N be such that $\phi\Gamma\psi \subseteq \lambda$ and $\phi \nsubseteq \lambda$ or $\psi \nsubseteq \lambda$. Then there exists $p, q \in N$ such that $\overline{apr}_{\phi}(p) > \overline{apr}_{\lambda}(p)$, $\underline{apr}_{\phi}(p) < \underline{apr}_{\lambda}(p)$ and $\overline{apr}_{\psi}(q) > \overline{apr}_{\lambda}(q)$, $\underline{apr}_{\psi}(q) < \underline{apr}_{\lambda}(q)$. This implies that $\overline{apr}_{\lambda}(p) = \overline{apr}_{\lambda}(q) = \alpha$ and $\underline{apr}_{\lambda}(p) = \underline{apr}_{\lambda}(q) = \beta$. Therefore $p, q \notin \mathscr{J}$. Since $\mathscr{J}$ is a prime ideal of M, $p\Gamma M\Gamma q \nsubseteq \mathscr{J}$ [17]. Then there exists $n \in N$, $\gamma_1, \gamma_2 \in \Gamma$ such that $p\gamma_1 n\gamma_2 q \notin \mathscr{J}$. Hence $\overline{apr}_{\lambda}(x\gamma_1 n\,\gamma_2 y) = \alpha$, $\underline{apr}_{\lambda}(x\gamma_1 n\,\gamma_2 y) = \beta$. Now, $\overline{apr}_{\phi\Gamma\psi}(p\gamma_1 n\,\gamma_2 q \geq \{\overline{apr}_{\phi}(p) \wedge \overline{apr}_{\psi}(n\,\gamma_2 q)\} \geq \{\overline{apr}_{\phi}(p) \wedge \overline{apr}_{\psi}(q)\} > \{\overline{apr}_{\lambda}(p) \wedge \overline{apr}_{\lambda}(q)\} = \alpha = \overline{apr}_{\lambda}(p\gamma_1 n\gamma_2 q)$ and $\underline{apr}_{\phi\Gamma\psi}(p\gamma_1 n\,\gamma_2 q) \leq \{\underline{apr}_{\phi}(p) \vee \underline{apr}_{\psi}(n\gamma_2 q)\} \leq \{\underline{apr}_{\phi}(p) \vee \underline{apr}_{\psi}(q)\} < \{\underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q)\} = \beta = \underline{apr}_{\lambda}(p\gamma_1 n\,\gamma_2 q)$, which is a contradiction. Thus λ is a RFPI of N. Conversely, Assume λ is a RFPI of N. To prove that $\mathscr{J}$ is a Prime ideal of N let λ be the RFPI of N and X, Y be two fuzzy ideals of N such that $X\Gamma Y \subseteq \mathscr{J}$. Let $X \nsubseteq \mathscr{J}$ and $Y \nsubseteq \mathscr{J}$ and let $x \in X\backslash\mathscr{J}$ and $y \in Y\backslash\mathscr{J}$. We define RFS ϕ , ψ of N as follows

$$\overline{apr}_{\phi}(p) = \begin{cases} 1 & p \in X \\ \alpha & p \notin X \end{cases} \quad , \quad \underline{apr}_{\phi}(p) = \begin{cases} 0 & p \in X \\ \beta & p \notin X \end{cases}$$

$$\overline{apr}_{\psi}(p) = \begin{cases} 1 & p \in Y \\ \alpha & p \notin Y \end{cases} \quad , \quad \underline{apr}_{\psi}(p) = \begin{cases} 0 & p \in Y \\ \beta & p \notin Y \end{cases}$$

Then ϕ, ψ are RFI of N. Since $\overline{apr}_{\phi}(p) = 1 > \alpha = \overline{apr}_{\lambda}(p)$ and $\underline{apr}_{\phi}(p) = 0 < \beta = \underline{apr}_{\lambda}(p), \phi \nsubseteq \lambda$. Similarly $\psi \nsubseteq \lambda$.
But $\phi\Gamma\psi \subseteq \lambda$. This is a contradiction, so $\mathscr{J}$ is a prime ideal of N.

4. Rough Fuzzy Semi Prime Ideals in Γ Rings

In this section, we discuss some theorems about RFSPI in gamma ring structure.

Definition 4.1: A non-constant RFI λ of a Γ Ring N is RFSPI if for any RFI $\phi\Gamma\phi \subseteq \lambda$ implies $\phi \subseteq \lambda$.

Theorem 4.2: Let N be a commutative Γ Ring and λ be a RFI of N, then the following are equivalent.
(i) $p_{(\alpha,\beta)}\Gamma\, p_{(\alpha,\beta)} \subseteq \lambda \Rightarrow p_{(\alpha,\beta)} \subseteq \lambda$
(ii) λ is a RFSPI of N
(iii) $\phi \circ \phi \subseteq \lambda$ implies $\phi \subseteq \lambda$

Proof: (i) implies (ii). $\phi\Gamma\phi \subseteq \lambda$ implies $\phi \nsubseteq \lambda$. Then there exists $p \in N$ such that $\overline{apr}_{\phi}(x) > \overline{apr}_{\lambda}(p)$ and $\underline{apr}_{\phi}(p) < \overline{apr}_{\lambda}(p)$. Let $\overline{apr}_{\phi}(p) = \alpha$ and $\underline{apr}_{\phi}(p) = \beta$. By (i) $p_{(\alpha,\beta)}\Gamma\, p_{(\alpha,\beta)} \subseteq \lambda$ implies $p_{(\alpha,\beta)} \subseteq \lambda$. This shows that $p_{(\alpha,\beta)}(x) \subseteq \lambda(x)$. This implies that $\overline{apr}_{\lambda}(p) \geq \overline{apr}_{\phi}(p)$ and $\underline{apr}_{\lambda}(p) \leq \overline{apr}_{\phi}(p)$. This is a contradiction, so $\phi \subseteq \lambda$. Therefore λ is a RFSPI of N. Obviously (ii) implies (iii).
(iii) implies (i). Assume $\phi \circ \phi \subseteq \lambda$ implies $\phi \subseteq \lambda$. To prove (i)
let $p_{(\alpha,\beta)}\Gamma\, p_{(\alpha,\beta)} \subseteq \lambda \Rightarrow p_{(\alpha,\beta)} \subseteq \lambda$, where $p_{(\alpha,\beta)}$ is a RFP of N.
$\phi \circ \phi \subseteq \lambda$ implies

$$\begin{cases} \vee\,[\wedge_i\,[\wedge\,[\overline{apr}_{\lambda_1}(u_i), \overline{apr}_{\lambda_2}(v_i)]]], 1 \leq i \leq n,\ x = \sum_{i=1}^{n} u_i\gamma_i v_i, u_i, v_i \in N, \gamma_i \in \Gamma \\ 0 \quad \text{otherwise} \end{cases}$$
$$\subseteq \begin{cases} \vee_{x=u\gamma v}\,[\wedge\,[\overline{apr}_{\lambda_1}(u_i), \overline{apr}_{\lambda_2}(v_i)]]],\ u_i, v_i \in N, \\ 0 \quad \text{otherwise.} \end{cases} \quad \text{and}$$
$$\begin{cases} \wedge\,[\vee_i\,[\vee\,[\underline{apr}_{\lambda_1}(u_i), \underline{apr}_{\lambda_2}(v_i)]]], 1 \leq i \leq n,\ x = \sum_{i=1}^{n} u_i\gamma_i v_i, u_i, v_i \in N, \gamma_i \in \Gamma \\ 1 \quad \text{otherwise} \end{cases}$$
$$\subseteq \begin{cases} \wedge_{x=u\gamma v}\left[\vee\left[\underline{apr}_{\lambda_1}(u_i), \underline{apr}_{\lambda_2}(v_i)\right]\right], u_i, v_i \in N, \\ 1 \quad \text{otherwise.} \end{cases}$$

Then it can be said that $\phi \circ \phi \subseteq \lambda$ implies $p_{(\alpha,\beta)}\Gamma\, p_{(\alpha,\beta)} \subseteq \lambda \Rightarrow p_{(\alpha,\beta)} \subseteq \lambda$, since $\phi \subseteq \lambda$ and ϕ can be obtained as $\phi = p_{(\alpha,\beta)} \subseteq \lambda$.

Theorem 4.3: let ξ be a semi prime ideal of a Γ Ring N and $\alpha, \beta \in [0,1]$ such that $\alpha + \beta \leq 1$. Then λ is a RFPI of N where
$\overline{apr}_{\lambda}(p) = \begin{cases} 1 & p \in \xi \\ \alpha & p \notin \xi \end{cases}$, $\underline{apr}_{\lambda}(x) = \begin{cases} 0 & p \in \xi \\ \beta & p \notin \xi \end{cases}$

Proof: For all $p, q \in N$. $\overline{apr}_{\lambda}(p) \wedge \overline{apr}_{\lambda}(q) = \alpha$ and $\underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q) = \beta$, then $\overline{apr}_{\lambda}(p-q) \geq \{\overline{apr}_{\lambda}(p) \wedge \overline{apr}_{\lambda}(q)\}$ and $\underline{apr}_{\lambda}(p-q) \leq \{\underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q)\}$. For all $p, q \in N$, if $\overline{apr}_{\lambda}(p) \wedge \overline{apr}_{\lambda}(q) = 1$ and $\underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q) = 0$, then $\overline{apr}_{\lambda}(p) = \overline{apr}_{\lambda}(q) = 1$ and $\underline{apr}_{\lambda}(p) = \underline{apr}_{\lambda}(q) = 0$. This implies $p, q \in \xi$. Since ξ is an ideal of M then $p - q \in \xi$. Therefore $1 = \overline{apr}_{A}(p-q) \geq \{\overline{apr}_{A}(p) \wedge \overline{apr}_{B}(q)\}$ and $0 = \underline{apr}_{A}(p-q) \leq \{\underline{apr}_{A}(p) \vee \underline{apr}_{B}(q)\}$. Similarly $\overline{apr}_{\lambda}(p\gamma q) \geq \overline{apr}_{\lambda}(q)$ [resp. $\overline{apr}_{\lambda}(p\gamma q) \geq \overline{apr}_{\lambda}(p)$] for all $p, q \in N$ and $\gamma \in \Gamma$. Let $\phi\Gamma\phi \subseteq \lambda$ and $\phi \nsubseteq \lambda$. Then there exists $p \in N$ such that $\overline{apr}_{\phi}(p) > \overline{apr}_{\lambda}(p)$ and $\underline{apr}_{\phi}(p) < \underline{apr}_{\lambda}(p)$. This implies $\overline{apr}_{\lambda}(p) = \alpha$, $\underline{apr}_{\lambda}(p) = \beta$. Since ξ is a semi prime ideal, $p\Gamma p \subseteq \xi$, then $\overline{apr}_{\lambda}(p\Gamma p) = \alpha$ and $\underline{apr}_{\lambda}(p\Gamma p) = \beta$. So, $\overline{apr}_{\phi\Gamma\phi}(p\Gamma p) \geq \overline{apr}_{\phi}(p) > \overline{apr}_{\lambda}(p) = \overline{apr}_{\lambda}(p\Gamma p)$ and $\underline{apr}_{\phi\Gamma\phi}(p\Gamma p) \leq \underline{apr}_{\phi}(p) < \underline{apr}_{\phi}(p) = \underline{apr}_{\phi}(p\Gamma p)$. This is a contradiction, hence λ is a RFSPI of N.

5. Rough Fuzzy Ideals in Γ Residue Class

This section examines rough fuzzy ideals in quotient rings and also investigates some of their properties.

Definition 5.1: (Barnes, 1966) Let $\mathscr{J}$ be an ideal of a Γ Ring N. If for each $p+\mathscr{J}$, $q+\mathscr{J}$ in the factor group $N/\mathscr{J}$ and each $\gamma \in \Gamma$, we define $(p+\mathscr{J})\,\gamma(q+\mathscr{J}) = p\gamma q + \mathscr{J}$, then $N/\mathscr{J}$ is a Γ Ring which we shall call the Γ residue class ring of N with respect to $\mathscr{J}$.

Proposition 5.2: Let $\mathscr{J}$ be an ideal of a Γ Ring N. If λ is a RFLI (resp.RFRI) of N, then the RFS $\tilde{\lambda}$ of $N/\mathscr{J}$ defined by $\overline{apr}_{\tilde{\lambda}}(p+\mathscr{J}) = \vee_{x_1\in\mathscr{J}}\overline{apr}_{\lambda}(p+x_1)$ and $\underline{apr}_{\tilde{\lambda}}(p+\mathscr{J}) = \wedge_{x_1\in\mathscr{J}}\underline{apr}_{\lambda}(p+x_1)$ is a RFLI (resp. RFRI) of the Γ residue class right $N/\mathscr{J}$ of N with respect to $\mathscr{J}$.

Proof: Let $p, q \in N$ be such that $p+\mathscr{J} = q+\mathscr{J}$. Then $q = p + x_2$ for some $x_2 \in \mathscr{J}$ and so
$\overline{apr}_{\tilde{\lambda}}(q+\mathscr{J}) = \vee_{x_1\in\mathscr{J}}\overline{apr}_{\lambda}(q+x_1) = \vee_{x_1\in I}\overline{apr}_{\lambda}(p+x_2+x_1) = \vee_{x_1+x_2=x_3\in\mathscr{J}}\overline{apr}_{\lambda}(p+x_3) = \overline{apr}_{\tilde{\lambda}}(p+\mathscr{J})$
$\underline{apr}_{\tilde{\lambda}}(q+\mathscr{J}) = \wedge_{x_1\in\mathscr{J}}\underline{apr}_{\lambda}(q+x_1) = \wedge_{x_1\in\mathscr{J}}\underline{apr}_{\lambda}(p+x_2+x_1) = \wedge_{x_1+x_2=x_3\in\mathscr{J}}\underline{apr}_{\lambda}(p+x_3) = \underline{apr}_{\tilde{\lambda}}(p+\mathscr{J})$
Hence $\tilde{\lambda}$ is well defined.
For any $x_1 + I$, $x_2 + \mathscr{J} \in N/\mathscr{J}$ and $\gamma \in \Gamma$, we have
$\overline{apr}_{\tilde{\lambda}}((x_1+\mathscr{J}) - (x_2+\mathscr{J})) = \overline{apr}_{\tilde{\lambda}}((x_1-x_2)+\mathscr{J}) = \vee_{x_3\in\mathscr{J}}\overline{apr}_{\lambda}((x_1-x_2)+x_3)$
$= \vee_{u-v=x_3\in\mathscr{J}}\overline{apr}_{\lambda}((x_1-x_2)+(u-v)) = \vee_{u,v\in\mathscr{J}}\overline{apr}_{\lambda}((x_1+u)-(x_2+v))$

$\geq \vee_{u,v\in\mathcal{J}}\{\overline{apr}_{\lambda}(x_1+u)) \wedge \overline{apr}_{\lambda}(x_2+v)\}=\{\vee_{u\in\mathcal{J}}\overline{apr}_{\lambda}(x_1+u)\} \wedge \{\vee_{v\in\mathcal{J}}\overline{apr}_{\lambda}(x_2+v)\}$

$= \overline{apr}_{\tilde{\lambda}}(x_1+\mathcal{J}) \wedge \overline{apr}_{\tilde{\lambda}}(x_2+\mathcal{J})$

$\underline{apr}_{\tilde{\lambda}}\big((x_1+\mathcal{J})-(x_2+\mathcal{J})\big) = \underline{apr}_{\tilde{\lambda}}((x_1-x_2)+\mathcal{J}) = \wedge_{x_3\in\mathcal{J}}\underline{apr}_{\lambda}((x_1-x_2)+x_3)$

$= \wedge_{x_3=u-v\in\mathcal{J}}\underline{apr}_{\lambda}((x_1-x_2)+(u-v)) = \wedge_{u,v\in\mathcal{J}}\underline{apr}_{\lambda}((x_1+u)-(x_2+v))$

$\leq \wedge_{u,v\in\mathcal{J}}\underline{apr}_{\lambda}\{(x_1+u)) \vee \underline{apr}_{\lambda}(x_2+v)\} =\{\wedge_{u\in\mathcal{J}}\underline{apr}_{\lambda}(x_1+u)\} \vee \{\wedge_{v\in\mathcal{J}}\underline{apr}_{\lambda}(x_2+v)\}$

$= \underline{apr}_{\tilde{\lambda}}(x_1+\mathcal{J}) \vee \underline{apr}_{\tilde{\lambda}}(x_2+\mathcal{J})$

$\overline{apr}_{\tilde{\lambda}}\big((x_1+\mathcal{J})\gamma(x_2+\mathcal{J})\big) = \overline{apr}_{\tilde{\lambda}}(x_1\gamma x_2)+\mathcal{J}) = \vee_{x_3\in I}\overline{apr}_{\lambda}((x_1\gamma x_2)+x_3)$

$\geq \vee_{x_3\in\mathcal{J}}\overline{apr}_{\lambda}(x_1\gamma x_2 + x_1\gamma x_3)$ because $x_1\gamma x_3 \in \mathcal{J}$

$= \vee_{x_3\in I}\overline{apr}_{\lambda}(x_1\gamma(x_2+x_3)) \geq \vee_{x_3\in I}\overline{apr}_{\lambda}(x_2+x_3) = \overline{apr}_{\tilde{\lambda}}(x_2+I)$ and

$\underline{apr}_{\tilde{\lambda}}\big((x_1+\mathcal{J})\gamma(x_2+\mathcal{J})\big) = \underline{apr}_{\tilde{\lambda}}(x_1\gamma x_2)+\mathcal{J}) = \wedge_{x_3\in I}\underline{apr}_{\lambda}((x_1\gamma x_2)+x_3)$

$\leq \wedge_{x_3\in\mathcal{J}}\underline{apr}_{\lambda}(x_1\gamma x_2 + x_1\alpha x_3)$ because $x_1\gamma x_3 \in \mathcal{J}$

$= \wedge_{x_3\in\mathcal{J}}\underline{apr}_{\lambda}(x_1\gamma(x_2+x_3)) \leq \wedge_{x_3\in\mathcal{J}}\underline{apr}_{\lambda}(x_2+x_3) = \underline{apr}_{\tilde{\lambda}}(x_2+\mathcal{J})$

Similarly, $\overline{apr}_{\tilde{\lambda}}\big((x_1+\mathcal{J})\gamma(x_2+\mathcal{J})\big) \geq \overline{apr}_{\tilde{\lambda}}(x_1+\mathcal{J})$ and $\underline{apr}_{\tilde{\lambda}}\big((x_1+\mathcal{J})\gamma(x_2+\mathcal{J})\big) \leq \underline{apr}_{\tilde{\lambda}}(x_1+\mathcal{J})$. Hence $\tilde{\lambda}$ is a RFLI (resp. RFRI) of $N/\mathcal{J}$.

Proposition 5.3: Let $\mathcal{J}$ be an ideal of a Γ Ring N. Then there exists a bijective mapping between the set of all RFLI of N such that $\overline{apr}_{\lambda}(0) = \overline{apr}_{\lambda}(u)$, $\underline{apr}_{\lambda}(0) = \underline{apr}_{\lambda}(u)$, for all $u \in \mathcal{J}$ and the set of all RFLI $\tilde{\lambda}$ of $M/\mathcal{J}$.

Proof: Let λ be a RFLI of N. we know that $\tilde{\lambda}$ is defined by $\overline{apr}_{\tilde{\lambda}}(a+\mathcal{J}) = \vee_{x_1\in\mathcal{J}}\overline{apr}_{\lambda}(a+x_1)$ and $\underline{apr}_{\tilde{\lambda}}(a+\mathcal{J}) = \wedge_{x_1\in\mathcal{J}}\underline{apr}_{\lambda}(a+x_1)$ is a RFLI of M/I. Since $\overline{apr}_{\lambda}(0) = \overline{apr}_{\lambda}(u)$, $\underline{apr}_{\lambda}(0) = \underline{apr}_{\lambda}(u)$, for all $u \in \mathcal{J}$, we get $\overline{apr}_{\lambda}(a+u) \geq \{\overline{apr}_{\lambda}(a) \wedge \overline{apr}_{\lambda}(u)\} = \overline{apr}_{\lambda}(a)$ and $\underline{apr}_{\lambda}(a+u) \leq \{\underline{apr}_{\lambda}(a) \vee \underline{apr}_{\lambda}(u)\} = \underline{apr}_{\lambda}(a)$, Again, $\overline{apr}_{\lambda}(a) = \overline{apr}_{\lambda}(a+u-u) \geq \{\overline{apr}_{\lambda}(a+u) \wedge \overline{apr}_{\lambda}(u)\} = \overline{apr}_{\lambda}(a+u)$ and $\underline{apr}_{\lambda}(a) = \underline{apr}_{\lambda}(a+u-u) \leq \{\underline{apr}_{\lambda}(a+u) \vee \underline{apr}_{\lambda}(u)\} = \underline{apr}_{\lambda}(a+u)$. Hence $\overline{apr}_{\lambda}(a+u) = \overline{apr}_{\lambda}(a)$ and $\underline{apr}_{\lambda}(a+u) = \underline{apr}_{\lambda}(a)$, for all $u \in \mathcal{J}$, that is $\overline{apr}_{\tilde{\lambda}}(a+\mathcal{J}) = \overline{apr}_{\lambda}(a)$ and $\underline{apr}_{\tilde{\lambda}}(a+\mathcal{J}) = \underline{apr}_{\lambda}(a)$. Therefore the correspondence $\lambda \to \tilde{\lambda}$ is injective. Now let $\tilde{\lambda}$ be any RFLI of $N/\mathcal{J}$ and define a RFS λ in N by $\overline{apr}_{\lambda}(p) = \overline{apr}_{\tilde{\lambda}}(p+\mathcal{J})$ and $\underline{apr}_{\lambda}(p) = \underline{apr}_{\tilde{\lambda}}(p+\mathcal{J})$ for all $p\in N$. For any $p, q \in N$ and $\gamma \in \Gamma$, we have $\overline{apr}_{\lambda}(p-q) = \overline{apr}_{\tilde{\lambda}}\big((p-q)+\mathcal{J}\big) = \overline{apr}_{\tilde{\lambda}}\big((p+\mathcal{J})-(q+\mathcal{J})\big) \geq \overline{apr}_{\tilde{\lambda}}(p+\mathcal{J}) \wedge \overline{apr}_{\tilde{\lambda}}(q+\mathcal{J}) = \overline{apr}_{\lambda}(p) \wedge \overline{apr}_{\lambda}(q)$, and $\underline{apr}_{\lambda}(p-q) = \underline{apr}_{\tilde{\lambda}}\big((p-q)+\mathcal{J}\big) = \underline{apr}_{\tilde{\lambda}}\big((p+\mathcal{J})-(q+\mathcal{J})\big) \leq \underline{apr}_{\tilde{\lambda}}(p+\mathcal{J}) \vee \underline{apr}_{\tilde{\lambda}}(q+\mathcal{J}) = \underline{apr}_{\lambda}(p) \vee \underline{apr}_{\lambda}(q)$ and $\overline{apr}_{\lambda}(p\gamma q) = \overline{apr}_{\tilde{\lambda}}\big((p\gamma q)+\mathcal{J}\big) = \overline{apr}_{\tilde{\lambda}}\big((p+\mathcal{J})\gamma(q+\mathcal{J})\big) \geq \overline{apr}_{\tilde{\lambda}}(q+\mathcal{J}) = \overline{apr}_{\lambda}(q)$, $\underline{apr}_{\lambda}(p\gamma q) = \underline{apr}_{\tilde{\lambda}}\big((p\gamma q)+\mathcal{J}\big) = \underline{apr}_{\tilde{\lambda}}\big((p+\mathcal{J})\gamma(q+\mathcal{J})\big) \leq \underline{apr}_{\tilde{\lambda}}(q+\mathcal{J}) = \underline{apr}_{\lambda}(q)$. Thus λ is RFLI of N. It is observed that, $\overline{apr}_{\lambda}(x_3) = \overline{apr}_{\tilde{\lambda}}(x_3+\mathcal{J}) = \overline{apr}_{\tilde{\lambda}}(\mathcal{J})$ and $\underline{apr}_{\lambda}(x_3) = \underline{apr}_{\tilde{\lambda}}(x_3+\mathcal{J}) = \underline{apr}_{\tilde{\lambda}}(\mathcal{J})$ for all $x_3 \in \mathcal{J}$, which shows that $\overline{apr}_{\lambda}(x_3) = \overline{apr}_{\lambda}(0)$ and $\underline{apr}_{\lambda}(x_3) = \underline{apr}_{\lambda}(0)$ for all $x_3 \in \mathcal{J}$.

6. Conclusions

Recently, there has been a notable rise in the frequency of academic events focused on the subject of RS theory. It may be utilized to improve many current soft computing techniques as well as deal with new uncertain information systems. Several algebraic structures have been integral to the development of RS theory, as they allow detailed analysis of set-theoretic properties. Our paper investigates the algebraic properties of RFS in gamma rings. We demonstrated the notion of rough fuzzy prime and semi-prime ideals in the gamma ring structure and proved some related properties of the quotient ring. A limitation of RS is that when partitioning the universe of objects in an information system, RS relies on equivalence relations. A future study will investigate rough fuzzy numerous ideals of gamma near rings, gamma fields, and gamma near fields. This study concludes that the results can broaden the application area of gamma rings and encourage additional research on related subjects. In addition, the structure can be used in various engineering applications in the near future.

List of Symbols and Abbreviations:

Γ - Gamma Rings
RS - Rough Set
IFS - Intuitionistic Fuzzy Set
RFS - Rough Fuzzy Set
RFI - Rough Fuzzy Ideal
IND (K) - Indiscernibility relation
$\overline{apr}$ - Upper approximation
$\underline{apr}$ - Lower approximation
RFLI - Rough Fuzzy Left Ideal
RFRI - Rough Fuzzy Right Ideal
RFPI - Rough Fuzzy Prime Ideal
RFSPI - Rough Fuzzy Semi Prime Ideal

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